A STUDY OF THE LARGE SCALE
COOLING IN THE BAY OF BOTHNIA

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AVKYLNINGEN I BOTTENVIKEN

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SMHI Rapporter
METEOROLOGI OCH KLIMATOLOGI

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SMHI

Sveriges meteorologiska och hydrologiska institut

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ABSTRACT

This report treats the large scale cooling in the Bay of Bothnia. From the heat conduction equation an expression is derived describing the changes in water temperature as a function of net longwave radiation, sensible heat flux and latent heat flux. The water temperature is defined as the mean water temperature in the central basin. The meteorological and water temperature data, needed for the heat flux calculations, have been extracted from analysed weather maps and analysed water temperature maps. Two cooling periods have been investigated. The first 1972/73 had a slow decrease in the water temperature while in the second, 1973/74, the cooling was rapid. The result shows a good agreement between calculated and analysed water temperature.

SAMMANFATTNING

Denna rapport behandlar den storskaliga avkylningen i Bottenviken. Från värmeledningsekvationen har ett förenklat uttryck tagits fram som beskriver vattentemperaturens förändring som en funktion av långvågsstrålning, sensibelt värme och latent värme. Vattentemperaturen är definierad som medeltemperaturen i centrala delen av Bottenviken. Meteorologiska data och vattentemperaturdata, som behövs för värmeflödesberäkningarna, har plockats fram från analyserade väderkartor och vattentemperaturkartor. Jämförelsen mellan analyserad och beräknad vattentemperatur är gjord för två olika avkylningsperioder. Den första, som sträcker sig från 1 november 1972 till 6 januari 1973, karaktäriseras av en långsam avkylning hos vattenmassan. Den andra perioden som sträcker sig från 1 november till 18 december 1973 uppvisade en betydligt snabbare avkylning. Resultaten från dessa två perioder visar en bra överensstämmelse mellan beräknad och analyserad vattentemperatur.

TABLE OF CONTENT

Nomenclature

T_w - sea water temperature

T_A - air temperature

ρ_w - sea water density

ρ_a - air density

C_w - specific heat of sea water

C_p - specific heat of air at constant pressure

C_c - sensible heat transfer coefficient

C_e - moisture transfer coefficient

F_s - net shortwave radiation

F_L - net longwave radiation

F_c - sensible heat flux

F_e - latent heat flux

F_p - heat flux due to precipitation

F_Q - heat flux from rivers

F_{-D} - deep water heat flux

α - sea surface albedo

S_o - solar constant

Z - zenit angle

T_R - transmission function

A_w - absorption function

T_i - cloudcover function

N - cloud coverage

ε - sea surface emissivity

σ - Stefan Bolzman's constant

ε_A - atmospheric water vapor pressure

ε_w - water vapor pressure at the sea surface

$\overline{U}$ - mean wind velocity

L - latent heat of evaporation

R - relative humidity

1. INTRODUCTION

Sea transports are very important for the trade between Sweden and other countries and between different parts of Sweden. One important obstacle for sea transports is, in winter time, the sea ice. Shipping and ice breaking service need, in order to plan their work in the most optimized way, daily ice information and ice forecasts. Today SMHI distributes both ice charts, of the actual ice situation, and ice drift forecasts. However, forecasts of ice formation and melting is today only made in a subjective way by meteorologists.

In 1979 a work on this subject started with the intention of getting a better understanding of the important processes that are governing the thermal changes of sea water. The purpose with the project is to forecast the date of the first ice formation and thereafter changes in the horizontal ice coverage until the sea is completely covered. The project has been divided in two separate parts. The reason for this separation is based on observations of the behaviour of the sea temperature during the period before freezing. Assume that the sea surface temperature in an arbitrary position i; $T_i(t)$, can be divided into:

$$T_i(t) = \overline{T}_i(t) + T_i'(t)$$

where

$\overline{T}_i(t)$ is a mean water temperature in a well mixed layer down to the depth D. $\overline{T}_i(t)$ varies on the time scale of days.

$T_i'(t)$ is a water temperature in a thin surface layer. $T_i'(t)$ varies on a time scale of hours.

This first report is concerning the cooling of the mean water temperature in the central basin of the Bay of Bothnia. It is necessary on this early stage to emphasize that some of the approaches have been chosen for the reasons that it shall be possible to use the results in an operational way and to make forecasts on a time scale of 5-10 days.

2. THE BAY OF BOTHNIA

The Bay of Bothnia has an area of 36.260 km^2 and a volume of 1.481 km^3. The mean depth is 41 m. It has a connection with the Sea of Bothnia through the Northern Quark. The sill depth, between the Swedish coast and Holmön, is about 30 m and between Holmön and the Finnish coast less than 20 m. (Figure 1).

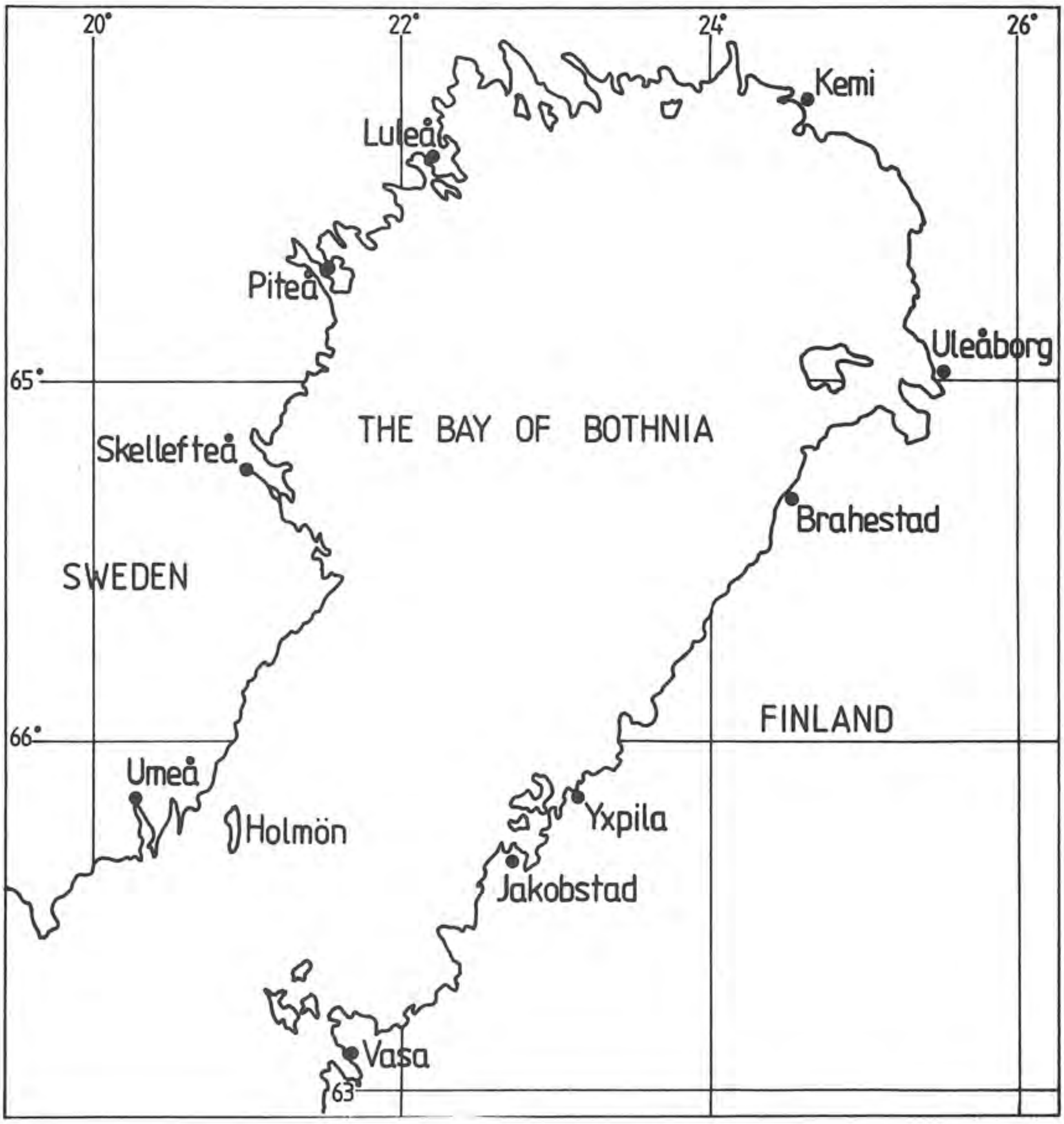


Figure 1 A map over the Bay of Bothnia.

In late autumn and in winter time the sea consists mainly of two layers. The salinity of the surface layer is approximately $3.5^{o}/oo$ and of the lower layer $4-4.5^{o}/oo$ (Kullenberg 1968).

Figure 2 shows the water balance in the Bay of Bothnia according to Dahlin (1976). The letters E, P and Q stands for evaporation, precipitation and water flux from rivers.

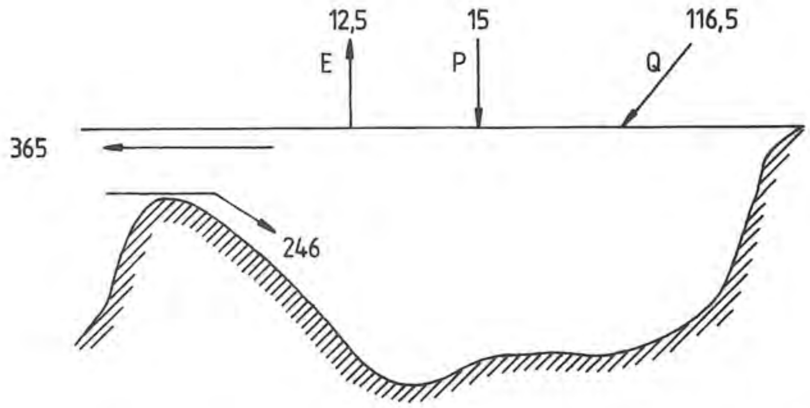


Figure 2 *The figure shows the cross section of the Bay of Bothnia. All water balance figures have the unit km^3 $year^{-1}$.*

3. THEORY

3.1 Heat fluxes

Consider the water mass in the Bay of Bothnia in late autumn. Heat fluxes to and from this water mass contrôl the heat content. The following section contains a description and magnitude estimation of the following fluxes

- net short wave radiation, F_s
- net long wave radiation, F_L
- sensible heat flux, F_c
- latent heat flux, F_e
- precipitation, F_p
- heat flux from rivers, F_Q
- deep water heat flux, F_{-D}

3.1.1 Short wave radiation, F_s. The magnitude of short wave radiation (F_s) penetrating a surface depends on

- Zenith angle of the sun
- albedo of the surface
- amount of clouds and water vapour in the atmosphere.

The formulation of the short wave radiation through the water surface is, according to Bodin (1979),

$$F_s = (1-\alpha) S_o \cos Z (T_R - A_W) \prod_{i=1}^{3} (1 - N_i (1 - T_i))$$

where

α - albedo of the water surface
S_o - solar constant (2.0 $\mathrm{cal \cdot cm^{-2} \cdot min^{-1}}$)
Z - Zenith angle
T_R - transmission function
$T_R = 1.0141 - 0.16 (\mathrm{SEC}\ Z)^{0.5}$
A_W - absorption by the water content in the atmosphere
$A_W = 0.077 (U \cdot \mathrm{SEC}\ Z)^{0.3}$
where
U - amount of water in the atmosphere
$\mathrm{SEC}\ Z = 1/\cos Z$

The amount of clouds and their height is also affecting the short wave radiation. The "cloud function" T_i is

$T_{LOW} = 0.35 - 0.015\ \mathrm{SEC}\ Z$

$T_{MIDDLE} = 0.04 - 0.01\ \mathrm{SEC}\ Z$

$T_{HIGH} = 0.9 - 0.04\ \mathrm{SEC}\ Z$

N_i is the amount of clouds of the different categories (low, middle, high).

The maximum value of F_s, over the Bay of Bothnia (latitude $\sim 65^o$) during the months Nov-Jan, is less than 25 $\mathrm{cal \cdot cm^{-2} \cdot day^{-1}}$.

3.1.2 Net long wave radiation, F_L. The net long wave radiation consists of two parts. One going from the sea surface to the atmosphere ($F_L\uparrow$) and one from the atmosphere to the sea surface ($F_L\downarrow$).

$$F_L = F_L\uparrow - F_L\downarrow$$

where

$$F_L\uparrow = \varepsilon\sigma T_W^4$$

ε - emissivity of the sea surface (.97)

σ - Stefan - Bolzman´s constant ($1.189 \cdot 10^{-7}$ cal$\cdot$ $\cdot$cm$^{-2}\cdot$ dag$^{-1}\cdot$K^{-4})

T_W - water temperature (K).

The major problem in determining F_L is to get a proper estimation of $F_L\downarrow$.
In this study the formulation from Washington & Semtner (1976) was chosen;

$$F_L\downarrow = \sigma T_A^4 \ (a+b\sqrt{e_A})(1+dN)$$

which follows from Brunt´s formula modified with a cloud factor
where

T_A - air temperature (K)

N - cloudiness in tenths

e_A - atmospheric water vapour pressure (mb)

a = .67

b = .05

d = .25

The net absorbed atmospheric radiation is computed based on a 97% absorptivity of the water surface.

F_L is of the order of 10^2 cal$\cdot$ cm$^{-2}\cdot$ day^{-1}.

3.1.3 Sensible heat flux, F_c. The formulation and the values of constants are taken from Friehe & Schmitt (1976). They used a bulk aerodynamic formula

$$F_c = \rho_a C_p C_c \overline{U}(T_w - T_A)$$

where

ρ_a - air density (1293 g m^{-3})

C_p - specific heat at constant pressure (.241 cal$\cdot g^{-1} \cdot {}^oC^{-1}$)

C_c - sensible heat transfer coefficient ($1.41 \cdot 10^{-3}$)

$\overline{U}$ - mean wind velocity at height h (m s^{-1})

T_w - sea surface temperature (K)

T_A - air temperature at height h (K)

F_c is of the order 10^2 cal$\cdot cm^{-2} \cdot day^{-1}$.

3.1.4 Latent heat flux, F_e. The latent heat flux is also taken from Friehe & Schmitt (1976) using a bulk aerodynamic formula

$$F_e = LC_e\overline{U}(Q_w - Q_A)$$

where

L - latent heat of evaporation (597 cal$\cdot g^{-1} \cdot {}^oC^{-1}$)

C_e - moisture transfer coefficient ($1.32 \cdot 10^{-3}$)

$\overline{U}$ - mean wind velocity at height h (m s^{-1})

Q_w - water vapour density close to the water surface ($g \cdot m^{-3}$)

Q_A - water vapour density at height h ($g \cdot m^{-3}$)

Measurements of Q_w and Q_A are not directly available. Therefore a transformation to the variables of water vapour pressure, e_w and e_A, is done.

$$Q_i = \frac{m_\nu \bullet e_i}{R_u \bullet T_i} \bullet 10^6 \qquad (g \bullet m^{-3})$$

where

m_ν = 18.016 (molecul weight of water vapour)

R_u = $8.314 \bullet 10^4$ (when e is in mb)

e - water vapour pressure (mb)

T - absolute temperature

This leads to the following expressions

$$Q_w = \frac{216.7}{T_w} \bullet e_w$$

$$Q_A = \frac{216.7}{T_A} \bullet e_A$$

Assume that the air close to the sea surface is saturated and has the same temperature as the sea surface. The saturated vapour pressure is a function of temperature.

$$e_w = 6.11\ e^{K(\frac{1}{T_K} - \frac{1}{T_w})}$$

$$e_A = R \bullet 6.11\ e^{K(\frac{1}{T_K} - \frac{1}{T_A})}$$

where

R - relative humidity in percent

The final expression for the latent heat flux, in $cal \bullet cm^{-2} \bullet day^{-1}$, is

$$F_e = P \bullet \overline{U} \bullet \left[\frac{e^{K(\frac{1}{T_K} - \frac{1}{T_w})}}{T_w} - \frac{R \bullet e^{K(\frac{1}{T_K} - \frac{1}{T_A})}}{T_A} \right] \quad (cal \bullet cm^{-2} \bullet day^{-1})$$

where

$P = 9.0 \cdot 10^3$

$K = 5418.1$

$T_K = 273$ (K)

T_w - water temperature (K)

T_A - air temperature (K)

$\overline{U}$ - mean wind velocity at height h ($m \cdot s^{-1}$)

F_e is of the order 10^2 $cal \cdot cm^{-2} \cdot day^{-1}$.

3.1.5 Precipitation, F_p. The heat flux due to precipitation depends mainly on the form (rain or snow). One major problem is that F_p is not a continuous flux. Maximum values of the order of 10^2 ($cal \cdot cm^{-2} \cdot day^{-1}$) appears in heavy snow fall, but most of the time F_p is zero. To get an estimation of the mean value during the months November and December precipitation data over 30 years were used. The mean value of the precipitation over these two months is 100 mm. The maximum heat flux arises when all this precipitation is snow, due to latent heat of melting. F_p (max) is then approximately 15 $cal \cdot cm^{-2} \cdot day^{-1}$.

3.1.6 Heat flux from rivers, F_Q. The fresh water flow from rivers to the Bay of Bothnia is, according to Dahlin (1976), 116.6 km^3 per year. Assume that this fresh water flow is equally spread over the whole sea surface area (36260 km^2). In late autumn the maximum temperature difference between the sea water and river water is approximately $5^{\circ}C$. The heat flux will then be 5 $cal \cdot cm^{-2} \cdot$ $\cdot day^{-1}$.

3.1.7 Deep water heat flux, F_{-D}. The deep water heat flux is very difficult to estimate. It varies a lot from year to year depending on the varying heat content in the deep water and the intensity of mixing processes. A way to make a rough estimation of F_{-D}, as a yearly mean value, is to use water balance considerations. Assume as a mean over the year, the salinity content to be constant in the whole basin. In order to keep the salinity constant in the upper layer there is a vertical water flux transporting salt and heat from below. According to a water balance consideration made by Fonselius (1971) the flux F1 (see figure 3) is 393 km^3/year and according to Dahlin 246 km^3/year. To get an estimation of the maximum mean vertical transport the value according to Fonselius have been used. Assume that all of F1 is going through the interface (A) down to the bottom water creating a circulation pattern shown in figure 3.

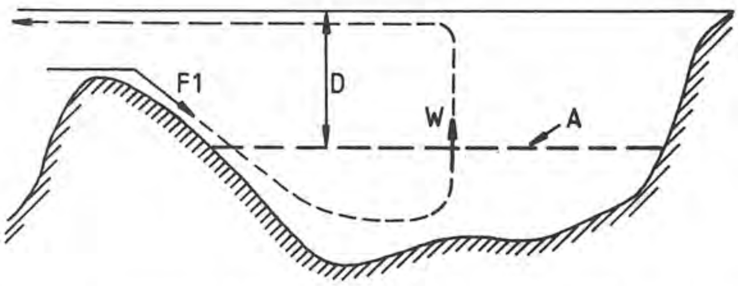


Figure 3 *A schematic figure showing the mean vertical circulation in the Bay of Bothnia. F1 is the water transport from the Sea of Bothnia. D, W and A are the mixed layer depth, vertical velocity and the interface area.*

Assume the following relationship between F1 and w.

$$F1 = w \cdot A$$

where

A is the total interface area between the two layers; $\sim 1500\ km^2$

w is the mean vertical velocity through the interface

$$w = \frac{F1}{A} = 7\ (cm \cdot day^{-1})$$

Typical temperature difference between surface and bottom layer, in November and December, is 2-4°C. As a mean then F_{-D} varies from 10-30 $cal \cdot cm^{-2} \cdot day^{-1}$.

3.2 Summary of the heat fluxes

A summary of the heat fluxes and their magnitude over the Bay of Bothnia, for the months November and December is shown in table 1.

Table 1

	HEAT FLUXES						
	F_s	F_L	F_c	F_e	F_p	F_Q	F_{-D}
MAGNITUDE $cal \cdot cm^{-2} \cdot day^{-1}$	~ 10	$\sim 10^2$	$\sim 10^2$	$\sim 10^2$	~ 10	~ 10	~ 10

The conclusion from table 1 is that the dominating heat fluxes over the Bay of Bothnia, for the months November and December, are

- net long wave heat flux, F_L
- sensible heat flux, F_c
- latent heat flux, F_e.

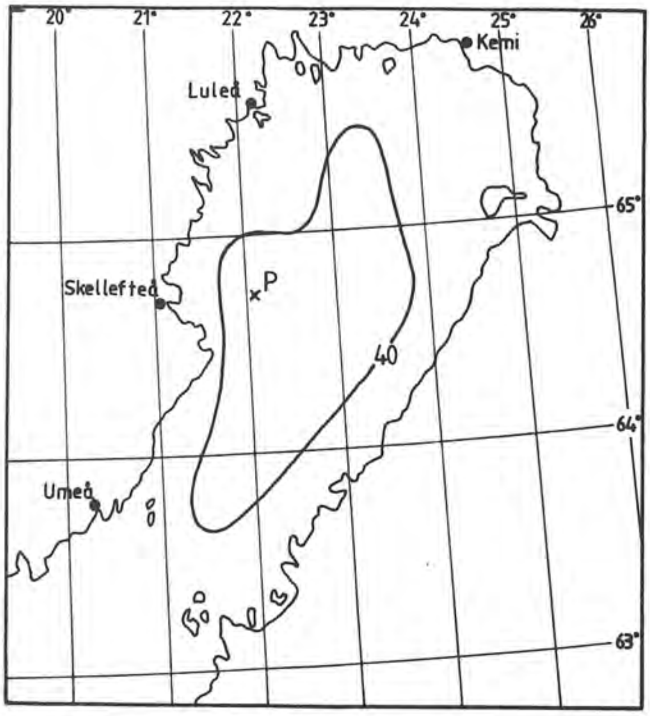
3.3 The mixed layer D

Once a month, the Swedish Coast Guard make oceanogrphical observations in P (figure 4). The temperature and salinity are measured at different depths from the surface down to 100 m. Measurements from several years show, in late autumn, a well mixed layer down to 40 m. An example of the changes in the temperature- and σ_t (2) profiles during three different observations is shown in figure 5.

Is this mix-layer depth found in P, representative for the whole central basin? Observations, made by the Finnish research vessel Aranda, in the beginning of November 1970 indicates a mean mixed layer depth of 40-50 m in the whole basin (Makkonen 1977). Therefore, a constant mixed layer down to 40 m in late autumn seems to be a reasonable assumption.

Note (2) Definition $\sigma_t = (\rho_w - 1) \cdot 10^3$ e.g. another way to express the water density.

Figure 4
A map over the Bay of Bothnia. The 40 m depth curve is drawn in the figure. P is the position where the Swedish Coast Guard make oceanographical observations.



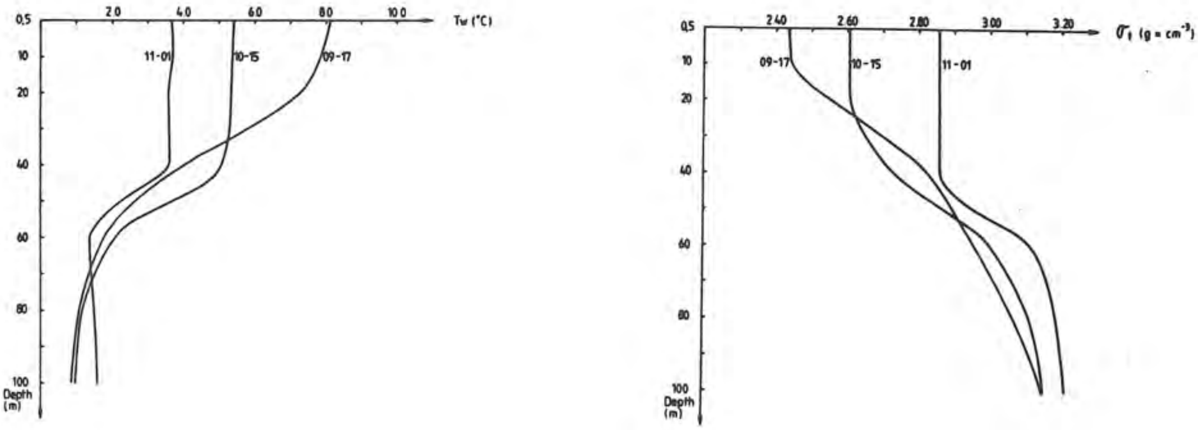


Figure 5 *The figure shows the temperature (to the left) and density profiles according to three different measurements at station P (see figure 4) the year 1973.*

3.4 The heat conduction equation

Changes in temperature of a body, for example water, can be described by heat fluxes to and from the body. The relation between temperature and heat flux is called the heat conduction equation (1).

$$\frac{dT}{dt} = - \frac{1}{\rho C} \nabla \bullet F \qquad (1)$$

where

T - temperature of the body

ρ - density of the body

C - specific heat of the body

F - heat flux vector

Consider the water mass in the Bay of Bothnia in late autumn. Using the following two assumptions

- the Bay of Bothnia consists of two layers, where the upper layer of depth D is well mixed
- no horizontal heat flow to or from the upper layer

equation (1) will be modified to

$$\frac{\partial T_w}{\partial t} = - \frac{1}{\rho_w C_w} \frac{\partial F}{\partial Z} \qquad (2)$$

where

T_w - water temperature of the upper layer

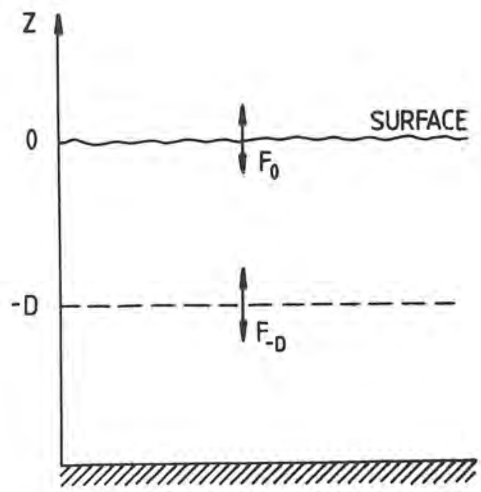
ρ_w - water density of the upper layer

C_w - specific heat of water

F - heat flux

Integrate equation (2) from the depth - D to the surface

$$\int_{-D}^{0} \frac{\partial T_w}{\partial t} dZ = - \frac{1}{\rho_w C_w} \int_{-D}^{0} \frac{\partial F}{\partial Z} dZ$$



$$\frac{\partial T_w}{\partial t} = - \frac{1}{\rho_w C_w D} (F_o - F_{-D}) \qquad (3)$$

where

F_o is the sum of fluxes through the water surface

- net short wave radiation, F_s
- net long wave radiation , F_L
- sensible heat flux , F_c
- latent heat flux , F_e
- precipitation , F_p
- heat flux from rivers , F_Q

F_{-D} is the heat flux through the interface.

In chapter 3.2 there is a summary of the magnitudes of these fluxes. According to this summary there are three dominating fluxes, F_L, F_c and F_e. The final formulation of equation (3) in this study is

$$\frac{\partial T_w}{\partial t} = - \frac{1}{\rho_w C_w D} (F_L+F_c+F_e) \qquad (3)$$

4. DATA

Meteorological and water temperature data has been collected from two periods. From November 1 1972 until March 3 1973 and November 3 1973 until February 1 1974. These two periods were chosen because they showed a considerable difference in the rate of water cooling. By using a mild period as a basic data sample for establishing relations between meteorological data and the rate of water cooling and afterwards using the chilly period as a test period, the result ought to be significant from a statistical point of view.

Mean rate of cooling	
72/73	73/74
$.056^{\circ}C$/day	$.080^{\circ}C$/day

Water temperature data has been extracted from analysed water temperature maps (figure 6). These maps are based on measurements from coastal stations and ships. The maps contain information from two days which means that the time resolution in the water temperature data is two days. Sometimes the analysis is based on less than 5 measurements and sometimes 20-30 measurements. Consequently the quality of data varies from one day to the other but in the long run or as a mean the cooling is well described. Temperature observations by ships, are taken at a depth of 5-6 m. Due to the assumption of a constant mixed layer of 40 m the observations are interpreted as representative in a water column from the surface down to 40 m.

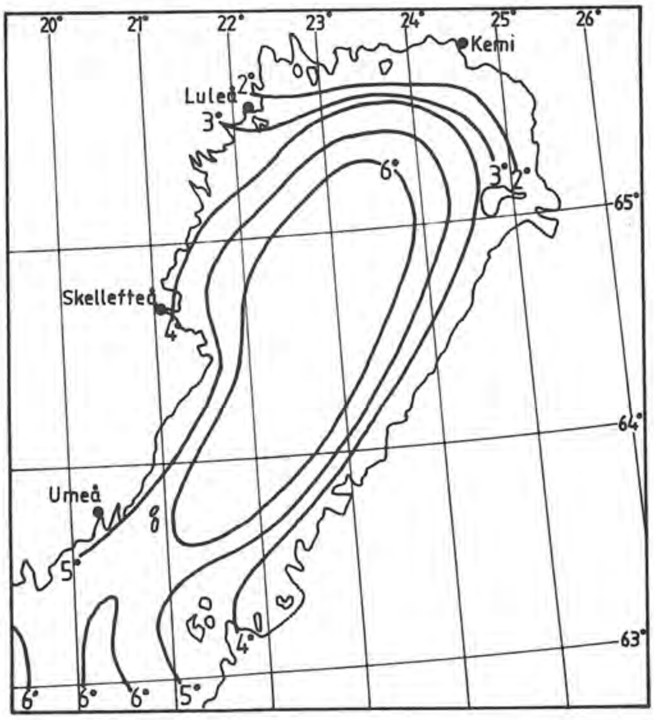


Figure 6 *This figure is a part of an analysed water temperature map. It shows the analysed water temperature in the Bay of Bothnia 1972-11-01.*

The water mass under consideration (M) is restricted to be inside the volume limited by the 40 m depth curve and its intersection with the bottom (figure 7).

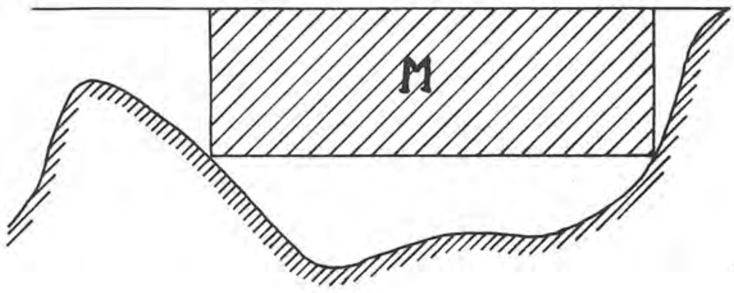


Figure 7 *The figure shows the water mass (M) over which the mean water temperature is calculated.*

When examining a lot of analysed maps of surface water temperature in the Bay of Bothnia a striking structure in the pattern of isotherms appears. To a large extent the isotherms seem to be well correlated with the bottom topography. In order to describe the temperature distribution in a convenient way it was assumed that the temperature distribution at a certain instant t can be separated into:

$$T_W(x,y,z,t) = \tau(t) \cdot F(x,y,z)$$

Since we earlier made an assumption of a constant mixing layer and a restriction of the considered water mass we find:

$$T_W(x,y,z,t) = \tau(t) \cdot F(x,y)$$

This separation seems to be a rather good assumption on the sample we have used.

Our problem of describing the temperature distribution at a certain instant t is now limited to find $\tau(t)$, which is easy:

$$\iiint_V T_w(x,y,z,t)\,dV = \iint_A T_w(x,y,t)\,dA = \iint_A \tau(t)F(x,y)\,dA$$

i.e.:
$$\tau(t) = \frac{\iint_A T_w(x,y,t)\,dA}{\iint_A F(x,y)\,dA}$$

where:

$$\iint_A F(x,y)\,dA = A$$

i.e. the quantity $\tau(t)$ represent a volume weighted mean temperature of the sea. In the following the denotation T_w is used but it should be referred as τ.

Meteorological data. In order to determine the heat fluxes in the system sea - atmosphere, using the earlier mentioned equation 3, values of the air temperature, wind velocity and a measure of cloudiness in the area of Bothnian Sea had to be known.

The only archived data base which could provide this necessary information was the plotted and by meteorologists subjectively analysed weather maps. From such a map an extraction of temperature and surface pressure was made manually. Besides, these data were interpolated to a grid net according to the present grid net used in operational numerical weather forecasts, see figure 8. The reason for extracting pressure information is of course that wind systems on the scale we are interested in may very well be described by the pressure pattern, and more important, it is the only way today and in the nearest future to make forecasts of the wind.

The cloudiness was extracted as an average in 4 regions. The uncertainty in this parameter is of course larger than in the other because of the "sometime discontinuous character" in the cloudiness. Lack of observations in the sea area had to be compensated with a physical insight in the nature of cloudiness, depending upon the weather situation given by the land observations. No classification of the cloudiness depending on altitude was made. It is important to point out that this is probably a rough estimate of cloudiness, which is possible to predict by using numerical weather forecasts.

In order to describe the diurnal variation of the weather parameters it was necessary to extrapolate data for every 6th hour 00, 06, 12, 18 GMT. This tremendous amount of data (44.720 values) were digitized and carefully controlled in a computer.

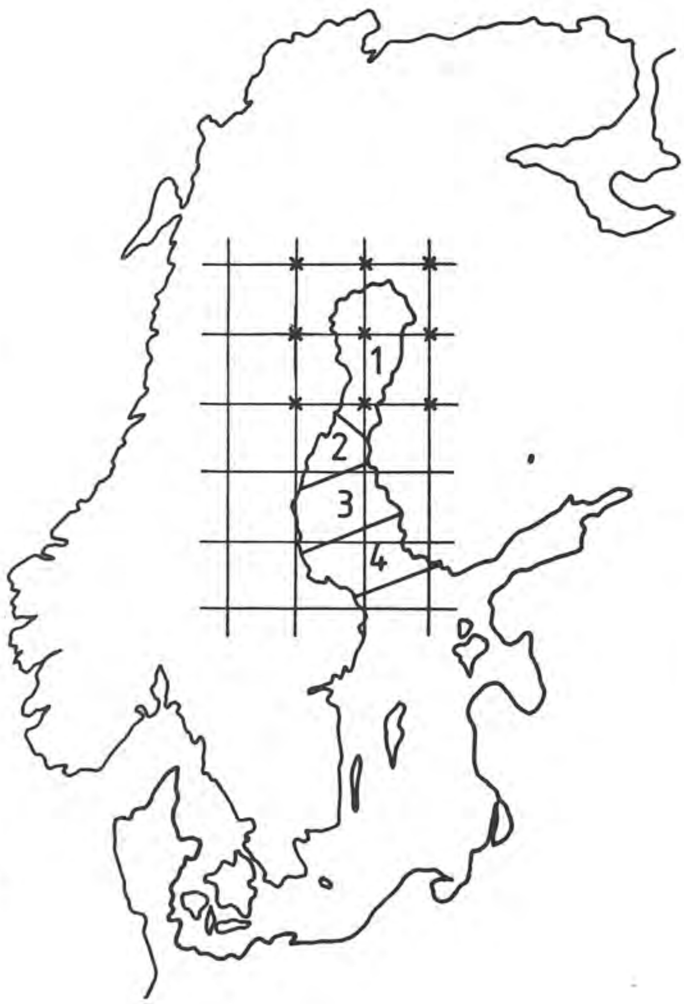


Figure 8
The meteorological parameters were extracted according to the shown grid net. Figures 1-4 describe the regions over which the cloudiness was estimated. The nine grid points marked with crosses are the points where mean values of air temperature and gradients in the pressure field were calculated.

5. CALCULATION

For both cooling periods, winter 72/73 and 73/74, equation 3 was used with following parameters,

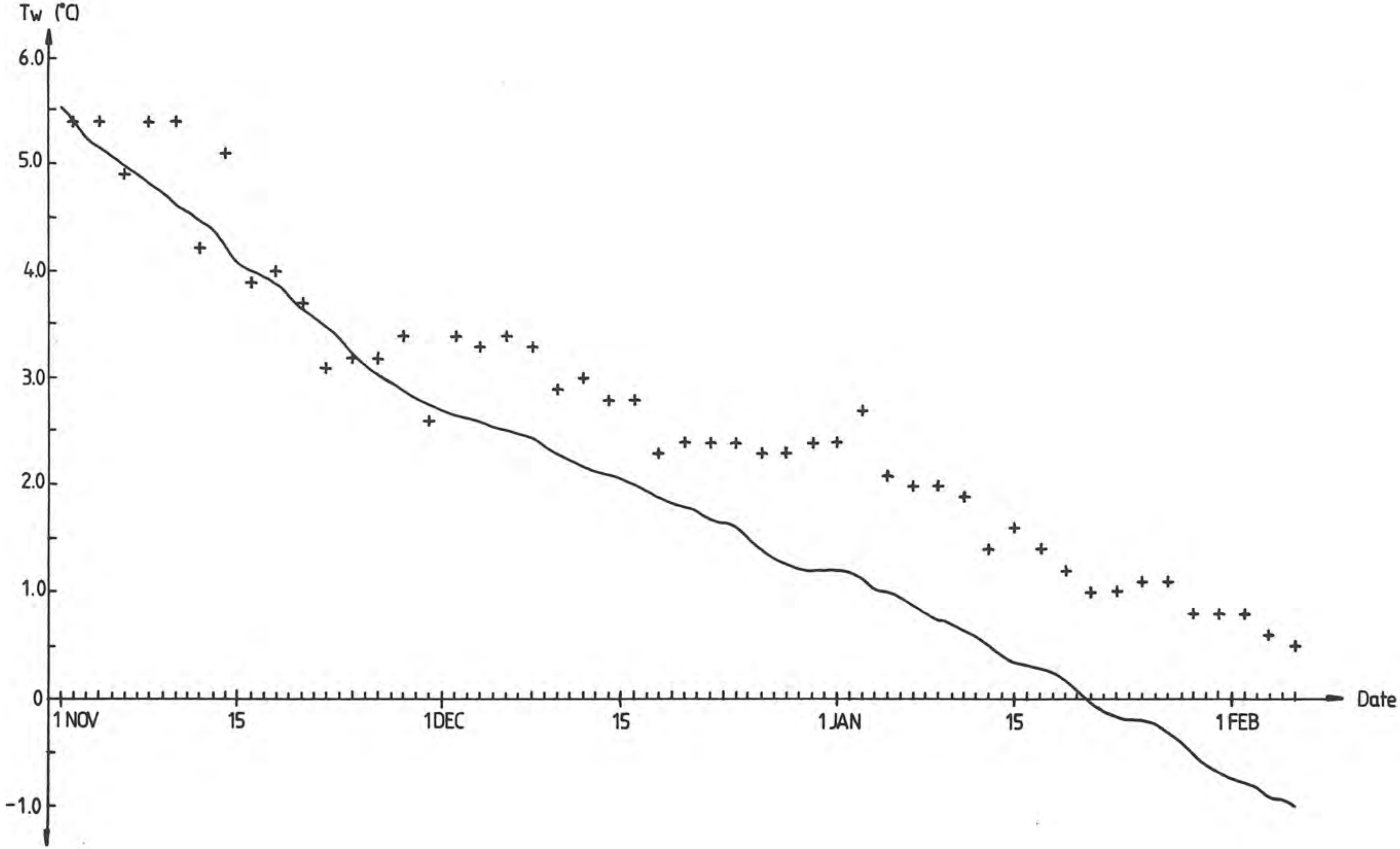
- timestep = 1 day
- air temperature and geostrophic wind determined as areal mean based on 9 respectively 12 grid points (see figure 8) were comprised to daily mean quantities (denotation T_A,V).
- relative air humidity assumed to be constant = 90%
- length of forecast: 1972/73 = 98 days, 1973/74 = 48 days[1)]

The results are given in figure 9 (a and b). The drawn lines represent the calculated temperatures for each time step. Crosses represent the water temperature determinated from the analysis mentioned earlier.

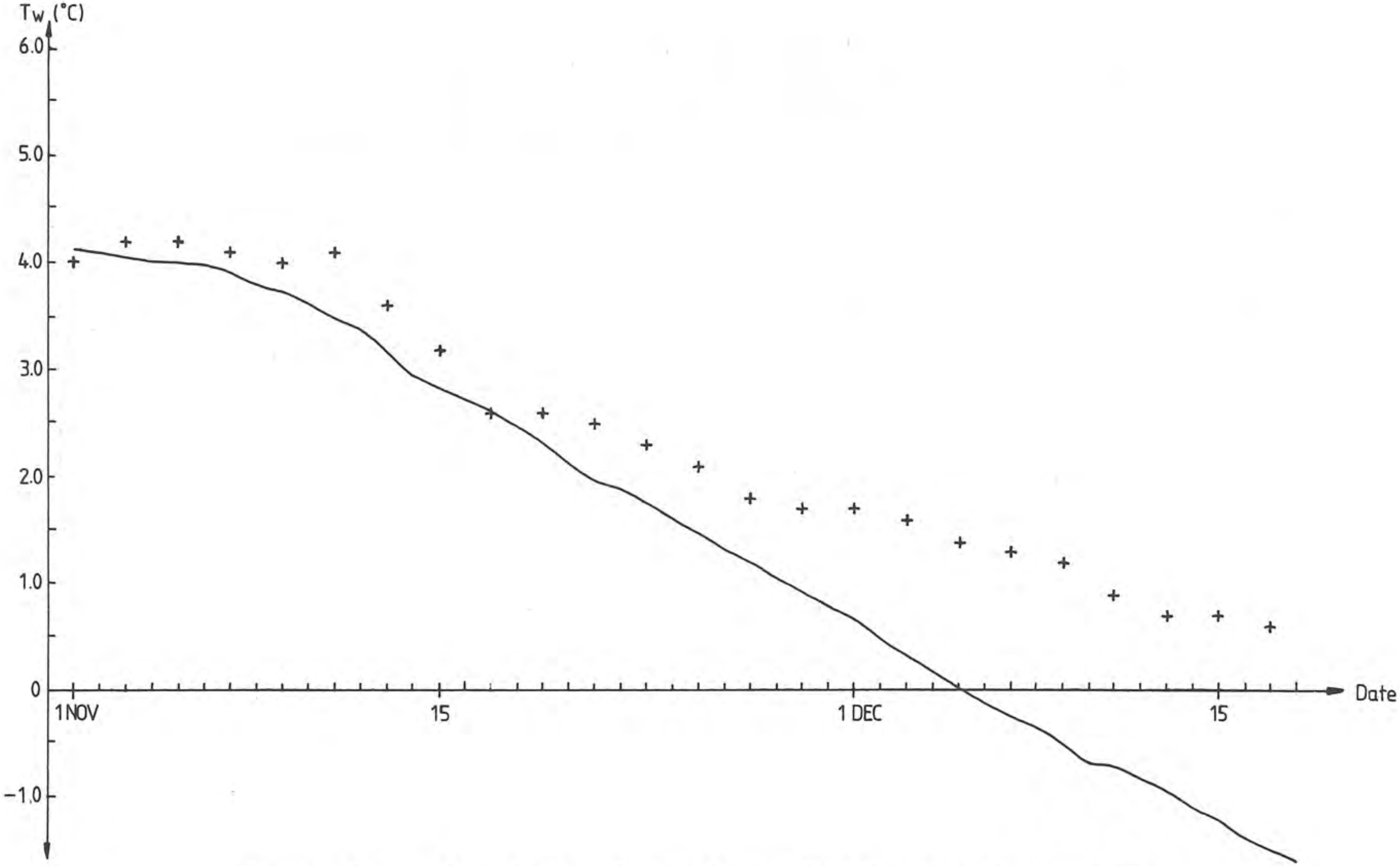
It is a rather good agreement between calculated and analysed temperature at the beginning of the seasons. In 1972/73 the analysed temperature in the beginning of the season is based on a few observations and it is hard to say if the plateau between Nov 1 and Nov 10 really did exist.

After approximately 20 days there is a plain divergence between the curves in both seasons. The modelled temperature decreases too fast compared to reality.

1) The accuracy of the analysed water temperature determined at an arbitrary instant t depends of course upon the number of observation which the analysis is based on, but roughly it is of the order 10^{-1} °C. The average cooling rate is as we already have seen approximately 10^{-1} °C/2 days. So the accuracy of the analysis is of the same order as the expected error. By making the forecast as long as possible we minimize the relative error in the analysis.



<u>*Figure 9a*</u> *The figure shows the analysed (+) and calculated (the drawn line) water temperature during the cooling period 1972/73.*



<u>Figure 9b</u> *The figure shows the analysed (+) and calculated (the drawn line) water temperature during the cooling period 1973.*

In 1972/73 we can distinguish between two different periods in the cooling of the sea. First period until Nov 21 shows a rather fast decrease while the next period until Feb 5 represent a cooling rate of approximately 1/3 of the first period. This main feature is also present in the calculated cooling.

The divergence between the curves seem to be caused by the incapacity of the model to describe the plateaus between Nov 21 to Dec 7 and Dec 17 to Jan 2.

In 1973/74 the model seems to be able to describe the first plateau between Nov 1 to Nov 11 but again the divergence between the curves starts at the second plateau after Nov 17.

Let us examine the potential sources of error in order to make approaches for improving the model. These are;

- Unsufficient description of the true heat fluxes.

  The heat fluxes has been discussed in chapter 3.2 and 3.3. Although we know very little about the deep water heat flux (F_{-D}), there were no extreme conditions during the periods where modelled and observed temperature diverge indicating the importance of the neglected fluxes.

- Incorrect treatment of the equations for heat fluxes.

  Those equations are intended to be used under homogeneous conditions i.e. giving a momentanuous flux from the ocean surface in a specific point using information about oceanographical and meteorological parameters in that point and in the vertical above and below.

  What would be the consequences when, as in our case, the input parameters are averaged in the time domain and in the space.

A well known theorem known as Jensen's inequalaty tells us:

If the function $g(x)$ is convex in the interval I where $X_K \varepsilon I$ and λ_K is a non-negative number so that $\Sigma\lambda_K = 1$ then:

$$g(\Sigma\lambda_K x_K) \leq \Sigma\lambda_K g(x_K)$$

where the summation is taken from 1 to n.

If we define $\lambda_K = \frac{1}{n}$ we find that

$$g(\overline{x}) \leq \overline{g(x)}$$

In the formulation of the fluxes we have used, we find:

$$F_L \sim T_A^{\ 4}$$

$$F_E \sim \overline{U}\, e^{T_A}$$

$$F_C \sim \overline{U}\, T_A$$

We see that F_L is a convex function of T_A.

By making an expansion into empirical orthogonal functions:

$$T_A(t+\tau) = \sum_{n=1}^{N} \alpha_n(t)\; h_n(\tau) \qquad \tau = 6,12,18,24 \text{ hours}$$

$$\overline{U}(t+\tau) = \sum_{n=1}^{N} \alpha_n(t)\; \gamma_n(\tau) \qquad N = 8$$

it was found that for the first 4 terms in the series expression $\frac{dh_n(\tau)}{d\tau} \sim K \frac{d\gamma_n(\tau)}{d\tau}$ when K is a non-negative constant. Those first 4 modes in the series expression explaned 97% of the total variation in T_A and $\overline{U}$ during the two periods. So as a good assumption we also regard F_C and F_E as convex functions of T_A (or $\overline{U}$).

The conclusion is that the total heat flux (F_L+ $+F_e+F_c$) is a convex function of T_A. Making time and areal averages of T_A, in this case daily mean values, will always give a total heat flux that is smaller than using momentanuous values of T_A. The overrated cooling in the model is therefore not due to averaging procedure, this would rather give an opposit effect.

- Not representative meteorological parameters.

  a) Wind
  The total heat flux increases with increasing wind. By comparing the computed geostrophic wind with observed winds at three coastal stations no evidence was found that the computed wind was too high.

  b) Cloudiness
  The total heat flux increases with decreasing cloudiness. It is well known by meteorologists that a weather observator has a tendence of overestimate the cloud coverage (coulisse-effect).

  Since the measure of cloudiness is based on observations made from the earth surface, this would indicate that the outgoing long wave radiation in the model is underestimated, rather than overestimated.

  c) Air temperature
  The total heat flux increases with decreasing air temperature. The analysis of T_A is mainly based on observations over land. These observations are affected by the temperature of the earths surface. The surface temperature, during this part of the year, depends mainly upon the outgoing radiation and the heat conduction. Examining the vertical air temperature profiles made at the radiosond station at Luleå (measured every 12th hour) during the two seasons 72/73

and 73/74 it was found that inversions occured in nearly 50% of all cases. This is also verified by the "Handbook of climatology for Defence" (1975). The mean temperature profile for the whole season 72/73 showed an inversion where the temperature increased $0.5^{\circ}C$ from the surface up to 100 m.

The conclusion of the previous discussion must be that the most probable error source is the description of the air temperature above the sea.

To correct for this, the following assumption was made: Equation 3 is a correct description of the cooling but among the observed parameters T_A is incorrect.

With regard to this assumption equation 3 can be solved in order to find the air temperature T_A' which would give the observed cooling. Equation 3 is a fourthorder equation in T_A. It was solved in an iterative way.

Also assume that this relationship is valid

$$T_A'(t) = f(\overline{U}(t),\ T_A(t),\ T_W(t),\ N(t))$$

Several empirical formulations were tested and the most successful was a linear expression:

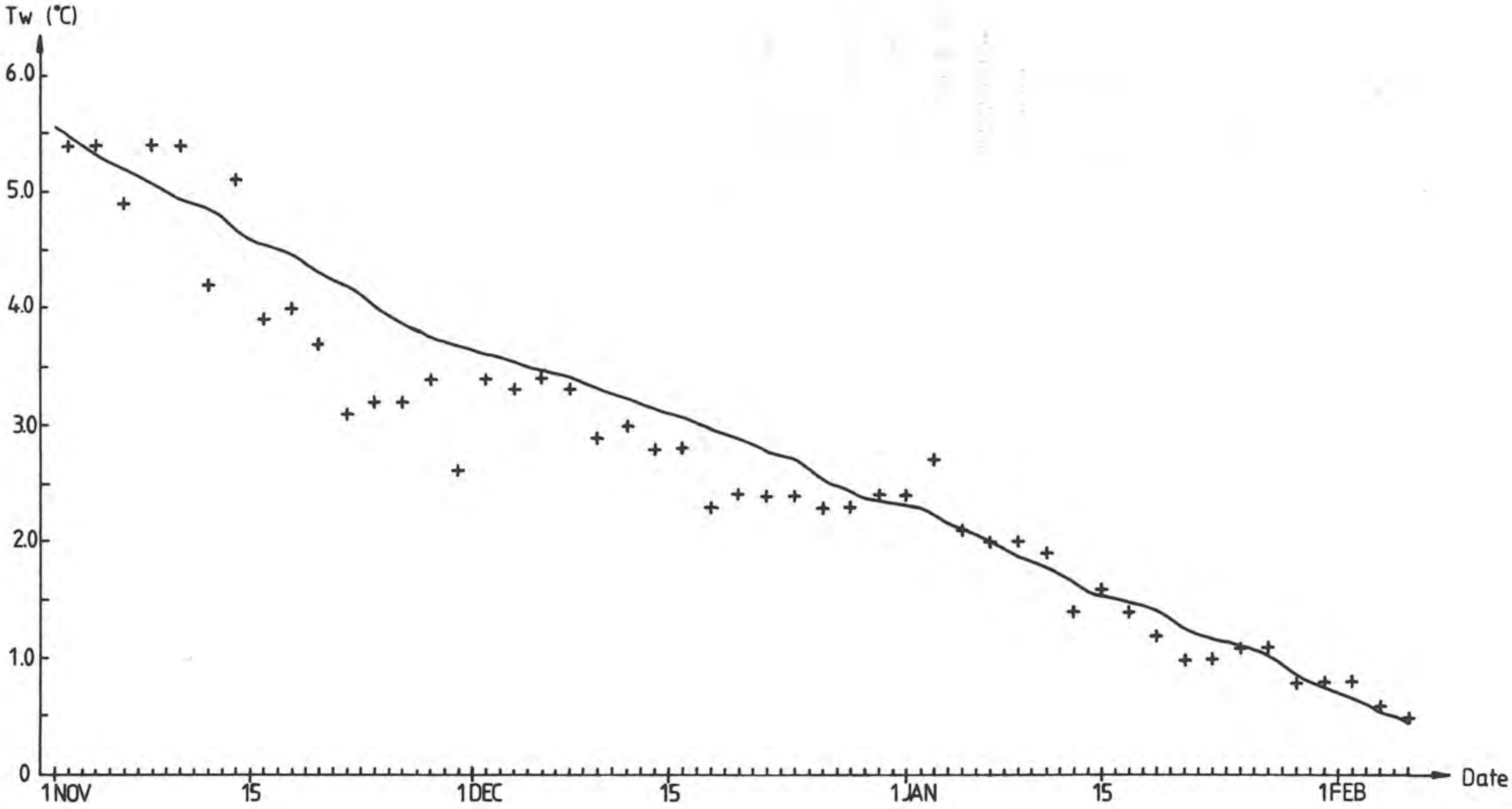
$$T_A'(t) = T_A + K\cdot(T_W - T_A) \qquad (4)$$

The value of the constant K was determined on the data base from the period 1972/73 and found to be

K = 0.44

6. RESULT

Equation (3) was then solved with the air correction (4) and the result is given in figure 10 a. Also the cooling during 73/74 was determined with the same constant K. The result from the period 73/74 is shown in figure 10 b. With this treatment of T_A the calculated cooling agrees very well with reality. This agreement is remarkable as the only time the analysed water temperature is used in the calculations is as an initial value November 1. However, the small deviations are not well described. As we have said before (see chapter 4) the quality of the analysed water temperature data varies a lot from day to day so it is hard to say if these small deviations really do exist. If they do exist the only term in equation 3 which could explain these deviations are variations in the mixed layer depth (D). These variations have not been investigated depending upon the lack of hydrographical data.



<u>*Figure 10a*</u> *The figure shows the analysed (+) and calculated (the drawn line) water temperature during the cooling period 1972/73.*

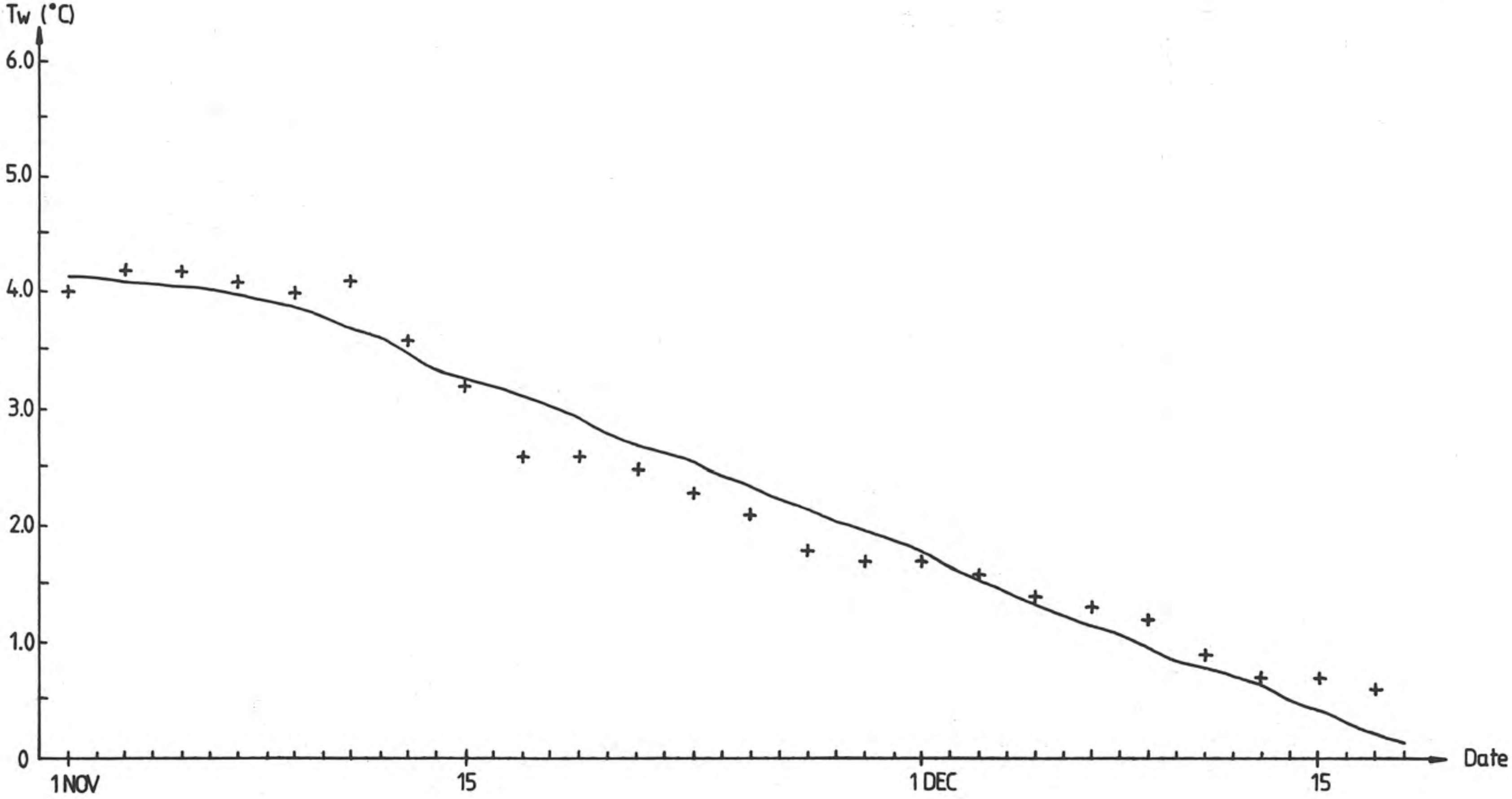


Figure 10b *The figure shows the analysed (+) and calculated (the drawn line) water temperature during the cooling period 1973.*

7. SUMMARY AND CONCLUSION

This report describes a method to compute the large scale water cooling in the Bay of Bothnia for the months November - January as a function of meteorological parameters. During these months the water temperature normally decreases from 5°C to 0°C. The definition of water is - the mean water temperature in the central basin from the surface down to the depth D, where D is the depth of the mixed layer. Hydrological measurements have shown that the mean mixed layer depth is approximately 40 m.

From the heat conduction equation, an expression describing the changes in water temperature as a function of heat fluxes is derived. An estimation of the magnitudes of the different heat fluxes shows that there are three dominant fluxes, during the months November - January,

- net longwave radiation
- sensible heat flux
- latent heat flux.

Two different cooling periods have been investigated. In the first period the water temperature, November 1, was 5.6°C and the cooling was very slow with a water temperature of .8°C on February 1. In the second period the water temperature, November 1, was 4.2°C and for this period the cooling was rapid with a water temperature of .7°C on December 13.

The meteorological data, required for solving the heat fluxes (air temperature, air pressure and cloudiness) has been extracted in grid points from analysed weather maps. The water temperature data are calculated from analysed water temperature maps.

Solving equation 3 with a timestep of one day and the assumption of a constant mixed layer and only using the analysed water temperature as an initial value the calculated cooling for the two periods agrees very well with the analysed water temperature (see figure 10 a and b).

This is remarkable as the cooling periods are 98 and 48 days long.

The conclusion is that it is possible to calculate the large scale cooling in the Bay of Bothnia using daily mean values of the air temperature, wind speed and cloud coverage and assuming a constant mixed layer depth of 40 m.

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