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AN OPERATIONAL BALTIC SEA CIRCULATION MODEL

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An operational Baltic Sea circulation model.
Part 1. Barotropic version.

Abstract

A vertically integrated barotropic model for the Baltic Sea is described both in dynamic and numerical terms. The model has been run operationally since March 1993 on a 5 x 5 km grid and is forced by interpolated 10 m winds and surface pressure from the 50 x 50 km HIRLAM atmospheric model. The water volume is not conserved but depends on inflow from rivers and in- or outflow through the Belt Sea and the Öresund.

Key words

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An operational Baltic Sea circulation model

Part 1. Barotropic version

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Introduction

This report describes the application of a barotropic circulation model to the Baltic Sea. The model is designed to be run operationally and is easily modified for specific circulation studies. In the operational mode, it will give daily forecasts on water level and water transports. Coupled to an ice-growth model, it will also be possible to make forecasts of ice-cover change and ice drifts. Further, it will act as a basic part of a modelling system for oil drift and spreading of chemicals.

The model has a horizontal resolution of 5 km and is forced by a high resolution atmospheric model, which provides the wind stress and sea surface pressure gradient forcing. The water balance is controlled by updating the river inflow and the water exchange through the Danish Sounds.

A verification study will be presented in the next part of the report, where water level observations will be compared with results from the model. Easily recognizable barotropic features like topographically generated eddies in the model will also be compared with estimates from analytical expressions. A non-linear version of the model including a Smagorinsky-type of eddy viscosity (Smagorinsky, 1963) has been run on a finer grid (in the order of hundreds of meters). Results from these experiments will also be presented for limited areas of the Baltic Sea in the next-coming report.

Governing equations

We assume the water body to be homogeneous and incompressible with density ρ. We apply a spherical polar coordinate system (λ, φ, z) where λ and φ denote longitude and latitude, respectively and z is the vertical distance above the surface of the earth. The sphere is rotating about its axis with constant angle velocity ω.

Starting from the linearized depth integrated transport equations, we can write the conservation equations for mass and momentum as

$$
\begin{aligned}
\eta_t &= -\frac{1}{R\cos\varphi}(U_\lambda + (V\cos\varphi)_\varphi) \\
U_t &= fV - \frac{gh}{R\cos\varphi}\eta_\lambda - \frac{h}{\rho R\cos\varphi}p_\lambda + \frac{1}{\rho}\tau^{s_\lambda} - \frac{1}{\rho}\tau^{b_\lambda} + A_H\nabla^2 U \\
V_t &= -fU - \frac{gh}{R}\eta_\varphi - \frac{h}{\rho R}p_\varphi + \frac{1}{\rho}\tau^{s_\varphi} - \frac{1}{\rho}\tau^{b_\varphi} + A_H\nabla^2 V
\end{aligned}
\tag{1}
$$

where η is the surface elevation deviation from the equilibrium level and U and V are the volume fluxes in the λ- and φ-direction. τ^s and τ^b denote the surface and bottom stress, ρ is the mean density of water, p is the atmospheric sea level pressure, h is the equilibrium water depth, A_H is the horizontal eddy viscosity coefficient, f is the Coriolis parameter equal to $2\omega sin\varphi$, R is the radius of the earth and g is the gravity acceleration. The subscripts on the $\eta-$, $U-$ and V-terms represent partial differentiations. The ∇^2-operator is defined as

$$
\nabla^2 = \frac{1}{R^2\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2} + \frac{1}{R^2\cos\varphi}\frac{\partial}{\partial\varphi}(\cos\varphi\frac{\partial}{\partial\varphi}) \tag{2}
$$

The boundary condition at the shore is no volume flux except where the inflow from main rivers is specified. To maintain the right water balance, also the water exchange through the Danish Sounds is specified and the evaporation is assumed to be equal to the precipitation.

The bottom stress is modelled by a quadratic friction law and the bottom stress components are written

$$\begin{aligned} \tau^{b_\lambda} &= \rho C_b (U^2 + V^2)^{1/2} U \\ \tau^{b_\varphi} &= \rho C_b (U^2 + V^2)^{1/2} V \end{aligned} \tag{3}$$

where C_b is a friction coefficient with a typical value of 0.0025.

The horizontal eddy diffusivity term has been included to suppress any kind of computational noise that can emerge from the nonlinear terms when the model is used in operational mode, meaning integration periods of the order of months. However, in this application the only nonlinear term is the bottom stress term, why the eddy viscosity can be held at a relatively low value. For the first period of application of this model, a value of 50 $m^2 s^{-1}$ has been chosen.

For shallow water it is common to make both the lateral and bottom friction term dependent on the local depth. This option is included in the model and an evaluation of including this dependence will be made in the verification report.

The vertical mean density in the Baltic Sea has a significant horizontal variation. This creates a horizontal gradient of the mean sea level, the so called steric effect, and amounts to a difference of approximately 0.25 m between the northern and southern ends of the basin (Ekman and Mäkinen, 1991). To include this effect in the model, the sea level output is modified according to a constant gradient in the $\varphi-$direction.

Atmospheric forcing

The difference between the average precipitation and evaporation in the Baltic Sea is of the order of 60 km^3yr^{-1}. It is an order of magnitude less than the mean annual river inflow and has been neglected in a first version of the model. It is possible to correct the total water volume at the end of the year when precipitation and evaporation data are available.

All atmospheric forcing is given by the high resolution atmospheric model HIRLAM (ref. HIRLAM) with a typical resolution of 40 km. The forcing fields are at 3 hour interval and are held constant during that time of the time integration. Bicubic interpolation is used to produce the 5 km wind and pressure gradient fields from the HIRLAM grid. The wind stress is then computed using the relations:

$$\begin{aligned} \tau^{s\lambda} &= \rho_a C_D \mid W \mid W^{\lambda} \\ \tau^{s\varphi} &= \rho_a C_D \mid W \mid W^{\varphi} \end{aligned} \tag{4}$$

where W^{λ} and W^{φ} are the wind velocity components at a given height above sea level, ρ_a is the density of air and C_D is a non-dimensional empirical drag coefficient. As a value on C_D the following relation suggested by Wu (1982) has been used.

$$C_D = (0.8 + 0.065 \mid W_{10} \mid) * 10^{-3} \tag{5}$$

where W_{10} is the wind velocity at 10 m height.

Hydrological forcing

The runoff regions around the Baltic Sea have been divided into 21 areas, each supplemented with a river outlet in the model. In lack of daily forecasts and even real-time values of fresh water inflow, monthly averages of runoff values from the years 1951 to 1990 (Bergström and Carlsson, 1993) have been used as boundary values after being scaled in order to account for the total runoff to the Baltic Sea.

In- and outflow through the Danish Sounds

To complete the control of the water budget in the whole system, the water exchange through the Danish Sounds has to be specified. This is done by relating the flow in both the Belt Sea and the Oresund to the water level difference between the gauge stations Viken and Klagshamn in the Oresund. The validity of this assumption, which is based upon a quadratic bottom friction law, has been demonstrated by Jacobsen (1980) and Stigebrandt (1980), and from the latter the following formula has been taken

$$Q_0 = \frac{(h_0 - h_1)}{\sqrt{|h_0 - h_1|}} A_0 \sqrt{\frac{2g}{c}} \tag{6}$$

where Q_0 is the total flow, h_0 and h_1 is the water level in the Kattegat and the south-west Baltic Sea, respectively, A_0 is the mean vertical cross section area, g is the gravitational constant and c is a constant with a typical value of 13. The division of the total flow into the both straits is given by the assumption that 27% is flowing through the Oresund.

Numerical methods

We use the finite difference form of the conservation equations. This means that a given function is defined only at discrete points in the region R where it is defined. Thus, for the one-dimensional case, we divide R into a set of intervals having equal length Δx, called the grid length. The approximation of the function $u(x)$ is denoted by u_j which stands for $u(j\Delta x)$. Derivatives are approximated by so called finite differences and as an example the $x-$derivative of u can be approximated by $(u_{j+1} - u_j)/\Delta x$.

However, there exist various possibilities to represent the variables in a horizontal grid. Here we have adopted the commonly used C-grid (Mesinger and Arakawa, 1976), according to Figure 1. This grid has the disadvantage that the Coriolis term has to be approximated by a mean value at four grid points. On the other hand, the C-grid has in general much better dispersion and phase speed properties than other grid configurations.

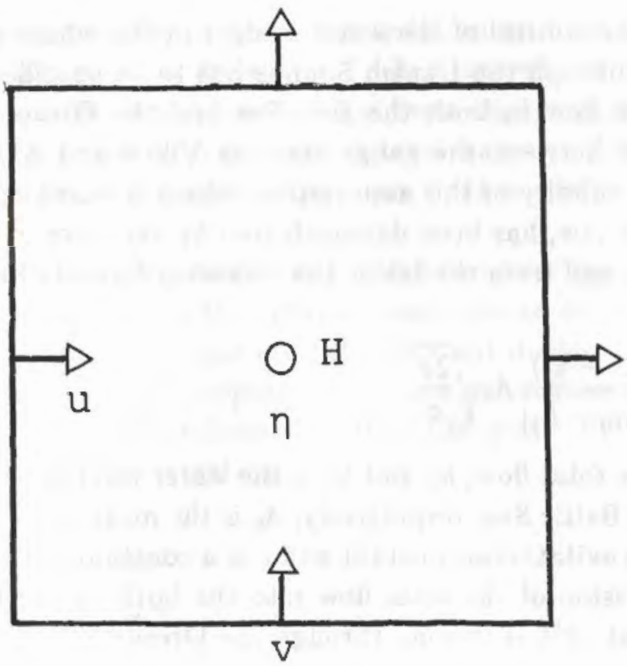

Figure 1. Position of variables within a C grid element.

We define the following operators:

$$
\begin{aligned}
(\delta_x \alpha)_{ij} &= (\alpha_{i+\frac{1}{2},j} - \alpha_{i-\frac{1}{2},j})/\Delta x \\
(\bar{\alpha}^x)_{ij} &= (\alpha_{i+\frac{1}{2},j} + \alpha_{i-\frac{1}{2},j})/2
\end{aligned}
\tag{7}
$$

Corresponding definitions can be made for the y-direction. Applying the averaging operator in both directions we get the two-dimensional average operator

$$(\bar{\alpha}^{xy})_{ij} = (\bar{\alpha}^{x^y})_{ij} \tag{8}$$

We can also apply the difference operator twice and get for the x-direction

$$(\delta_{xx}\alpha)_{ij} = (\delta_x(\delta_x\alpha))_{ij} = (\alpha_{i+1,j} + \alpha_{i-1,j} - 2\alpha_{i,j})/(\Delta x)^2 \tag{9}$$

With the definitions $d_\lambda = \frac{1}{Rcos\varphi}$ and $d_\varphi = \frac{1}{R}$ and omitting the indices i and j, the conservation equations can then be written as

$$\begin{aligned}
\eta_t &= -d_\lambda(\delta_\lambda U + \delta_\varphi(V cos\varphi)) \\
U_t &= f\bar{V}^{\lambda\varphi} - d_\lambda gh\delta_\lambda\eta - d_\lambda\frac{h}{\rho}\delta_\lambda p + \frac{1}{\rho}\tau_\lambda^s - \frac{1}{\rho}\tau_\lambda^b + \\
&\quad A_H d_\lambda(d_\lambda\delta_{\lambda\lambda} + d_\varphi\delta_\varphi(cos\varphi\delta_\varphi))U \\
V_t &= -f\bar{U}^{\lambda\varphi} - d_\varphi gh\delta_\varphi\eta - d_\varphi gh\delta_\varphi e - d_\varphi\frac{h}{\rho}\delta_\varphi p + \frac{1}{\rho}\tau_\varphi^s - \frac{1}{\rho}\tau_\varphi^b + \\
&\quad A_H d_\lambda(d_\lambda\delta_{\lambda\lambda} + d_\varphi\delta_\varphi(cos\varphi\delta_\varphi))V
\end{aligned} \tag{10}$$

Thus, spatial derivatives have been approximated by central differences and variables not given at the same grid point have been approximated by mean values from the neighbouring points.

Time differencing is made using a variation of the forward-backward scheme according to Sielecki, 1968. This scheme has an implicit nature though the order in which the computations are done, makes it explicit. The time step limit can be estimated by von Neumann's method and results in

$$\Delta t \leq \frac{\Delta x}{\sqrt{2gH}} \tag{11}$$

where Δx is a true distance on the Earth.

The final set of difference equations then can be written as

$$
\begin{aligned}
\eta^{n+1} &= \eta^n - \Delta t d_\lambda(\delta_\lambda U^n + \delta_\varphi(V^n cos\varphi)) \\
U^{n+1} &= U^n + \Delta t(f\bar{V}^{\lambda\varphi^n} - d_\lambda gh\delta_\lambda\eta^{n+1} - d_\lambda\frac{h}{\rho}\delta_\lambda p^{n+1} + \\
&\quad \frac{1}{\rho}\tau_\lambda^{s^{n+1}} - \frac{1}{\rho}\tau_\lambda^{b^n} + A_H d_\lambda(d_\lambda\delta_{\lambda\lambda} + d_\varphi\delta_\varphi(cos\varphi\delta_\varphi))U^n) \\
V^{n+1} &= V^n - \Delta t(f\bar{U}^{\lambda\varphi^{n+1}} + d_\varphi gh\delta_\varphi\eta^{n+1} + d_\varphi gh\delta_\varphi e + d_\varphi\frac{h}{\rho}\delta_\varphi p^{n+1} - \\
&\quad \frac{1}{\rho}\tau_\varphi^{s^{n+1}} + \frac{1}{\rho}\tau_\varphi^{b^n} - A_H d_\lambda(d_\lambda\delta_{\lambda\lambda} + d_\varphi\delta_\varphi(cos\varphi\delta_\varphi))V^n)
\end{aligned}
\tag{12}
$$

where U^n stands for $U(n\Delta t)$.

Grid configuration and bathymetry

The model is set up on the same type of grid as the atmospheric model, a rotated latitude/longitude grid. This means that the poles of the computational grid differ from the geographical poles. To keep the variation of the grid size at a minimum, the equator has thus been moved to 60^o N. Normally, ocean models covering basins at the scale of the Baltic Sea use a cartesian grid on a polar stereographic projection, eventually supplemented with a map factor to account for the distortion of the grid size depending on the latitude. The reason to use a spherical grid in this application is firstly that it facilitates data exchange with the atmospheric model as the CPU time for the interpolation of the atmospheric forcing is decreased when the ocean model shares the atmospheric model grid points. Secondly, if the ocean model will be extended to cover the North Sea and maybe parts of the North Atlantic, the spherical grid has advantages before a cartesian one because of the grid distortion associated with the latter type of grid.

The Baltic Sea has been divided into a grid consisting of $165 * 301$ grid points for a grid length of approximately 5 km, (see Figure 2). For a baroclinic model, the grid size is normally guided by the size of baroclinic

Figure 2. *The horizontal grid with a grid length of 5 km for the Baltic Sea. Here only the southern part of the area is shown.*

eddies and waves with a typical size of the internal Rossby radius, while for a barotropic model, where the circulation is mainly governed by topography, the resolution will be decided by the depth gradient. For the Baltic Sea with its complicated topography, a resolution of the lowest modes of topographic waves in the largest subbasins will require a grid size down to the order of 1 *km*. However, in practice it is the available computer power that limits the grid size.

The depths are computed as mean values from a data base consisting of depths at approximately every second *km*. Figure 3 shows the depth distribution in intervals of 50 *m*.

Summary

A vertically integrated barotropic model has been described both in dynamic and numerical terms. The model is set up on a $5 * 5$ *km* grid fo the Baltic Sea and has been in operational mode since the beginning of March 1993. The model is forced by interpolated wind at the 10 *m* level and surface pressure from a high resolution (50 *km*) atmospheric model. The water volume is regulated through river runoff and in- or outflow through the Belt Sea and the Öresund.
A second part of reports on the barotropic model is planned to be released during 1994 and will cover verification and model experiments.

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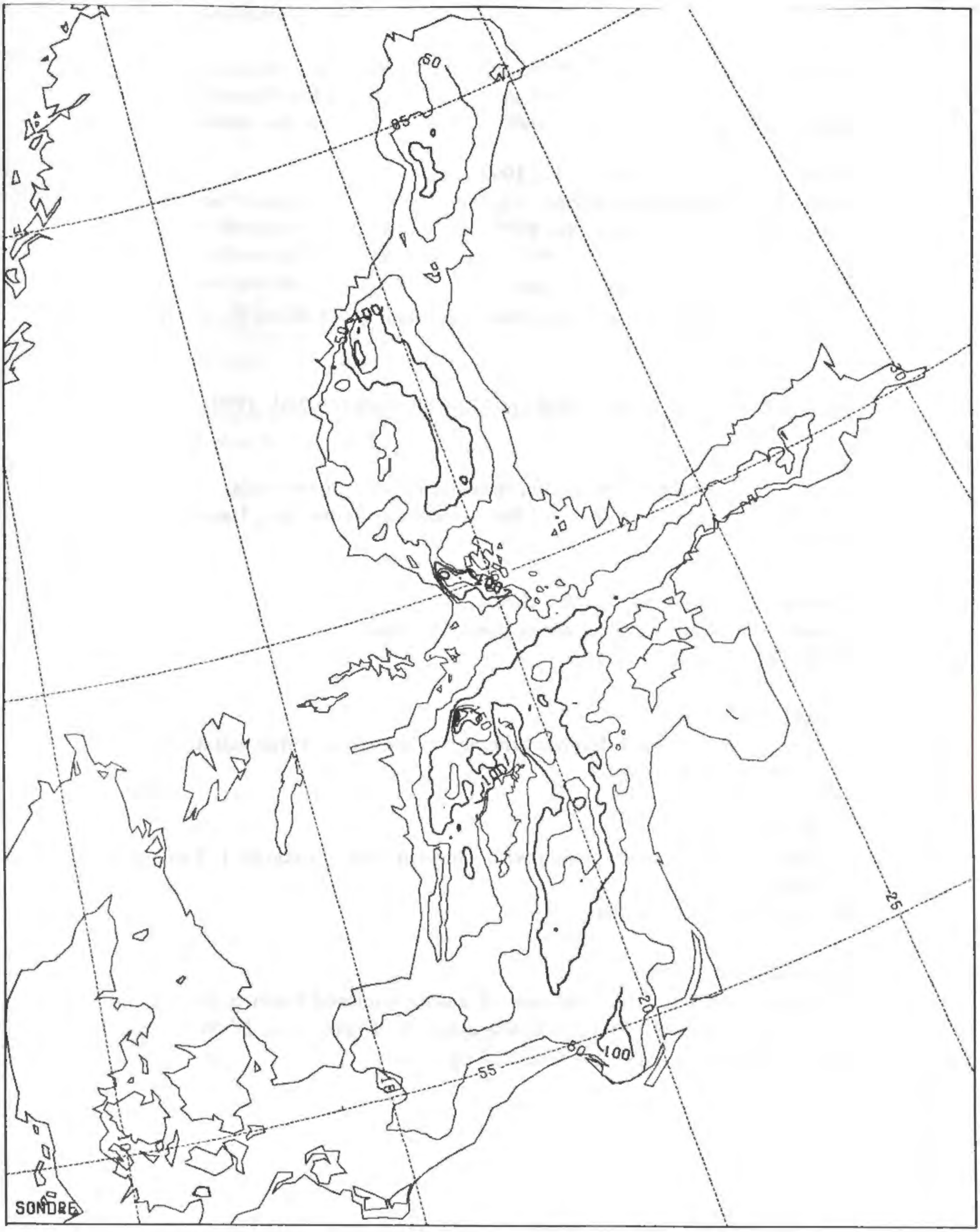

Figure 3. Depths for the Baltic Sea model. Isolines are shown for every 50 m.

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