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THE SWEDISH LIMITED AREA MODEL

by Per Undén

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Part A. Formulation

Abstract

This report is a description of the Swedish limited area model, at present (June 1982) tested operationally with encouraging results. The models is based on the ECMWF-model, although some simplifications have been done in the physic package. The major differences are that this model uses a simpler radiation scheme although a diurnal cycle is included, and for the convection scheme a simple adjustment is used.

To avoid problems with the poles the grid has been transformed, and a fictious north-pole is placed at 180° longitude and 30° latitude. A relaxation zone (Kållberg, 1977) is used with a width of 8 points, where the solution of the model and the boundaries are weighted together.

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INTRODUCTION

The Swedish Limited Area Model (LAM) has been developed from the ECMWF gridpoint model and particularily from the ECMWF limited area version of the same model. The code of the model eminates from versions at the centre during 1977-1979. Formulations follow Burridge (1977 and 1979) and for the physics Louis (1979).

The major differences from the ECMWF model are of course in the physics and the formulation on a transformed spherical grid.

1 THE DYNAMICAL EQUATIONS AND FINITE DIFFERENCES

The model is formulated in spherical coordinates and with a vertical $\sigma = p/p_s$ system. To facilitate conservation properties the equations are written in flux form.

1.1 The governing equations

The momentum equations take the form:

$$\frac{\partial u}{\partial t} - \frac{1}{\cos\Theta} Z \cdot p_s \, v\cos\Theta + \frac{1}{a\cos\Theta}\frac{\partial}{\partial\lambda}(\phi + E) + RT\frac{1}{a\cos\Theta}\frac{\partial}{\partial\lambda}(\ln p_s) + $$
$$+ \dot{\sigma}\frac{\partial u}{\partial\sigma} = F_u \qquad (1.1)$$

$$\frac{\partial v}{\partial t} + Z \cdot p_s \, u + \frac{1}{a}\frac{\partial}{\partial\Theta}(\phi + E) + RT\frac{1}{a}\frac{\partial}{\partial\Theta}(\ln p_s) + \dot{\sigma}\frac{\partial v}{\partial\sigma} = F_v \qquad (1.2)$$

where

$$Z = \frac{1}{p_s}\left(f + \frac{1}{a\cos\Theta}\left(\frac{\partial v}{\partial\lambda} - \frac{\partial}{\partial\Theta}(u\cos\Theta)\right)\right) \qquad (1.3)$$

and

$$E = \frac{1}{2} (u^2 + \frac{1}{\cos\Theta} v^2 \cos\Theta) \quad (1.4)$$

F_u and F_v represent the non-adiabatic terms which will be described in chapter 2.

The thermodynamic equation:

$$\frac{\partial T}{\partial t} + \frac{1}{p_s} \left[\frac{1}{a\cos\Theta} (p_s\, u \frac{\partial T}{\partial \lambda} + p_s\, v\cos\Theta \frac{\partial T}{\partial \Theta}) + p_s \dot{\sigma} \frac{\partial T}{\partial \sigma} - \frac{\kappa T \omega}{\sigma} \right] =$$
$$= \frac{1}{c_p} \frac{dQ}{dt} \quad (1.5)$$

where

$$\omega = \frac{dp}{dt} = \frac{d}{dt} (p_s\, \sigma) = p_s\, \dot{\sigma} + \sigma(\frac{\partial p_s}{\partial t} + \frac{1}{a\cos\Theta} (u \frac{\partial p_s}{\partial \lambda} +$$
$$+ v\cos\Theta \frac{\partial p_s}{\partial \Theta}) = p_s\, \dot{\sigma} + \sigma \frac{\partial p_s}{\partial t} + \frac{\sigma}{a\cos\Theta} (p_s\, u \frac{\partial}{\partial \lambda} (\ln p_s) +$$
$$+ p_s\, v\cos\Theta \frac{\partial}{\partial \Theta} (\ln p_s)) \quad (1.6)$$

and $\frac{dQ}{dt}$ represent effects from non-adiabatic heating.

The continuity equation in the σ-system is:

$$\frac{\partial p_s}{\partial t} + \frac{1}{a\cos\theta} (\frac{\partial}{\partial \lambda} (p_s\, u) + \frac{\partial}{\partial \Theta} (p_s\, v\cos\Theta)) + \frac{\partial}{\partial \sigma} (p_s\, \dot{\sigma}) = 0 \quad (1.7)$$

The boundary conditions for $\dot{\sigma}$ are:

$$(p_s\, \dot{\sigma}) = 0 \quad \text{at } \sigma = 0 \quad (1.8)$$

$$(p_s\, \dot{\sigma}) = 0 \quad \text{at } \sigma = 1 \quad (1.9)$$

Using these boundary condition the continuity equation can be integrated:

$$\sigma \frac{\partial p_s}{\partial t} + p_s\, \dot{\sigma} = - \int_0^\sigma \frac{1}{a\cos\Theta} (\frac{\partial}{\partial \lambda} (p_s\, u) + \frac{\partial}{\partial \Theta} (p_s\, v\cos\Theta)) d\sigma \quad (1.10)$$

Integration of (1.10) to $\sigma = 1$ gives the surface pressure tendency equation:

$$\frac{\partial p_s}{\partial t} = - \int_0^1 \frac{1}{a\cos\Theta} (\frac{\partial}{\partial \lambda} (p_s\, u) + \frac{\partial}{\partial \Theta} (p_s\, v\cos\Theta)) d\sigma \quad (1.11)$$

The geopotential is related to temperature through the hydrostatic equation:

$$\frac{\partial \phi}{\partial \ln\sigma} = - RT \tag{1.12}$$

The continuity equation for water vapor is:

$$\frac{\partial q}{\partial t} + \frac{1}{p_s} \left(\frac{1}{a\cos\Theta} \left(p_s\, u \frac{\partial q}{\partial \lambda} + p_s\, v\cos\Theta \frac{\partial q}{\partial \Theta}\right) + p_s\, \dot{\sigma} \frac{\partial q}{\partial \sigma}\right) = \frac{dC}{dt} \tag{1.13}$$

where $\frac{dC}{dt}$ represents non-adiabatic effects.

1.2 Finite differences and grid system

The forecast variables are staggered in a horizontal Arakawa - C grid (fig. 1.1)(see for example, Mesinger & Arakawa (1976)). The area of integration defined in the transformed spherical grid (see also section 4.2) is shown in fig. 1.2. The resolution there is $\Delta\Theta = \Delta\lambda = 1.5^{\circ}$ and the area contains 41x50 gridpoints.

There is also a vertical staggering of variables as shown in fig. 1.1 for 9 levels. The $\sigma_{k+\frac{1}{2}}$ levels are defined so that $(\Delta \ln\sigma) = \frac{\Delta\sigma_k}{\sigma_k}$.

The hydrostatic equation (1.12) is then

$$\frac{\Delta_\sigma \Phi}{\Delta_\sigma \ln\sigma} = - RT \tag{1.14}$$

or in integrated form

$$\phi_{k+\frac{1}{2}} = \phi_s + \sum_{\ell=k+1}^{NLEV} R\, T_\ell (\Delta_\sigma \ln\sigma)_\ell \tag{1.15}$$

The integrated forms of the continuity equation are:

$$\sigma_{k+\frac{1}{2}} \frac{\partial p_s}{\partial t} + p_s\, \dot{\sigma}_{k+\frac{1}{2}} = - \frac{1}{a\cos\Theta} \sum_{\ell=1}^{k} (\delta_\lambda U + \delta_\Theta (V\cos\Theta))_\ell \Delta\sigma_\ell \tag{1.16}$$

and

$$\frac{\partial p_s}{\partial t} = - \frac{1}{a\cos\Theta} \sum_{\ell=1}^{NLEV} (\delta_\lambda U + \delta_\Theta (V\cos\Theta))_\ell \Delta\sigma_\ell \tag{1.17}$$

where

$$U = \overline{p_s}^{\lambda}\, u \tag{1.18}$$

$$V = \overline{p_s}^{\Theta}\, v \tag{1.19}$$

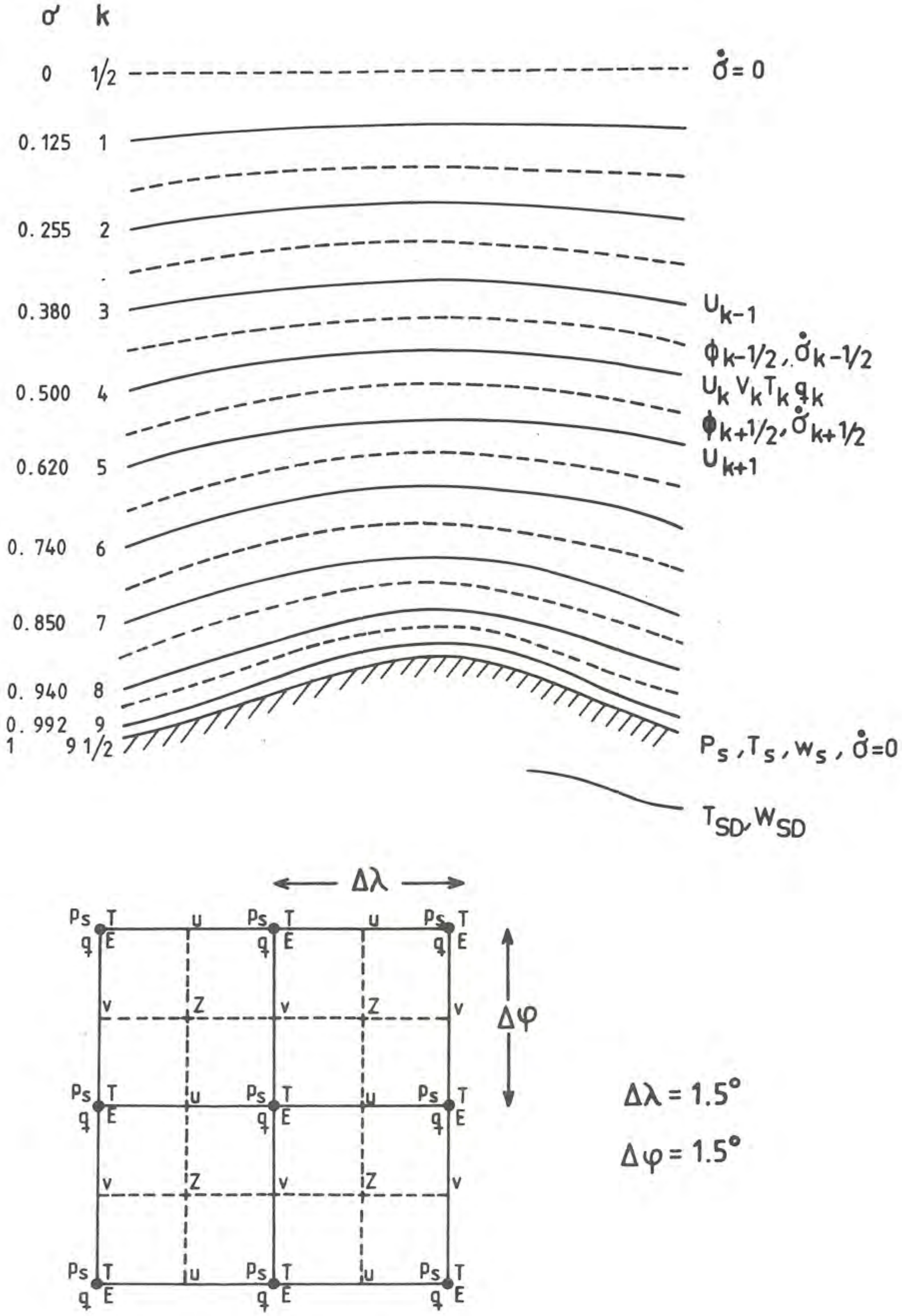

Figure 1.1 *Vertical structure and horizontal grid of the Swedish Limited Area Model (LAM).*

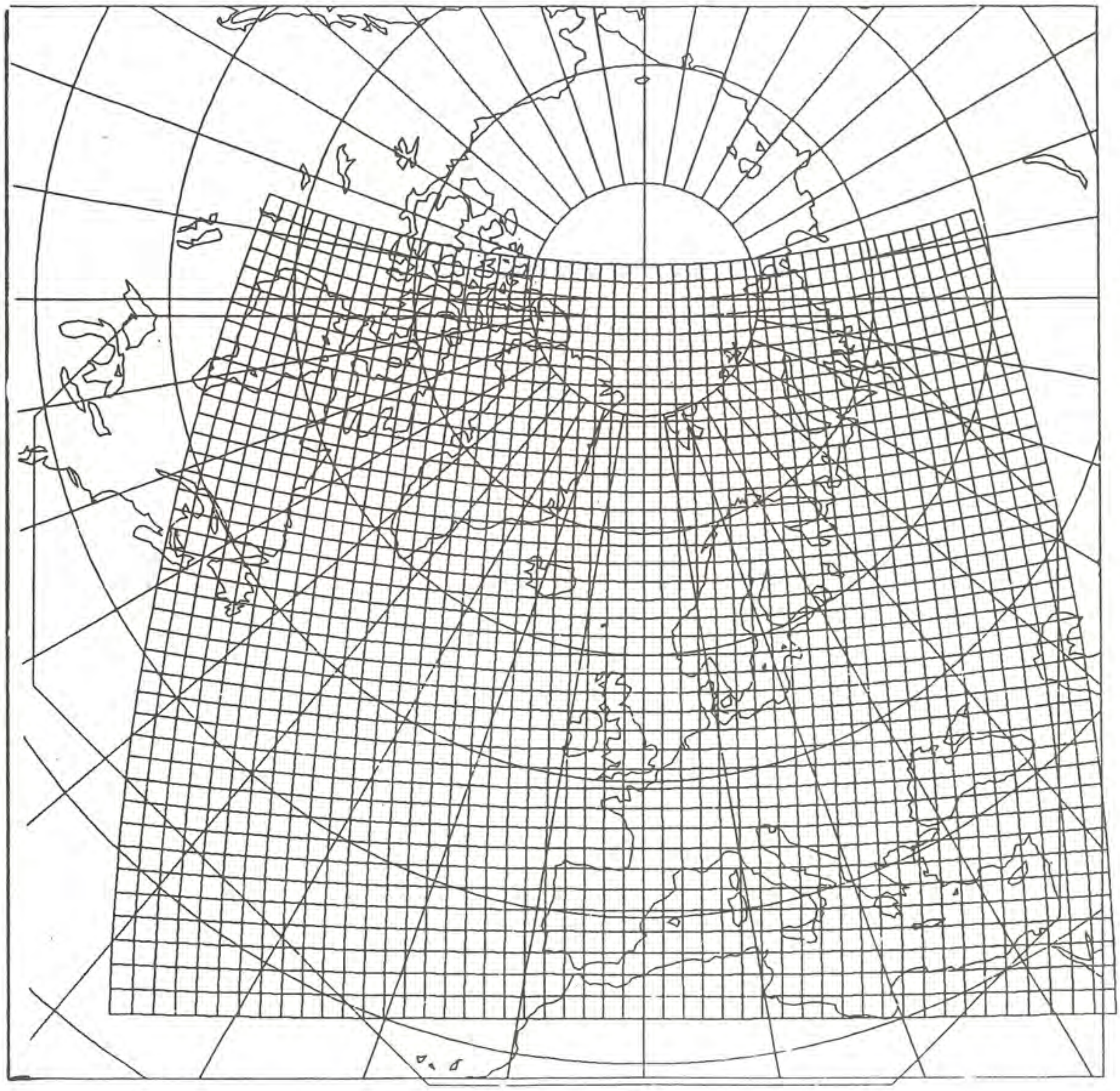

Figure 1.2 The area of integration

Then the thermodynamic equation (1.5) can be solved:

$$\frac{\partial T}{\partial t} + \frac{1}{p_s}\left[\frac{1}{a\cos\Theta}\left(\overline{U\delta_\lambda T}^{\lambda} + \overline{V\cos\Theta\delta_\Theta T}^{\Theta}\right) + \frac{\overline{p_s\dot{\sigma}\Delta_\sigma T}^{\sigma}}{\Delta_\sigma\sigma} - \frac{\kappa T\omega}{\sigma}\right] =$$

$$= \frac{1}{c_p}\frac{dQ}{dt} \tag{1.20}$$

where

$$\frac{\kappa T\omega}{\sigma} = \frac{\kappa T}{\sigma}\,\overline{\left(\sigma\frac{\partial p_s}{\partial t} + p_s\,\dot{\sigma}\right)}^{\sigma} + \frac{\kappa}{a\cos\Theta}\,\overline{\left(U\overline{T}^{\lambda}\,\delta_\lambda(\ln p_s)\right.}^{\lambda} +$$

$$\left. + \overline{v\cos\Theta\,\overline{T}^{\Theta}\,\delta_\Theta(\ln p_s)}^{\Theta}\right) \tag{1.21}$$

The continuity equation for water vapor (1.13) is in finite difference form

$$\frac{\partial q}{\partial t} + \frac{1}{p_s}\left(\frac{1}{a\cos\Theta}\left(\overline{U\delta_\lambda q}^{\lambda} + \overline{V\cos\Theta\ \delta_\Theta q}^{\Theta}\right) + \frac{\overline{p_s\dot{\sigma}\ \Delta_\sigma q}^{\sigma}}{\Delta_\sigma\sigma}\right) = \frac{dC}{dt} \tag{1.22}$$

The momentum equations (1.1 and 1.2) are formulated to conserve potential absolute enstrophy in their finite difference forms.

$$\frac{\partial u}{\partial t} - \frac{1}{\cos\Theta}\cdot\overline{Z}^{\Theta}\cdot\overline{V\cos\Theta}^{\lambda\Theta} + \frac{1}{a\cos\Theta}\,\delta_\lambda\left(\overline{\Phi}^{\sigma} + E\right) +$$

$$+ \frac{R\overline{T}^{\lambda}}{a\cos\Theta}\,\delta_\lambda\ (\ln p_s) + \frac{1}{\overline{p_s}^{\lambda}}\frac{\overline{\overline{p_s\dot{\sigma}}^{\lambda}\ \Delta_\sigma u}^{\sigma}}{\Delta_\sigma\sigma} = F_u \tag{1.23}$$

$$\frac{\partial v}{\partial t} + \overline{Z}^{\lambda}\cdot\overline{U}^{\lambda\Theta} + \frac{1}{a}\,\delta_\Theta\left(\overline{\phi}^{\sigma} + E\right) + \frac{R\overline{T}^{\Theta}}{a}\ \delta_\Theta(\ln p_s) +$$

$$+ \frac{1}{\overline{p_s}^{\Theta}}\frac{\overline{\overline{p_s\dot{\sigma}}^{\Theta}\ \Delta_\sigma v}^{\sigma}}{\Delta_\sigma\,\sigma} = F_v \tag{1.24}$$

where

$$E = \frac{1}{2}\left(\overline{u^2}^{\lambda} + \frac{1}{\cos\Theta}\,\overline{v^2\cos\Theta}^{\Theta}\right) \tag{1.25}$$

and

$$Z = \frac{1}{\overline{p_s\ a\cos\Theta}^{\lambda\Theta}}\cdot\left(af\cos\Theta + \delta_\lambda v - \delta_\Theta(u\cos\Theta)\right) \tag{1.26}$$

2 TIME INTEGRATION SCHEME

The model equations in chapter 1 can be integrated with the explicit leap-frog scheme, i e for $\frac{\partial u}{\partial t} = A_u; u(t+\Delta t) = u(t-\Delta t) + 2\Delta t \cdot A_u(t)$, except for the first time step. This is done for the moisture equation (1.22) but for the other prognostic equations a semi-implicit scheme is used.

2.1 The semi-implicit algorithm

In order to describe the semi-implicit algorithm the equations 1.23, 1.24 and 1.20 are written in a form, where terms not involved in the implicit treatment are transferred to the right hand side.

$$\frac{\partial u}{\partial t} + \frac{1}{a\cos\Theta}(\delta_\lambda \overline{\Phi}^\sigma + R\overline{T}^\lambda \delta_\lambda \ln p_s)) = a_u \quad (2.1)$$

$$\frac{\partial v}{\partial t} + \frac{1}{a}(\delta_\Theta \overline{\Phi}^\sigma + R\overline{T}^\Theta \delta_\Theta (\ln p_s)) = a_v \quad (2.2)$$

$$\frac{\partial T}{\partial t} + \frac{1}{p_s}\left(\overline{\frac{p_s \dot{\sigma} \Delta_\sigma T}{\Delta_\sigma \sigma}}^\sigma - \frac{\kappa T}{\sigma}\overline{(p_s \dot{\sigma} + \sigma \frac{\partial p_s}{\partial t})}^\sigma \right) = a_T \quad (2.3)$$

where a_u, a_v and a_T represent the remaining terms. Also the continuity (1.7) and hydrostatic (1.14) equations will be used:

$$\frac{\partial p_s}{\partial t} + \frac{1}{a\cos\Theta}(\delta_\lambda U + \delta_\Theta(V\cos\Theta)) + \frac{\Delta_\sigma p_s \dot{\sigma}}{\Delta_\sigma \sigma} = 0 \quad (2.4)$$

$$\frac{\Delta_\sigma \Phi}{\Delta_\sigma(\ln\sigma)} = -RT \quad (2.5)$$

The linear terms in the prognostic equations (2.1-2.4) are going to be treated implicitly. The non-linear terms will be linearized with a constant temperature profile $T_o(\sigma)$.

$$\frac{\partial u}{\partial t} + \frac{1}{a\cos\Theta}(\delta_\lambda \overline{\overline{\Phi}^\sigma}^{2t} + RT_o \delta_\lambda \overline{(\ln p_s)}^{2t}) + \frac{R(\overline{T}^\lambda - T_o)}{a\cos\Theta}\delta_\lambda(\ln p_s) = a_u$$
(2.6)

$$\frac{\partial v}{\partial t} + \frac{1}{a}(\delta_\Theta \overline{\overline{\Phi}^\sigma}^{2t} + RT_o \delta_\Theta \overline{(\ln p_s)}^{2t}) + \frac{R(\overline{T}^\Theta - T_o)}{a}\delta_\Theta(\ln p_s) = a_v$$
(2.7)

$$\frac{\partial T}{\partial t} + \overline{\frac{\overline{\dot{\sigma}}^{2t} \Delta_\sigma T_o}{\Delta_\sigma \sigma}}^\sigma - \frac{\kappa T_o}{\sigma}\overline{(\overline{\dot{\sigma}}^{2t} + \sigma \frac{\partial(\ln p_s)}{\partial t})}^\sigma + \overline{\frac{\dot{\sigma}\Delta_\sigma(T-T_o)}{\Delta_\sigma \sigma}}^\sigma -$$

$$- \frac{\kappa(T-T_o)}{\sigma}\overline{(\dot{\sigma} + \sigma\frac{\partial(\ln p_s)_E}{\partial t})}^\sigma = a_T \quad (2.8)$$

where

$\frac{\partial(\ln p_s)}{\partial t}$ E is the explicit tendency

$$\frac{\partial(\ln p_s)}{\partial t} E + \frac{1}{p_s \ a\cos\Theta} (\delta_\lambda \ U + \delta_\Theta(V\cos\Theta)) + \frac{\Delta_\sigma \dot{\sigma}}{\Delta_\sigma \sigma} = 0 \qquad (2.9)$$

$\frac{\partial(\ln p_s)}{\partial t}$ in 2.8 is the final tendency from the semi-implicit form of the continuity equation (2.4).

$$\frac{\partial(\ln p_s)}{\partial t} + \frac{1}{p_s \ a\cos\Theta} (\overline{u \ \delta_\lambda \ p_s}^\lambda + \overline{v\cos\Theta \ \delta_\Theta \ p_s}^\Theta) + \frac{\Delta_\sigma \ \overline{\dot{\sigma}}^{2t}}{\Delta_\sigma \sigma} +$$

$$+ \frac{1}{a\cos\Theta} (\delta_\lambda \ \overline{u}^{2t} + \delta_\Theta(\overline{v}^{2t} \cos\Theta)) = 0 \qquad (2.10)$$

The operator Δ_{tt} (second differential) will be used:

$$\Delta_{tt}\psi = \psi(t + \Delta t) + \psi(t - \Delta t) - 2\psi(t) = 2(\overline{\psi}^{2t} - \psi) \qquad (2.11)$$

With the aid of this definition $\frac{\partial(\ln p_s)}{\partial t}$ SI is defined as a part of $\frac{\partial(\ln p_s)}{\partial t}$ in (2.10).

$$\frac{\partial(\ln p_s)}{\partial t} SI + \frac{1}{a\cos\Theta} (\delta_\lambda(\tfrac{1}{2} \ \Delta_{tt} \ u) + \delta_\Theta(\tfrac{1}{2} \ \Delta_{tt} \ v\cos\Theta)) +$$

$$+ \frac{\Delta_\sigma(\frac{1}{2} \ \Delta_{tt} \ \dot{\sigma})}{\Delta_\sigma \sigma} = 0 \qquad (2.12)$$

Then

$$\frac{\partial(\ln p_s)}{\partial t} = \frac{\partial \ln p_s}{\partial t} E + \frac{\partial \ln p_s}{\partial t} SI \qquad (2.13)$$

where

$\frac{\partial \ln p_s}{\partial t}$ E is the explicit tendency from the explicit form of the continuity equation (2.9).

With the aid of definition (2.11) terms $\overline{\psi}^{2t}$ in eqs. 2.6-2.8 can be written $\frac{1}{2} \ \Delta_{tt} \ \psi + \psi$. The last term will be transferred to the right hand side and then all explicit terms are gathered there.

$$\frac{\partial u}{\partial t} + \frac{1}{a\cos\Theta}\left(\delta_\lambda\left(\frac{1}{2}\Delta_{tt}\ \overline{\Phi}^\sigma\right) + RT_o\ \delta_\lambda\left(\frac{1}{2}\Delta_{tt}\ \ln p_s\right)\right) = A_u \qquad (2.14)$$

$$\frac{\partial v}{\partial t} + \frac{1}{a}\left(\delta_\Theta\left(\frac{1}{2}\Delta_{tt}\ \overline{\Phi}^\sigma\right) + RT_o\ \delta_\Theta\left(\frac{1}{2}\Delta_{tt}\ \ln p_s\right)\right) = A_v \qquad (2.15)$$

$$\frac{\partial T}{\partial t} + \frac{\overline{\left(\frac{1}{2}\Delta_{tt}\ \dot{\sigma}\right)\Delta_\sigma\ T_o}^\sigma}{\Delta_\sigma \sigma} - \frac{\kappa T_o}{\sigma}\ \overline{\left(\frac{1}{2}\Delta_{tt}\ \dot{\sigma} + \sigma\frac{\partial \ln p_s}{\partial t}\right.}^\sigma \left. SI\right) = A_T \qquad (2.16)$$

In the last equation (2.13) has been used. Now A_T, A_v and A_u contain all explicit calculations and when the explicit tendencies are calculated it remains to solve the semi-implicit system on the left hand sides of eqs. 2.14-2.16 with the aid of the continuity equation (2.12, 2.13) plus the hydrostatic equation (2.5).

By time averaging of the gravity wave terms the semi-implicit scheme has a numerical stability criterion depending on maximum wind speed $|U|_{max}$:

$$\Delta t_{max} < \frac{\Delta x}{|U|_{max}}.$$

2.2 Solution of the semi-implicit system

A new pressure gradient variable, P, is defined by

$$P = \overline{\Phi}^\sigma + RT_o(\ln p_s) \qquad (2.17)$$

Then the momentum equations become:

$$\frac{\partial u}{\partial t} + \frac{1}{a\cos\Theta}\ \delta_\lambda\ \left(\frac{1}{2}\Delta_{tt}\ P\right) = A_u \qquad (2.18)$$

$$\frac{\partial v}{\partial t} + \frac{1}{a}\ \delta_\Theta\ \left(\frac{1}{2}\Delta_{tt}\ P\right) = A_v \qquad (2.19)$$

By integration of the continuity equation (2.12) from the highest level down to each level of the model, by use of the hydro-static equation (2.5) and by integration of the thermodynamic equation (2.16) from the surface up to each level of the model an equation for P can be derived.

$$\underline{\frac{\partial P}{\partial t}} + \underline{\underline{G}}\ \left(\frac{1}{2}\Delta_{tt}\ d\right) = \underline{A_P} \qquad (2.20)$$

where $d = \frac{1}{a\cos\Theta}\ (\delta_\lambda\ U + \delta_\Theta(V\cos\Theta))$

$\underline{\psi}$ denotes a vertical column vector and

$\underline{\underline{G}}$ a NLEV·NLEV matrix.

The matrix G has coefficients depending on the standard temperature profile $T_o(\sigma)$ and on the vertical discretization. A_p are all integrated explicit temperature tendencies and the explicit p_s tendencies.

Let $\wedge$ denote the explicit $t + \Delta t$ value which is, for example, for P:

$$\hat{P}(t + \Delta t) = P(t - \Delta t) + 2\Delta t\, A_p(t) \tag{2.22}$$

$$\hat{u}(t + \Delta t) = u(t - \Delta t) + 2\Delta t\, A_u(t) - \frac{\Delta t}{a\cos\Theta}\, \delta_\lambda(\hat{P}(t + \Delta t) + P(t - \Delta t) - 2P(t)) \tag{2.23}$$

$$\hat{v}(t + \Delta t) = v(t - \Delta t) + 2\Delta t\, A_v(t) - \frac{\Delta t}{a}\, \delta_\lambda\, (\hat{P}(t + \Delta t) + P(t - \Delta t) - 2P(t)) \tag{2.24}$$

$$\underline{P}(t + \Delta t) - \hat{\underline{P}}(t + \Delta t) + 2\Delta t\, \underline{\underline{G}}(\tfrac{1}{2}\, \Delta_{tt}\, d) = 0 \tag{2.25}$$

The momentum equations are then:

$$u(t + \Delta t) - \hat{u}(t + \Delta t) + \frac{\Delta t}{a\cos\Theta}\, \delta_\lambda(P(t + \Delta t) - \hat{P}(t + \Delta t)) = 0 \tag{2.26}$$

$$v(t + \Delta t) - \hat{v}(t + \Delta t) + \frac{\Delta t}{a}\, \delta_\Theta(P(t + \Delta t) - \hat{P}(t + \Delta t)) = 0 \tag{2.27}$$

Taking the divergence of these equations gives:

$$d(t + \Delta t) - \hat{d}(t + \Delta t) + \Delta t \left[\frac{1}{a\cos\Theta}\, \delta_\lambda\, \frac{1}{a\cos\Theta}\, \delta_\lambda(P(t + \Delta t) - \hat{P}(t + \Delta t)) + \frac{1}{a\cos\Theta}\, \delta_\Theta(\frac{\cos\Theta}{a}\, \delta_\Theta(P(t + \Delta t) - \hat{P}(t + \Delta t))\right] = 0 \tag{2.28}$$

Then $P(t + \Delta t) - \hat{P}(t + \Delta t)$ from 2.25 is inserted:

$$\underline{d}(t + \Delta t) - \hat{\underline{d}}(t + \Delta t) - \Delta t^2\, \underline{\underline{G}} \left[\frac{1}{a\cos\Theta}\, \delta_\lambda(\frac{1}{a\cos\Theta}\, \delta_\lambda(\Delta_{tt}\, d)) + \frac{1}{a\cos\Theta}\, \delta_\Theta(\frac{\cos\Theta}{a}\, \delta_\Theta(\Delta_{tt}\, d))\right] = 0 \tag{2.29}$$

This is a system of NLEV coupled partial differential equations in finite differences which can be written for $\Delta_{tt}\,\underline{d}$:

$$\Delta_{tt}\,\underline{d} - \Delta t^2\,\underline{\underline{G}}\left[\frac{1}{a\cos\Theta}\,\delta_\lambda\,\left(\frac{1}{a\cos\Theta}\,\delta_\lambda\,(\Delta_{tt}\,\underline{d})\right) + \frac{1}{a\cos\Theta}\,\delta_\Theta(\Delta_{tt}\,\underline{d})\right] =$$

$$= \underline{\hat{d}}(t+\Delta t) + \underline{d}(t-\Delta t) - 2\underline{d}(t)$$

The system 2.28 can be diagonalized and uncoupled using the matrix:

$$\underline{\underline{E}} = (\underline{\psi}_1, \ldots\ \underline{\psi}_{NLEV}) \tag{2.31}$$

whose columns are eigenfunctions of $\underline{\underline{G}}$.

The linear transformation

$$\underline{y} = \underline{\underline{E}}^{-1}(\Delta_{tt}\,\underline{d}) \tag{2.32}$$

is performed using the fact that

$$\underline{\underline{E}}^{-1}\cdot\underline{\underline{G}}\cdot\underline{\underline{E}} = \mathrm{diag}(C_k^2) \tag{2.33}$$

where the eigenvalues C_k^2 are squares of the models' gravity wave phase speeds.

This gives a set of NLEV Helmholtz equations

$$y_k - \Delta t^2\,C_k^2\left[\frac{1}{a\cos\Theta}\,\delta_\lambda\left(\frac{1}{a\cos\Theta}\,\delta_\lambda\,y_k\right) + \frac{1}{a\cos\Theta}\,\delta_\Theta\left(\frac{\cos\Theta}{a}\,\delta_\Theta\,y_k\right)\right] = f_k \tag{2.34}$$

where

$$\underline{f} = \underline{\underline{E}}^{-1}\,(\underline{\hat{d}}(t+\Delta t) + \underline{d}(t-\Delta t) - 2\underline{d}(t))$$

The boundary conditions are $y_k = 0$ at the boundaries of the limited area.

The right hand sides are Fourier transformed and in Fourier space the equations (2.34) form a tri-diagonal system and can be solved by Gaussian elimination in the latitudinal direction. Then $(\Delta_{tt}\,d)$ can be recovered from:

$$\Delta_{tt}\,\underline{d} = \underline{\underline{E}}\,\underline{y} \tag{2.35}$$

and inverse Fourier transform back to physical space.

Having determined $(\Delta_{tt}\ d), (\frac{1}{2}\ \Delta_{tt}\ \dot{\sigma} + \sigma\ \frac{\partial \ln\ p_s}{\partial t}\ SI)$ and $\frac{\partial \ln\ p_s}{\partial t}\ SI$ can be calculated by integrating the continuity equation (2.12). Then (2.13) is integrated to give $\ln p_s(t + \Delta t)$ and $T(t + \Delta t)$ is computed from eq. (2.16). $\dot{P}(t + \Delta t)$ is given by 2.17 and hydrostatic equation (2.5) and finally $u(t + \Delta t)$ and $v(t + \Delta t)$ are computed from eqs. 2.26 and 2.27.

The standard temperature profile $T_o(\sigma)$ is chosen $\equiv$ 300K, to ensure stability of the scheme (Simmons et al., 1978).

3. THE PHYSICAL PARAMETERIZATION

The physical parameterization used in the Swedish LAM is to a great extent developed at the ECMWF. However, radiation and convection are treated in a more simplified fashion although we also take into account a diurnal cycle. In this chapter the right hand sides of eqs. 1.1-1.2, 1.5 and 1.13 will be specified. These equations can be written formally:

$$\frac{\partial u}{\partial t} = A_u - \frac{1}{p_s}\frac{\partial \tau_\lambda^T}{\partial \sigma} + (F_M^T)_\lambda \tag{3.1}$$

$$\frac{\partial v}{\partial t} = A_v - \frac{1}{p_s}\frac{\partial \tau_\Theta^T}{\partial \sigma} + (F_M^T)_\Theta \tag{3.2}$$

$$\frac{\partial T}{\partial t} = A_T + \frac{L_1}{c_p}(C^L + C^c - E) - \frac{1}{p_s}\frac{g}{c_p}\frac{\partial}{\partial \sigma} H^T + F_T^T \tag{3.3}$$

$$\frac{\partial q}{\partial t} = A_q - (C^L + C^c - E) - \frac{g}{p_s}\frac{\partial}{\partial \sigma} R^T + F_q^T \tag{3.4}$$

where A_u, A_v, A_T and A_q stand for all the adiabatic calculations described in chapter 1. The other notations are listed in the appendix.

3.1 Turbulent fluxes of momentum, heat and moisture

A vertical diffusion equation is used to describe vertical turbulent fluxes. The turbulence is, however, treated differently in the surface layer (between ground and lowest model level) and in the rest of the model atmosphere.

3.1.1 Surface fluxes

The surface fluxes are based on Monin-Obukhov similarity theory so the profiles of wind and temperature depend on external parameters.

$$\frac{kz}{u_*}\frac{\partial u}{\partial Z} = \Phi_M\left(\frac{Z}{L}\right), \quad \frac{\partial v}{\partial Z} = 0$$

$$\frac{k}{\Theta_*}\frac{\partial \Theta}{\partial Z} = \Phi_H\left(\frac{Z}{L}\right)$$

where Φ_M and Φ_H are universal functions, $L = \frac{\Theta \; u_*^2}{kg \; \Theta_*}$ is the Monin-Obukhov length and

$$u_*^2 = -(\overline{w'u'})_s = \frac{1}{\rho_s}(\tau_\lambda^T)_s$$

$$u_*\Theta_* = -(\overline{w'\Theta'})_s = \frac{1}{c_p \cdot \rho_s}(H^T)_s$$

These relations are implicit,but analytical expressions explicitly describing the fluxes in terms of external parameters have been developed by Louis (1979).

The following equations for fluxes of momentum, sensible heat and moisture are used, in case of unstably stratified surface layer:

$$(\tau^T)_s = \rho_h \, a \cdot \left| |V_h| - \frac{b \cdot s_h}{|V_h| + b \, c \, a_h |s_h|^{1/2}} \right| \cdot V_h \qquad (3.5)$$

$$(H^T)_s = - c_p \, \sigma_h^\kappa \, \rho_h \frac{a}{d_H} \left| |V_h| - \frac{b \cdot s_h}{|V_h| + b \, c \, a_h |s_h|^{1/2}} \right| \cdot$$
$$\cdot \, (T_s - \frac{T_h}{\sigma_h^\kappa}) \qquad (3.6)$$

$$(R^T)_s = - \rho_h \frac{a}{d_q} \left| |V_h| - \frac{b \cdot s_h}{|V_h| + b \, c \, a_h |s_h|^{1/2}} \right| \cdot$$
$$\cdot \, (q_{sat}(T_s) - q_h) \cdot WET \qquad (3.7)$$

where

$$WET = \min \left[(\min(\frac{sn}{sn_{crit}}, 1) + (1 - \min(\frac{sn}{sn_{crit}}, 1)) \cdot \right.$$
$$\left. \frac{W_s}{0.75 \, W_{crit}}), \, 1 \right] \qquad (3.8)$$

W_{crit} is specified and sn_{crit} = 15 cm.

If the surface layer is stably stratified the following relations are used:

$$(\tau^T)_s = \rho_h \, a (1 + \frac{b}{2} \frac{s_h}{V_h^2})^{-2} \cdot |V_h| V_h \qquad (3.9)$$

$$(H^T)_s = - c_p \, \sigma_h^\kappa \, \rho_h \frac{a}{d_H} (1 + \frac{b}{2} \frac{s_h}{V_h^2})^{-2} \cdot |V_h| (T_s - \frac{T_h}{\sigma_h^\kappa}) \qquad (3.10)$$

$$(R^T)_s = - \rho_h \frac{a}{d_q} (1 + \frac{b}{2} \frac{s_h}{V_h^2})^{-2} \cdot |V_h| (q_{sat}(T_s) - q_h) \cdot WET \qquad (3.11)$$

where

s_h is the thermal stability

$$s_h = gh \frac{\Theta_h - \Theta_s}{\theta_s} + 0.6077(q_h - q_{sat}(T_s)) \cdot \text{WET} \qquad (3.12)$$

and

$$a = k^2 (\ln \frac{h}{z_o})^{-2} \qquad (3.13)$$

$$a_h = a \cdot (\frac{h}{z_o})^{\frac{1}{2}}$$

where h is the height of lowest model level above ground. The constants used are:

$b = 9.4$

$c = 5.3$

$d_H = 0.74$

$d_q = \frac{d_H}{\alpha}$ where α is a tuning parameter (at present = 0.35)

The surface roughness length z_o is computed over sea:

$$z_o = 0.032 \frac{u_*^2}{g} \qquad (3.15)$$

but not allowed to drop below 0.0015 cm. Over ice z_o = 0.01 cm and over land z_o is specified in the grid points.

3.1.2 Fluxes above the surface layer

The fluxes above the surface layer are calculated on the basis of mixing length theory

$$\tau^T_{k+\frac{1}{2}} = - \frac{g}{p_s} (\rho^2 K_m \frac{\Delta V}{\Delta \sigma})_{k+\frac{1}{2}} \qquad (3.16)$$

$$H^T_{k+\frac{1}{2}} = - g \frac{c_p}{p_s} (\rho^2 \sigma^\kappa K_H \frac{\Delta (T/\sigma^\kappa)}{\Delta \sigma})_{k+\frac{1}{2}} \qquad (3.17)$$

$$R^T_{k+\frac{1}{2}} = - \frac{g}{p_s} (\rho^2 K_q \frac{\Delta q}{\Delta \sigma})_{k+\frac{1}{2}} \qquad (3.18)$$

The diffusion coefficients depend on thermal stability and windshear so that the fluxes also match the surface fluxes in section 3.1.1.

The diffusion coefficients are for unstable stratification:

$$(K_M)_{k+\frac{1}{2}} = \ell^2_{k+\frac{1}{2}} \left(\left|\frac{\Delta V}{\Delta z}\right|_{k+\frac{1}{2}} - \frac{b\, s_{k+\frac{1}{2}}}{\left|\frac{\Delta V}{z}\right|_{k+\frac{1}{2}} + b \cdot c \cdot a_{k+\frac{1}{2}} \left| s_{k+\frac{1}{2}} \right|^{1/2}} \right) \qquad (3.19)$$

and in stable case:

$$(K_M)_{k+\frac{1}{2}} = \ell^2_{k+\frac{1}{2}} \left(\left|\frac{\Delta V}{\Delta z}\right|_{k+\frac{1}{2}} \left(1 + \frac{b}{2} \frac{s_{k+\frac{1}{2}}}{(\frac{\Delta V}{\Delta z})^2_{k+\frac{1}{2}}} \right)^{-2} \right) \qquad (3.20)$$

K_H and K_q are in both cases:

$$(K_H)_{k+\frac{1}{2}} = \frac{\ell}{d_H} (K_M)_{k+\frac{1}{2}} \qquad (3.21)$$

$$(K_q)_{k+\frac{1}{2}} = \frac{\ell}{d_q} (K_M)_{k+\frac{1}{2}} \qquad (3.22)$$

The stability function is now:

$$s_{k+\frac{1}{2}} = g \left| \frac{\ell}{\Theta} \left(\frac{\Delta\Theta}{\Delta z}\right) + 0.6077 \left(\frac{\Delta q}{\Delta z}\right) \right|_{k+\frac{1}{2}} \qquad (3.23)$$

a is specified

$$a_{k+\frac{1}{2}} = \left(\frac{\ell}{\Delta z}\right)^2_{k+\frac{1}{2}} \left(\left(\frac{z_{k+1}}{z_k}\right)^{1/3} - 1 \right)^{3/2} \cdot \left(\frac{z}{z_k}\right)^{1/2} \qquad (3.24)$$

The mixing length ℓ is

$$\ell = \frac{k \cdot z}{1 + \frac{k \cdot z}{\lambda}} \qquad (3.25)$$

with the asymptotic mixing length λ = 300 m.

Constants b, c, d_H and d_q are the same as in 3.1.1.

3.1.3 Numerical calculations and solution of the diffusion equation

The flux calculations need information from u, v and T/q-points in the staggered grid (fig. 1.1). To facilitate this, velocity components are first interpolated to T/q gridpoints linearily. This is done for time level τ-1 (t-Δt) which is used for the diffusion coefficients.

$$u^{\tau-1}_{i,j} = 0.5 \; (u^{\tau-1}_{i,j} + u^{\tau-1}_{i-1,j}) \qquad (3.26)$$

$$v^{\tau-1}_{i,j} = 0.5 \; (v^{\tau-1}_{i,j} + v^{\tau-1}_{i,j+1}) \qquad (3.27)$$

(j southwards)

The system 3.1-3.4 with inserted fluxes from sections 3.1.1 and 3.1.2 form vertical diffusion equations with respect to the tendencies due to turbulent fluxes. Because the time steps are rather long they are solved implicitly in time. In finite differences they take the form

$$u_k^{\tau+1} - u_k^{\tau-1} = A_k \quad (u_{k+1}^{\tau+1} - u_k^{\tau+1}) - C_k(u_k^{\tau+1} - u_{k-1}^{\tau+1}) \tag{3.28}$$

and similarily for v, T and q.

The coefficients A_k and C_k are

$$A_k = \frac{g^2 \ \sigma_k \ K_{k+\frac{1}{2}}}{p_s \ R \ T_k \ \Delta\sigma_k \ \Delta\sigma_{k+\frac{1}{2}}} \tag{3.29}$$

$$C_k = \frac{g^2 \ \sigma_k \ K_{k-\frac{1}{2}}}{p_s \ R \ T_k \ \Delta\sigma_k \ \Delta\sigma_{k-\frac{1}{2}}} \tag{3.30}$$

where K is calculated from $u^{\tau-1}$, $v^{\tau-1}$, $T^{\tau-1}$ and $q^{\tau-1}$.

The boundary conditions are zero flux at the top ($c_1 = 0$) and u_s, $v_s = 0$ and q_s predicted from surface calculations. For T the diffusion coefficient in the surface layer is known but the flux depends on the difference between T_{NLEV} and T_s. The surface fluxes determining T_s are also dependent on T_{NLEV} so $T_s^{\tau+1}$ and $T_{NLEV}^{\tau+1}$ are solved simultaneously in the Gaussian elimination procedure which is used to solve the diffusion equation. This implicit way of calculating T_s guarantees that the same fluxes are used both for the vertical diffusion scheme and in the surface temperature prediction equation (see 3.3).

Finally the tendencies due to turbulent fluxes are re-interpolated:

$$\frac{\partial u}{\partial t} i,j = 0.5 \ (\frac{\partial u}{\partial t}^T \ i,j + \frac{\partial u}{\partial t}^T \ i+1,j) \tag{3.31}$$

$$\frac{\partial v}{\partial t} i,j = 0.5 \ (\frac{\partial v}{\partial t}^T \ i,j + \frac{\partial v}{\partial t}^T \ i,j-1) \tag{3.32}$$

3.2 Radiation

The radiation scheme is a simplified scheme suggested by Bodin (1980). It is only used to predict the surface temperature which then determines the state of the boundary layer. Radiation processes in the free atmosphere are not considered to be of vital importance for short range forecasting.

3.2.1 Long-wave radiation

The long-wave radiation treatment is based on a Brunt-type of formula for the return radiation from a clear sky, i e

$$\varepsilon_{ao} = a + b\sqrt{e} \tag{3.33}$$

where $_{ao}$ is the apparent emissivity of a clear sky so that

$$L\downarrow = \varepsilon_{ao} \ \sigma_B \ T^4_{2m} \tag{3.34}$$

An evaluation of such formulas can be found in Arnfield (1979). His examination yields values of a and b due to Andersson (1954)

$a = 0.68$
$b = 0.0036$

if water vapor pressure e is in Pascal.

Eq. (3.33) is extended to cloudy conditions by

$$\varepsilon_a = \varepsilon_{ao}\{1 + (\sum_{i=1}^{N} k_i \cdot \max(c_i - c_{sum}, 0))\} \tag{3.35}$$

where c_i is fractional cloud amount in layer i and k_i is a cloud specific re-emission parameter according to

$$c_{sum} = \sum_{m=1}^{i-1} c_m \tag{3.36}$$

	k_i	i
High clouds	0.06	4 = N
Middle clouds	0.18	3
Low clouds	0.21	2
Fog and stratus	0.24	1

The return radiation from a cloudy sky can be written:

$$L\downarrow = \varepsilon_a \sigma_B T^4_{2m} \tag{3.37}$$

which yields the net long wave radiation:

$$\begin{aligned} L_{NET} &= \varepsilon_s \sigma_B T^4_s - \varepsilon_a \sigma_B T^4_{2m} + (1 - \varepsilon_s)\ \sigma_B T^4_{2m} \cdot \varepsilon_a \\ &= \varepsilon_s \sigma_B T^4_s - \varepsilon_s \varepsilon_a \sigma T^4_{2m} \end{aligned} \tag{3.38}$$

where the last term is the reflected radiation and ε_s is surface emissivity. L_{NET} is positive when upward.

In the present (1982) version of the model T_{2m} and e in the formulas are taken as the values at the top of the surface layer.

3.2.2 Short-wave radiation

The short-wave radiation package is the same as in Bodin (1979) and accounts for absorbtion and scattering in clear air and in clouds.

The incoming solar radiation at the edge of the atmosphere, the solar constant, is set at

$$F_o = 1395 \text{ w/m}^2.$$

Absorbtion by water vapor

Absorbtion is given by McDonald (1960) as

$$A_w = 0.077 \, (u \cdot \sec z)^{0.3} \tag{3.39}$$

where u is the optical path length ('precitable water') and sec z = $1/\cos z$, where z is the zenith angle.

Scattering

Scattering in a cloudless atmosphere is given by Kondratyev (1969) as a scattering transmission function

$$\tau = 1.041 - 0.16 \, (\sec z)^{1/2} \tag{3.40}$$

Clouds

Accounting for absorbtion and scattering Pandalfo et al. (1971) give the following transmission functions for clouds

Type	T_i	i
High clouds	$0.9-0.04 \cdot \sec z$	4
Middle clouds	$0.45-0.01 \cdot \sec z$	3
Low clouds	$0.35-0.015 \cdot \sec z$	2
Stratus and fog	$0.25-0.01 \cdot \sec z$	1

The cloud absorbtion factor is then calculated from

$$A_{c\ell} = \sum_{i=1}^{4} (1 - c_i(1 - T_i)) \tag{3.41}$$

where c_i is fractional cloud cover, computed from the cloud parameterization scheme of the LAM.

Assuming a surface albedo A and a turbidity Tu we get

$$F_{surf} = (1 - A)\cdot Tu\cdot A_{c\ell}(\tau - A_w) \cos z\ F_o \tag{3.42}$$

Tu is chosen as 0.95 and the albedo A is given as an empirical function of the zenith angle z. A correction is made if sea ice or snow cover is present.

3.2.3 Auxiliary relations

The necessary astronomical information is computed from

$$\cos z = \sin \delta \cdot \sin \phi + \cos \delta \cdot \cos \phi \cdot \cos t \tag{3.43}$$

where the solar radiation is

$$\delta = 23.5 \sin \left(\frac{2\pi}{365} (\mathrm{Day}-80)\right) \tag{3.44}$$

with D = day of year.

The local hour angle is

$$t = (\mathrm{time}_{GMT} - \mathrm{Noon}_{GMT})\cdot 15 + \lambda \tag{3.45}$$

where λ is negative to the west.

The path length is calculated from

$$u = \int_o^\infty \rho q dz = -\frac{1}{g}\int_{p_o}^{o} g dp = -\frac{p_s}{g}\int_1^o q\cdot d\sigma \tag{3.46}$$

assuming a dry atmosphere above level 2 in the model, we get

$$u = \frac{p_s}{g} \cdot \sum_{\ell=2}^{NLEV} q_\ell (\Delta_\sigma \sigma)\ell \tag{3.47}$$

3.3 Surface calculations

3.3.1 Surface temperature

A predictive equation is used for the surface temperature over land. Over sea it is held constant during the forecast.

$$c_s \frac{\partial T_s}{\partial t} - F_{surf} + \varepsilon_s \sigma_B T_s^4 - \varepsilon_s \varepsilon_a \sigma_B T_{2m}^4 - H_s^T - L_1 R_s^T +$$

$$+ L_2 \rho_w M_{sn} + B_T = 0 \tag{3.48}$$

where c_s is the heat capacity per unit area of soil and F_s, ε_s, ε_a are defined in 3.2. Turbulent fluxes of sensible heat (eq. 3.9) and moisture (eq. 3.10) are included in H_s^T and R_s^T.

The heat condiction in the soil is given by B_T:

$$B_T = \lambda_T \frac{T_s - T_{SD}}{\Delta z} \tag{3.49}$$

where λ_T is the conductivity of the soil, T_{SD} is a fixed ground temperature. At present it is chosen as the arithmetic mean of monthly mean air temperature and T_{NLEV}. In an operational cycle one could use a running last 24h mean air temperature as suggested by Deardorff (1978).

Melting of snow (unit m H_2O/s) is considered when snow is present and the surface temperature exceeds T_o = 273.16K. In that case the surface temperature is reduced to melting point and the heat is used to melt the snow.

$$\begin{aligned} T_s &= T_o \\ M_{sn} &= - \frac{1}{L_2 \rho_w} \left(c_s \frac{\partial T_s}{\partial t}\right)_R \end{aligned} \tag{3.50}$$

where $\left(c_s \frac{\partial T_s}{\partial t}\right)_R$ stands for the tendency computed from (3.48) without snow melt taken into account.

The soil parameters c_s, λ_T and Δz are based on the proposal by Deardorff (1978). They are based on a soil diffusivity $\kappa_s = 7.0 \cdot 10^{-6} \cdot$ TUNGRC m^2/s (tunable) and a diurnal cycle with T_{PER} = 86400s. Then $\Delta z = \sqrt{\kappa_s T_{PER}}$ and $c_s = c_{soil} \cdot \rho_{soil} \cdot \Delta z / 2\sqrt{\pi}$. $\lambda_T / \Delta z = 2 / T_{PER}$ which will be the fact if tuning constant TUNGRC is set = 10. Whether to use this value or not depends on how reliable T_{SD} values are considered to be.

3.3.2 Surface moisture

The equation for soil moisture w_s (in m H_2O/ is:

$$\frac{\partial w_s}{\partial t} = (1 - \delta^{sn}) \frac{(R^T)_s}{\rho_w} + (1 - \delta^{P_r}) \frac{P_s}{\rho_w} B_w + M_{sn} - RO \qquad (3.51)$$

where p_s is total precipitation (kg/m^2)

$$\delta^{sn} = 0 \quad \text{if } sn < 0$$
$$\delta^{sn} = 1 \quad \text{if } sn > 0 \qquad (3.52)$$

$$\delta^{P_r} = 0 \quad \text{if } T_s > T_o = 273.16\text{K}$$
$$\delta^{P_r} = 1 \quad \text{if } T_s < T_o \qquad (3.53)$$

B_w represents the moisture flux due to diffusion in the soil.

$$B_w = \lambda_w \frac{W_s - W_{SD}}{\Delta z} \qquad (3.54)$$

where $\lambda_w/\Delta z$ is specified following Deardorff (1978) but multiplied by a tuning parameter (TUNGRC), at present equal to 10. W_{SD} is the 'deep' soil water content kept constant (can be related to T_{SD}).

Run-off takes place when W_s exceeds W_{crit} (tuning parameter ASSAT, at present = 0.15). W_s is then kept constant.

3.4 Horizontal diffusion

To parameterize the horizontal turbulence (the F's in eq. 3.1-3.4) a non-linear second order diffusion scheme is used.

For a variable x (=u, v, T or q) it is:

$$F_x = k|\nabla^2 x|\nabla^2 x \qquad (3.55)$$

where ∇^2 is the Laplacian operator. The constant k can be used to tune the diffusion. At present a weak diffusion is used with $k = 2.5 \cdot 10^4$. With $\Delta\lambda = \Delta\Theta = 1.5^o$ it means an e-folding time of a $2\Delta x$ temperature wave with amplitude 2.5K of about 60 hours and 10 days for a $4\Delta x$ wave.

However, the effective diffusion coefficient $k|\nabla^2 x|$ is rather different depending on the type of variable.

For horizontal diffusion of specific humidity, q, k is to take into account the vertical variations of q. This means that the multiplication factor varies from $0.8 \cdot 10^3$ at the lowest level to $4 \cdot 10^3$ at the top of the model atmosphere.

3.5 Condensation and precipitation

A large scale precipitation scheme is used to take care of moisture converging due to horizontal and vertical advections. Also vertical and to a small extent horizontal diffusions affect the moisture. To parameterize convective precipitation in unstable saturated air a moist convective adjustment scheme is used.

3.5.1 Large scale precipitation

Condensation of water vapor takes place when the air is super-saturated and precipitation falls out immediately. Then humidity (q') and temperature (T') are changed:

$$q = q' - C^L \Delta t \tag{3.56}$$

$$T = T' + \frac{L_1}{c_p} C^L \cdot \Delta t \tag{3.57}$$

also

$$q = q_{sat}\,(T) \tag{3.58}$$

By assuming a linear dependence

$$q_{sat}(T) = q_{sat}(T') + \frac{dq_{sat}}{dT}\,(T')\cdot(T-T') \tag{3.59}$$

but

$$T-T' = \frac{L_1}{c_p} C^L \Delta t \text{ and } q_{sat}(T) = q' - C_L \Delta t \tag{3.60}$$

so the condensation rate $C^L \Delta t$ follows from

$$C^L \Delta t = \frac{q' - q_{sat}(T')}{1 + \frac{L_1}{c_p} \frac{dq_{sat}}{dT}(T')} \tag{3.61}$$

$q_{sat}(T')$ is interpolated from a saturation water vapor pressure table.

Evaporation can optionally take place:

$$E_k^L = \alpha_1 (q_{sat} - q)\left(\frac{1}{\alpha_2} \sqrt{\sigma}_k \; P^L_{k+\frac{1}{2}}\right) \tag{3.62}$$

where $\alpha_1 = 5.44 \cdot 10^{-4}$ and $\alpha_2 = 5.09 \cdot 10^{-3}$.

$P^L_{k+\frac{1}{2}}$ is the precipitation falling through level k:

$$P^L_{k+\frac{1}{2}} = \sum_{\ell=1}^{k} ((C^L\Delta t)_L - E^L_\ell) \cdot \frac{p_s \; \Delta\sigma_\ell}{g} \tag{3.63}$$

The condensation rate $C^L\Delta t$ can be modified by a tuning parameter (TUNCON < 1).

3.5.2 Moist adiabatic adjustment

In unstable saturated air columns the temperatures and humidities are adjusted immediately:
The conditions are

$q > q_{sat}$ in a column

$\frac{\partial T}{\partial \sigma} > \Gamma_e$ in a column

where

$$\Gamma_e = \frac{R}{c_p} \frac{T}{\sigma} \cdot \frac{(p + \frac{0.622L_1}{RT} e_{sat})}{(p + \frac{0.622L_1}{c_p} \frac{\partial e_{sat}}{\partial T}} \tag{3.64}$$

The temperatures are then adjusted by ΔT to an average moist adiabat so that

$$c_p\Delta T + L\Delta q \; d\sigma = 0 \tag{3.65}$$

and

$$q = q_{sat}(T) \tag{3.66}$$

$$\frac{\partial T}{\partial \sigma} = \Gamma_e \tag{3.67}$$

The surplus humidity is immediately precipitated. The tuning parameter TUNCON can also here be used to reduce condensation at each time step.

3.5.3 Adjustment of negative humidity values

Negative humidity values, which may occur due to truncation errors in the finite difference scheme, are corrected in the following way:

if $q'_k < 0$

$$q_k = 0 \tag{3.68}$$

and at a lower level

$$q_{k+1} = q'_{k+1} + \frac{q'_k \cdot \Delta\sigma_k}{\Delta\sigma_{k+1}} \tag{3.69}$$

Moisture is thus diffused from beneath and total moisture in the column is conserved except for a fictious upward surface flux if $q_{nlev} < 0$.

3.6 Clouds

Clouds are included only as diagnostic variables. The amount is determined from an empirical linear function of relative humidity. The model atmosphere is divided into four regions and the maximum relative humidity at any level in each region determines the cloud amount. In this way cloud amounts of high, medium and low clouds plus stratus/fog cover are computed for use in the radiation scheme (see section 3.2).

Present formulation:

High clouds	$\sigma < 0.47$	$N_H = 25 \cdot U_H - 15$
Medium clouds	$0.47 < \sigma < 0.80$	$N_M = 30 \cdot U_M - 21$
Low clouds	$0.80 < \sigma < \sigma_{NLEV}$	$N_L = 50 \cdot U_L - 41$
Stratus/fog	NLEV	$N_S = 150 \cdot U_{NLEV} - 141$

where U_J (J = H, M, L) is

$$\max\left((q/q_{sat})_{k_{min}}, (q/q_{sat})_{k_{min+1}} \ldots, (q/q_{sat})_{k_{max}}\right)$$

and k_{min}, k_{max} are the levels in each region.

N are in octas (8 = total cover).

Those specifications are based on the assumption that ACRIT = 1 (critical relative humidity). If a lower value is chosen the formulas have to be altered.

3.7 Interaction of the physics with the semi-implicit scheme

The time tendencies from the vertical and horizontal diffusion schemes are included in the terms on the right hand sides of equations 2.6-2.8. This is a problem for the condensation scheme, which should satisfy rather precise conditions at $t + \Delta t$, $q = q_{sat}$ and $\frac{\partial T}{\partial \sigma} = \Gamma_e$. If those routines are included in the semi-implicit scheme these conditions will no longer be satisfied when the implicit correction has been performed. Therefore large scale condensation, moist adiabatic adjustment and corrections of negative values are done in a final step <u>after</u> the implicit corrections have been made.

4. THE LIMITED AREA

The area of integration is shown in fig. 1.1 but the modifications of the equations in chapter 1 due to the boundaries and the transformed grid will be described here.

4.1 The boundary scheme

The interior variables are smoothly adjusted towards their prescribed boundary values using s relaxation technique developed by Davies (1976) and modified by Kållberg (1977). Within the boundary zone for a dependent variable s:

$$s^{\tau+1} = (1 - \alpha)\, s_I^{\tau+1} + \alpha\, s_B^{\tau+1} \tag{4.1}$$

where $s_I^{\tau+1}$ is the interior value which is the final one with respect to the semi-implicit scheme. $S_B^{\tau+1}$ is a value valid at $t + \Delta t$ linearily interpolated in time from the boundary data.

α is a function chosen as

$$\alpha = 1 - \tanh\,(a \cdot j) \tag{4.2}$$

$$a = \frac{2}{N-4} \tag{4.3}$$

where j is the distance in gridpoints to the nearest boundary and N=8 (the width of the boundary zone). The values of α can be seen in table 4.1.

j	$\alpha = 1 - \tanh\,(j/2)$
0	1.0
1	0.538
2	0.238
3	0.095
4	0.036
5	0.013
6	0.005
7	0.002

Table 4.1
Relaxation factors

As can be seen above the boundary zones to the west and south consist of a band 8 gridpoints wide where the outermost gridpoint value is taken completely from a boundary value because these values cannot be computed from equations in ch. 1 due to centered finite differences.

Also due to the staggered grid, the two outer lines to the north and east cannot be computed.

4.2 The transformed spherical grid

Conversion between the normal latitudes-longitudes and the transformed spherical grid are needed when interpolating data, for astronomical constants, for the Coriolis-parameter and for dissemination of forecasts.

4.2.1 Transformation formulae

We consider a new coordinate system defined by the longitude of the new North Pole, λ_{NP}, and the latitude ϕ_{NP}. The radius vector of this new North Pole, n, is also normal to the new equator plane. In a Cartesian system (fig. 4.1) the coordinates will be:

$$\mathbb{n} = (\cos\lambda_{NP} \cos\phi_{NP}, \sin\lambda_{NP} \cos\phi_{NP}, \sin\phi_{NP})$$

$\mathbb{c}$ is a radius vector to a point (λ,ϕ) on the earth's surface (fig. 4.2).

$$\mathbb{c} = (\cos\lambda\cos\phi, \sin\lambda\cos\phi, \sin\phi)$$

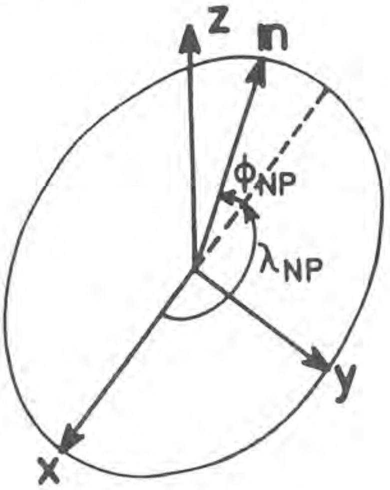

Figure 4.1

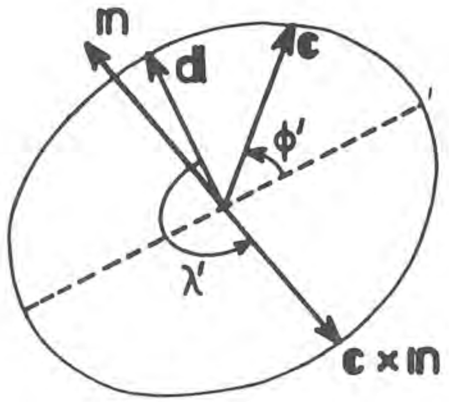

Figure 4.2

The latitude of $\mathbb{c}$ in the new system, ϕ', can then be found from the scalar product between vectors $\mathbb{c}$ and $\mathbb{n}$.

$$\phi' = \pi/2 - \text{arc cos } (\mathbb{c}\cdot\mathbb{n}), \quad |\mathbb{c}| = |\mathbb{n}| = 1$$

To avoid too extensive writing we hereby only consider the pole somewhere on the 180°E-meridian, which makes the grid suitable for European users. This is no principal restriction, it only means dropping a lot of factors and terms in the formulae.

$$\lambda_{NP} = \Pi \Rightarrow$$

$$\mathbb{n} = (-\cos\phi_{NP},\ 0,\ \sin\phi_{NP})$$

$$\phi' = \Pi/2 - \arccos\left|-\cos\phi_{NP}\cos\lambda\cos\phi + \sin\phi_{NP}\sin\phi\right| \tag{4.4}$$

Then the vector product $\mathbb{c} \times \mathbb{n}$ (see fig. 4.2) can be used to obtain the new longitude λ'. $\mathbb{c} \times \mathbb{n}$ is perpendicular to both $\mathbb{n}$ and $\mathbb{c}$. Therefore it lies in the new equator plane and it is perpendicular to $\mathbb{c}$'s projection on this plane. The angle λ' is obtained through the scalar product between $\mathbb{c} \times \mathbb{n}$ and a vector $\mathbb{d}$ in the negative y-direction.

$$\mathbb{d} = (0,\ -1,\ 0)$$

$$\lambda' = \arccos\left|\frac{(\mathbb{c} \times \mathbb{n})\cdot\mathbb{d}}{|\mathbb{c} \times \mathbb{n}|}\right|,\quad |\mathbb{d}| = 1$$

$$\mathbb{c} \times \mathbb{n} = \begin{vmatrix} e_x & e_y & e_z \\ \cos\lambda\cos\phi & \sin\lambda\cos\phi & \sin\phi \\ -\cos\phi_{NP} & 0 & \sin\phi_{NP} \end{vmatrix} =$$

$$= (\sin\phi_{NP}\sin\lambda\cos\phi,\ -\cos\phi_{NP}\sin\phi - \sin\phi_{NP}\cos\lambda\cos\phi,\ \cos\phi_{NP}\sin\lambda\cos\phi)$$

$$(\mathbb{c} \times \mathbb{n})\cdot\mathbb{d} = \cos\phi_{NP}\sin\phi + \sin\phi_{NP}\cos\lambda\cos\phi$$

$$|\mathbb{c} \times \mathbb{n}| = |\mathbb{c}| \cdot |\mathbb{n}| \cdot \sin(\Pi/2-\phi')$$

but with the aid of eq. (4.4)

$$|\mathbb{c} \times \mathbb{n}| = \sin|\arccos(-\cos\phi_{NP}\cos\lambda\cos\phi + \sin\phi_{NP}\sin\phi) \tag{4.5}$$

$$\lambda' = \arccos\left|\frac{(\cos\phi_{NP}\sin\phi+\sin\phi_{NP}\cos\lambda\cos\phi)}{\sin(\alpha)}\right| \tag{4.6}$$

where α is the argument in eq. (4.5)

APPENDIX

List of symbols and notations

Independent variables

λ	longitude
Θ	latitude
$\sigma = p/p_s$	vertical coordinates
t	time

Dependent variables

T	temperature
V	horizontal wind vector with
u	longitudinal and latitudinal velocity
v	components
q	specific humidity
p_s	surface pressure
T_s	surface temperature
w_s	soil moisture
s_n	snow cover

Diagnostic variables

$p = \sigma p_s$	pressure
ϕ	geopotential
$\dot{\sigma} = \frac{d}{dt}$	vertical velocity in σ-system
$\omega = \frac{dp}{dt}$	vertical velocity in p-system
$\rho = \frac{P}{RT}$	density of air
ρ_w	density of water
$\Theta = T(\frac{p}{p_o})^{R/c_p}$	potential temperature (p_o = 1000mb)
$q_{sat} = 0.622\ \frac{e_{sat}}{p}$	saturation specific humidity
e_{sat} =	saturation water vapor pressure
h =	height of lowest model level
z =	geometric height

Surface specification fields

$_s$ surface geopotential

z_o surface roughness length

T_{SD} ground temperature

w_{SD} ground moisture

Constants and parameters

g acceleration due to gravity

$f = 2\Omega \sin\Theta_{real}$ Coriolis parameter (Θ_{real} = real latitude on earth)

Ω angular velocity of the earth

a radius of the earth

R gas constant for dry air

c_p specific heat for dry air at constant pressure

$\kappa = R/c_p$

L_1 latent heat of condensation

L_2 latent heat of ice melting

$K = 0.35$ von Karman's constant

$\sigma_B = 5.6 \cdot 10^{-8}$ Stefan-Boltzmann constant

Finite difference notations

$$(\Delta\sigma)_k = \sigma_{k+\frac{1}{2}} - \sigma_{k-\frac{1}{2}}$$

$$(\Delta\sigma)_{k+\frac{1}{2}} = \sigma_{k+1} - \sigma_k$$

$$\overline{A}^x = \frac{A(x-\frac{\Delta x}{z})+A(x+\frac{\Delta x}{z})}{2}$$

$$\overline{A}^{xy} = |A(x - \frac{\Delta x}{z}, y - \frac{\Delta y}{z}) + A(x + \frac{\Delta x}{z}) + A(x - \frac{\Delta x}{z}, y + \frac{\Delta y}{z}) + $$
$$+ A(x + \frac{\Delta x}{z}, y + \frac{\Delta y}{z})|/4$$

$$\Delta_x A = A(x + \frac{\Delta x}{z}) - A(x - \frac{\Delta x}{z})$$

$$\Delta_{xx} A = A(x + \Delta x) + A(x - \Delta x) - 2A(x)$$

Indices

i	longitude gridpoints
j	latitude gridpoints
k	model level
τ	time level
s	surface value
s_o	surface deep, i.e. in ground
h	value at top of the surface layer
$2m$	value at 2 m height above surface
sat	saturation value
crit	critical value

Notations in physical parameterization

$\tau^T = (\tau^T_\lambda, \tau^T_\theta)$	downward momentum flux due to turbulent motion
$F^T_M = ((F^T_M)_\lambda, (F^T_M)_\theta)$	horizontal frictional force (due to turbulent motion)
H^T	downward flux of sensible heat
R^T	downward flux of moisture due to turbulent motion
F^T	horizontal flux of sensible heat
F^T_q	horizontal flux of moisture
C^L	rate of large scale condensation
C^C	rate of convective condensation
E	rate of evaporation of precipitation

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Box 923, S-601 19 Norrköping, Sweden. Phone +46 11 10 80 00. Telex 644 00 smhi s

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