A SENSITIVITY ANALYSIS OF STEADY, FREE FLOATING ICE

EN KÄNSLIGHETSANALYS AV STATIONÄR, FRIFLYTANDE IS

by Anders Omstedt

SMHI Rapporter
METEOROLOGI OCH KLIMATOLOGI

Nr RMK 19 (1980)

SMHI

Sveriges meteorologiska och hydrologiska institut

A SENSITIVITY ANALYSIS OF STEADY, FREE FLOATING ICE

EN KÄNSLIGHETSANALYS AV STATIONÄR, FRIFLYTANDE IS

by Anders Omstedt

SMHI Rapporter
METEOROLOGI OCH KLIMATOLOGI

Nr RMK 19 (1980)

ISSN 0347-2116

Norrköping 1980

ABSTRACT

The equation for steady, free floating ice is derived and analysed for a shallow sea. The analysis treats how accurate the free ice drift can be computed when variations in the ingoing parameters are introduced. Besides errors due to unperfect winds, areas with large currents cause bad accuracy. If further more the bottom depth is neglected in these areas the accuracy become worse. Variable ice roughness and variable friction velocity introduce errors which are less important but still noticable in the computed ice drift.

SAMMANFATTNING

Ekvationen för stationär, friflytande is härleds och analyseras för ett grunt hav. Analysen behandlar hur noggrant den stationära, friflytande isens rörelse kan beräknas då variationer införs i ingångsparametrarna. Förutom fel på grund av icke perfekta vindar, orsakar områden med starka strömmar dålig noggrannhet. Om dessutom bottendjupet försummas i dessa områden blir noggrannheten sämre. Varierande isskrovlighet och varierande friktionshastighet medför fel som är mindre viktiga men ändock märkbara i den beräknade ishastigheten.

TABLE OF CONTENT

Nomenclature

C	Coriolis force
C^a	Wind stress coefficient
C^w, D^w	Water stress coefficients
C^w_∞	Water stress coefficient in a deep sea
$C^{w'}_\infty$	Water stress coefficient in a deep sea without a logarithmic layer under ice surface
D_E	Ekman depth
f	Coriolis parameter
G	Gravitational force
H	Bottom depth
H/D_E	Normalized bottom depth
h^a	Logarithmic height
h^i	Mean ice thickness
h^w	Logarithmic depth
i	$\sqrt{-1}$
K^a	Turbulent exchange coefficient in the air
K^w	Turbulent exchange coefficient in the water
m^i	Ice mass
N	Ice concentration
$\Delta S/L$	Water level slope
U^a_*	Air friction velocity
U^w_*	Water friction velocity
W^a	Wind velocity
Wg^a	Geostrophic wind velocity
W^i	Ice drift velocity
W^w	Current velocity
W^w_g	Geostrophic current velocity
$\left\lvert \frac{W^i}{W_g{}^a} \right\rvert$	Geostrophic wind factor

$\left\lvert \frac{W_g^{\ w}}{W_g^{\ a}} \right\rvert$	Geostrophic current factor
$z_o^{\ a}$	Ice roughness height parameter
$z_o^{\ w}$	Ice roughness depth parameter
$\theta_g^{\ a}$	Geostrophic wind direction, towards which the wind is blowing
θ^i	Ice drift direction
$\theta_g^{\ w}$	Geostrophic current direction
κ	von Karmans constant
ρ^a	Air density
ρ^i	Ice density
ρ^w	Water density
τ^a	Air stress
τ^w	Water stress
$\lvert\ \rvert$	Absolute value

1. INTRODUCTION

In sea ice dynamic modelling work several approximations and parameter choices have to be done before ice drift can be computed, errors are therefore introduced and an important question is to what degree this influence the computed ice drift itself? One way of answering this question is to observe and analyse ice drift situations and study the discreapancy between model and field data. Another more idealized way is to study how sensitive the ice drift equations are for variations in the entering parameters.

In this paper the second method is used for the case when the ice is non-accelerating and free floating in a shallow sea like the Gulf of Bothnia. This means that the acceleration force and the force due to internal friction in the ice are omitted and just variations in air stress, water stress, coriolis force and gravitational force due to tilting sea surface are considered. The analysis first treats a case called the standard case which assumes constant stress parameters, a deep sea and negligible currents. Secondly the stress parameters are varied between estimated extreme values and the currents are negligible. This case is called the stagnant sea case. Thirdly the importance of currents on non-accelerating and free floating ice is studied. This case is called the current case.

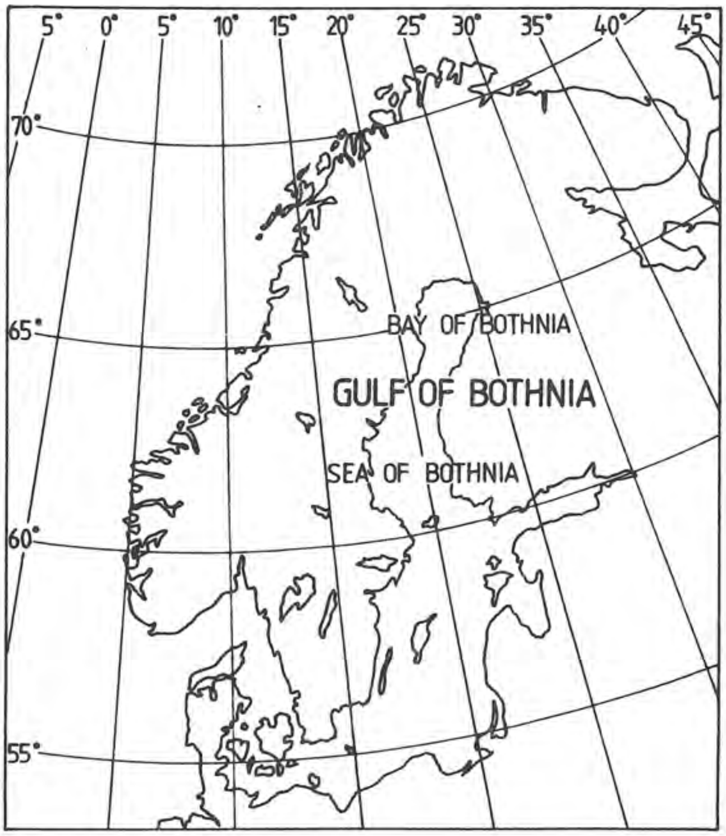


Figure 1. Map of Scandinavia with surrounding waters.

2. FREE ICE DRIFT EQUATION

When acceleration and internal friction are negligible, the ice drift equations become:

$$\tau^{a} + \tau^{w} + C + G = 0 \qquad (1)$$

where τ^{a} is the wind stress, τ^{w} the water stress, C the coriolis force and G the gravitational force due to tilting surface.

All forces in equation (1) are external forces written in complex form and the equation is called the free ice drift equation. In figure 2 a schematic sketch of free ice drift is shown. The figure illustrates how the wind and the current affects the ice and how the ice and the bottom depth modify the wind and current profiles.

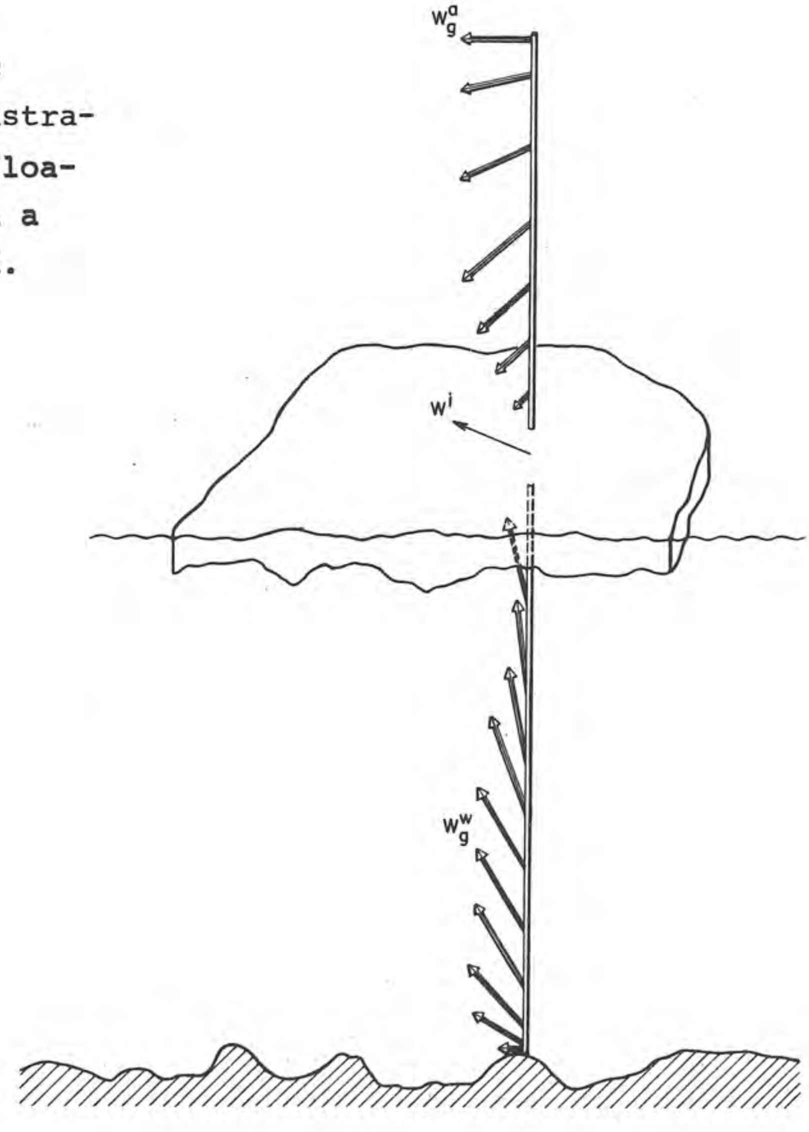


Figure 2. A schematic sketch illustrating free floating ice in a shallow sea.

Even if the internal friction and the acceleration of the ice are important, it seems possible to isolate ice drift periods where the internal forces and the acceleration are less important, McPhee (1977). In the Gulf of Bothnia this is the case when the ice concentration is less than 80%, Leppäranta (1979), and the time scale is larger than a few hours, Udin and Omstedt (1976). For a sensitivity analysis the free ice drift equation is also a good starting point as the ice drift mostly never exceed the free ice drift.

Different ice drift models use different explicit expressions for the external forces. In this analysis a logarithmic - Ekman approach is used which gives linear stress expressions. The explicit form of the external forces are;

$$\tau^a = C^a W_g{}^a \tag{2}$$

$$\tau^w = C^w (W_g{}^w - W^i) + D^w W_g{}^w \tag{3}$$

$$C = - i\, m^i\, f\, W^i \tag{4}$$

$$G = i\, m^i\, f\, W_g{}^w \tag{5}$$

where m^i is the ice mass, $W_g{}^a$ is the geostrophic wind, $W_g{}^w$ is the geostrophic current and W^i the ice velocity all written in complex form.

The complex constant C^a depends mainly on roughness height on upper ice surface ($Z_o{}^a$), air friction velocity ($U_*{}^a$) and logarithmic height (h^a). The complex constants C^w and D^w depends mainly on roughness depth on lower ice surface ($Z_o{}^w$), water friction velocity ($U_*{}^w$), logarithmic depth (h^w) and bottom depth (H). The derivation of all forces are given in Appendix A.

With above expressions equation (1) can be written;

$$C^a W_g{}^a + C^w (W_g{}^w - W^i) + D^w W_g{}^w + i\, m^i\, f\, (W_g{}^w - W^i) = 0 \tag{6}$$

which are made non-dimensional by dividing the equation with geostrophic wind.

$$\frac{W^i}{W_g{}^a} = \frac{C^a + (C^w + D^w + i\, m^i\, f)\, W_g{}^w / W_g{}^a}{C^w + i\, m^i\, f} \tag{7}$$

The absolute values and arguments of the two non-dimensional velocities can be written in complex form as:

$$\frac{W^i}{W_g^a} = \left|\frac{W^i}{W_g^a}\right| e^{-i(\theta_g^a - \theta^i)} \quad ; \quad \frac{W_g^w}{W_g^a} = \left|\frac{W_g^w}{W_g^a}\right| e^{-i(\theta_g^a - \theta_g^w)}$$

where θ_g^a, θ^i and θ_g^w is the geostrophic wind direction, the ice drift direction and the geostrophic current direction. The absolute values of the non-dimensional velocities are called the geostrophic wind factor $|W^i/W_g^a|$ and the geostrophic current factor $|W_g^w/W_g^a|$. The arguments are called the geostrophic wind deflecting angle $(\theta_g^a - \theta^i)$ and the geostrophic current deflecting angle $(\theta_g^a - \theta_g^w)$ respectively. Equation (7) is the starting point for the sensitivity analysis.

3. SENSITIVITY ANALYSIS

3.1 General remarks

Free ice drift can be calculated from equation (7). Though several simplifications in the derivation of the external forces are made, the free ice drift depends on two velocities (W_g^a, W_g^w), ten parameters which enter the stress coefficients (Z_o^a, U_*^a, h^a, Z_o^w, U_*^w, h^w, H, f, ρ^a, ρ^w) and the ice mass (m^i). All varies and some can be chosen in many ways.

The sensitivity analysis is therefore concentrated essentially to three problems:

1. How should a proper parameter choice be made?
2. How does ice velocity change when parameters vary between estimated extreme values?
3. Which parameters influence the free ice drift most?

These problems are treated in the following chapters.

3.2 Standard case

To make a proper stress parameter choice the stagnant sea case for a deep sea with constant stress parameters is first studied. The stagnant sea assumption means that currents are negligible. Absolute value and argument of equation (7) therefore become;

$$\begin{cases} \left|\dfrac{W^i}{W_g{}^a}\right| = \dfrac{|C^a|}{|C^w + i\ m^i\ f|} & \text{a)} \\ \theta_g{}^a - \theta^i = \arg(C^w + i\ m^i\ f) - \arg(C^a) & \text{b)} \\ C^j = \dfrac{\rho^j \sqrt{if\ K^J}}{1+|h^J|\ln|h^J/Z_o^J|\sqrt{if/K^J}} \qquad J = a \text{ or } w & \text{c)} \end{cases} \tag{8}$$

From this set of equations one notices that geostrophic wind factor and deflecting angle only depend on ice mass and stress parameters. Before solving the equations, stress parameters must be chosen in a proper way. If the ice roughness is first analysed a reasonable assumption is that, taken as a mean, the ice is in hydrostatic balance. If this assumption also is converted to roughness height ($Z_o{}^a$) and roughness depth ($Z_o{}^w$) which not has to be true, see chapter 3.31, the relation between roughness parameters is;

$$Z_o{}^w \sim \frac{\rho_i}{\rho_w - \rho_i} * Z_o{}^a \tag{9}$$

where $\sim$ stands for the same order of magnitude.

Field measurements in the Gulf of Bothnia, Udin and Omstedt (1976), has shown that the air and water stresses were of the same magnitude in the case of free floating ice. According to the definition of friction velocity it follows;

$$U_*{}^w \sim \sqrt{\frac{\rho^a}{\rho^w}} * U_*{}^a \tag{10}$$

The logarithmic height (h^a) and depth (h^w) in the neutral case can according to Tennekes (1973) and equation (10) be written as;

$$h^a \sim 0.03\ U_*{}^a/f \tag{11}$$

$$h^w \sim 0.03 \sqrt{\frac{\rho^a}{\rho^w}}\ U_*{}^a/f \tag{12}$$

If air density (ρ^a), water density (ρ^w), ice density (ρ^i) and coriolis parameter (f) are treated as constants, the stress parameters are now reduced to depend just on two parameters, roughness height (Z_o^a) and air friction velocity (U_*^a) which here are treated as independent variables. Physically this is not true, because air friction velocity depends on ice roughness. This will however be considered later in chapter 3.32.

Equations (8) are now solved with constants given in table 1 and equations (9-12). The solution for the geostrophic wind factor and deflecting angle becomes 2.0% and 14.5^o respectively.

Table 1 Constants used in the standard case

Constant	Standard value	Unit
f = Coriolis parameter	0.00013	s^{-1}
ρ^a = Air density	1.3	$Kg\ m^{-3}$
ρ^i = Ice density	917.	$Kg\ m^{-3}$
ρ^w = Water density	1003.	$Kg\ m^{-3}$
κ = von Karman´s constant	0.4	--
Z_o^a = Roughness height	0.001	m
U_*^a = Air friction velocity	0.25	$m\ s^{-1}$
m^i/ρ^i = Ice volume	0.5	m^3

Before comparing these values with field data geostrophic wind has to be reduced to the standard meteorological height of 10 meters. According to values during ice condition in the Baltic, Joffre (1978), geostrophic wind should be reduced by a factor 0.766 and turned 17.1 degrees to left. This gives 2.6% in the wind factor and 31.6 degrees in the deflecting angle. The calculated wind factor is higher than those measured at the ice-breaker TOR during the SEA ICE -75 experiment, Udin and Omstedt (1976). That could be due to influence from the ship and the fact that the icebreaker´s anemometer was placed 24 meters above the ice. The deflecting angle is in accordance with data from the above mentioned field experiment but is larger than the angle reported by Leppäranta (1979). As there are several possible ways to choose stress parameter values and reach the same solution, an order of magnitude check of the friction coefficients has also been made. Force balance calculations on ice drift in the Bay of Bothnia, Udin and Omstedt (1976), has shown that wind stress and water stress are of the same order of magnitude and one order larger than coriolis force for free ice drift. With the same notations as in chapter 2 this means that;

$$|\tau^a|/|\tau^w| \sim 1 \quad \text{or} \quad |C^a|/|C^w| \sim |W^i/W_g{}^a|$$

$$|\tau^a|/|C| \sim 10 \quad \text{or} \quad |C^a| \sim 10m^i f|W^i/W_g{}^a|$$

With constants according to the standard case the magnitude of the stress coefficients become

$$|C^a| \sim 10^{-2} \quad \text{and} \quad |C^w| \sim 5\cdot10^{-1}$$

These estimated values are compared with corresponding calculated values. From equation (8c) the friction coefficients in the air and the sea can be calculated. With values according to table 1 this gives;

$$|C^a| = 9.4\cdot10^{-3} \quad \text{and} \quad |C^w| = 0.44$$

which agrees well with above estimated values. It seems therefore that parameters are chosen in a proper way. It must be pointed out that different values of the stress coefficients are reached when square law stresses are used, McPhee (1977).

3.3 Stagnant sea case

The normalized free drift equations for a shallow sea with negligible currents become;

$$\left\{\begin{aligned}
&\left|\frac{W^i}{W_g{}^a}\right| = \frac{|C^a|}{|C^w+im^i f|} \\
&\theta_g{}^a - \theta_i = \arg(C^w+im^i f) - \arg(C^a) \\
&C^a = \frac{\rho^a \sqrt{ifK^a}}{1+h^a \ln(h^a/Z_o{}^a)\sqrt{if/K^a}} \\
&C^w = \frac{\rho^w \quad \sqrt{ifK^w}}{\tanh(\sqrt{if/K^w}(|H|-|h^w|))+|h^w|\ln(|h^w/Z_o{}^w|)\sqrt{if/K^w}}
\end{aligned}\right. \tag{13}$$

In the stagnant sea case this equations are studied for different parameters and with the relations between air and sea stress parameters given by equations (9-12). Although the parameters are reduced the geostrophic wind factor $|W_i/W_g{}^a|$ and the geostrophic deflecting angle $(\theta_g{}^a - \theta_i)$ still depends on upper ice surface roughness $(Z_o{}^a)$, air friction velocity $(U_*{}^a)$, ice mass (m^i) and bottom depth (H). In this chapter they are treated one by one by letting each parameter vary between estimated extreme values, see table 2, while the other parameters are constants and the same as in the standard case.

Table 2 Intervals used in the stagnant sea case.

Constant		Standard value	Range	Unit
$Z_o{}^a$	= Roughness height	0.001	0.0001-0.01	m
$U_*{}^a$	= Air friction velocity	0.25	0.1-1.0	ms^{-1}
$H/D_e{}^w$	= Normalized depth	1.0	0.1-2.0	---
m^i/ρ^i	= Ice volume	0.5	0.05-1.0	m^3

3.31 Ice roughness

From measurements in the atmosphere several values of ice surface roughness height $(Z_o{}^a)$ has been reported. The scatter in the data are large even if measurements are carried out over same ice type. According to Ling and Untersteiner (1974) the reported average roughness heights in the Arctic vary between $2 \cdot 10^{-4}$ and $3.5 \cdot 10^{-3}$ meters. This interval is probably much larger, particular as the atmosphere measurements in general are performed on rather level ice where the average roughness height ought to be small. The estimated interval is therefore taken to $(10^{-4}-10^{-2})$ meter, with a mean roughness of 10^{-3} meter. According to equation (9) this corresponds to a roughness depth $(Z_o{}^w)$ ranging from $(10^{-3}-10^{-1})$ meter, an interval which is in accordance with roughness depths measured below different ice types in the Gulf of St. Lawrence, Johannessen (1970). If also a thumbrule is used, which states that the roughness height is equal to about 1/30 of the height of the dominant roughness elements, Untersteiner and Badgley (1965), the corresponding upper surface heights ranges from $(3 \cdot 10^{-3}-3 \cdot 10^{-1})$ meter and the corresponding lower surface depths ranges from $(3 \cdot 10^{-2}-3)$ meter. This seems to be resonable values even for the Gulf of Bothnia, why above estimated roughness intervals are used below.

As the roughness parameters probably not are in hydrostatic balance, at least not on level ice, they are first treated as independent variables. With air roughness heights ($Z_o{}^a$) ranging from 10^{-4} to 10^{-2} meters and water roughness depths ranging from 10^{-3} to 10^{-1} meters, equations (13) are computed with all other parameters according to standard values, see table 1 and 2. The solutions are illustrated in figure 3.

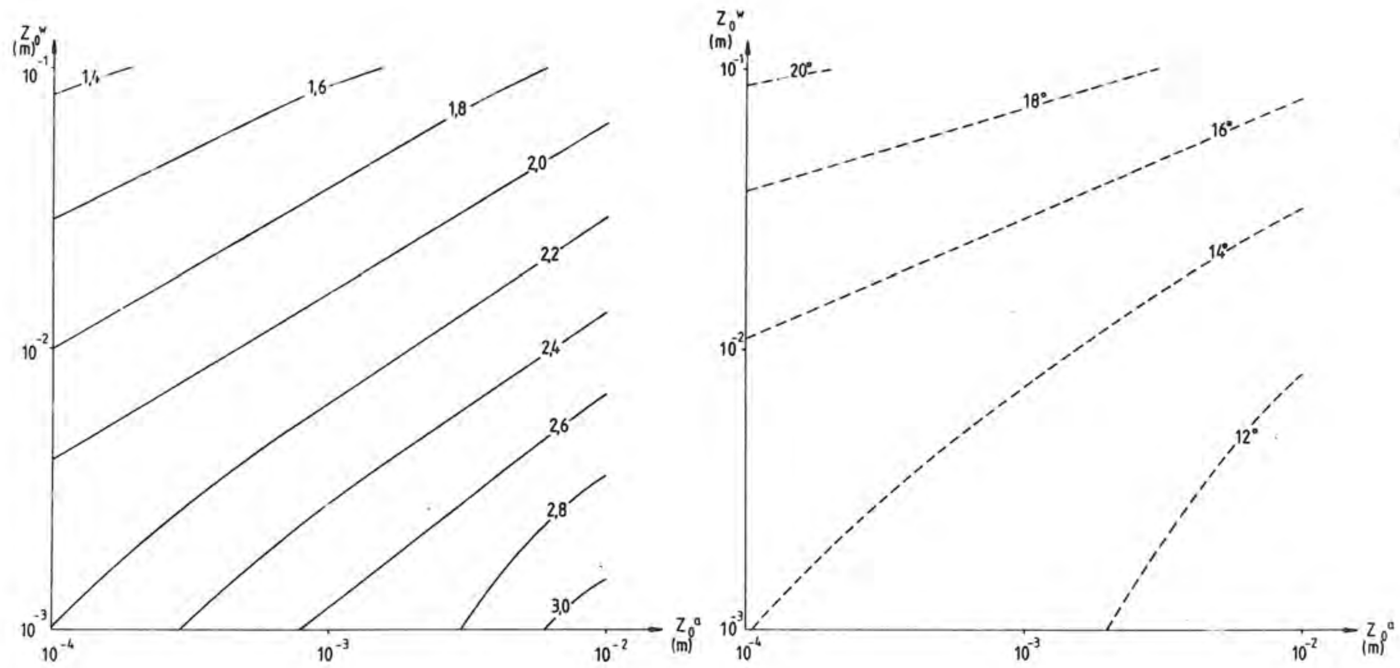


Figure 3. Geostrophic wind factors (fully drawn lines) and geostrophic deflecting angles (dashed lines) computed for different air roughness heights ($Z_o{}^a$) and different water roughness depths ($Z_o{}^w$).

The solutions of equations (13) particularly deviate from the standard case when air roughness heights are large and water roughness depths are small and also when air roughness heights are small and water roughness depths are large. This two ice situations probably never exist, why expected sensitivity for different ice roughness values should become smaller than figure 3 indicates.

If hydrostatic balance between the roughness parameters are assumed, water roughness depth is not independent to air roughness height. With equation (9) and a variable roughness height, the solutions of equation (13) deviate less from the standard case, see figure 4. The geostrophic wind factor and the geostrophic deflecting angle could now be calculated with an accuracy of (2.0 ± 0.2) % respectively (14.5 ± 2.5) degrees. The deviations from the standard case are therefore rather small. This is due to the fact that a high roughness height leads to a high roughness depth which counter act each other when free ice drift is calculated. The analysis indicates that variations in ice roughness over above estimated interval do not influence free ice drift in any drastic way.

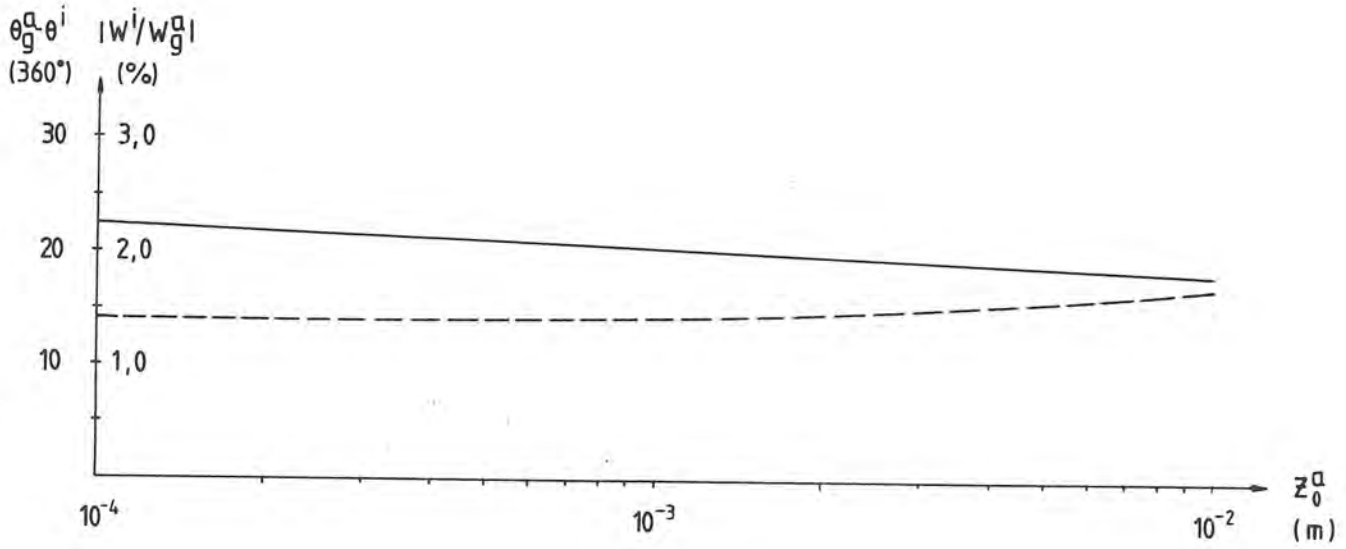


Figure 4. Geostrophic wind factor (fully drawn line) and geostrophic deflecting angle (dashed line) as a function of ice surface roughness height (Z_o^a).

3.32 Friction velocity

The range over which air friction velocity (U_*^a) vary also determines the ranges over which water friction velocity (U_*^w), logarithmic height (h^a) and logarithmic depth (h^w) vary according to equation (10), (11) respectively (12). In the atmosphere a reasonable interval for air friction velocity seems to be (10^{-1}-1) meter per second which according to equation (11) gives a logarithmic height interval of (23 - 230) meters. The corresponding water friction velocity and logarithmic depth intervals then become ($4\cdot10^{-3}$ $-4\cdot10^{-2}$) meter per second and (0.8 - 8) meters respectively, intervals which are in good agreement with experimental data found in the Bay of Bothnia and the Arctic. The solutions of equations (13) with an air friction velocity which varies in the interval (10^{-1} - 1) meter per second and with all other parameters according to standard values are illustrated in figure 5.

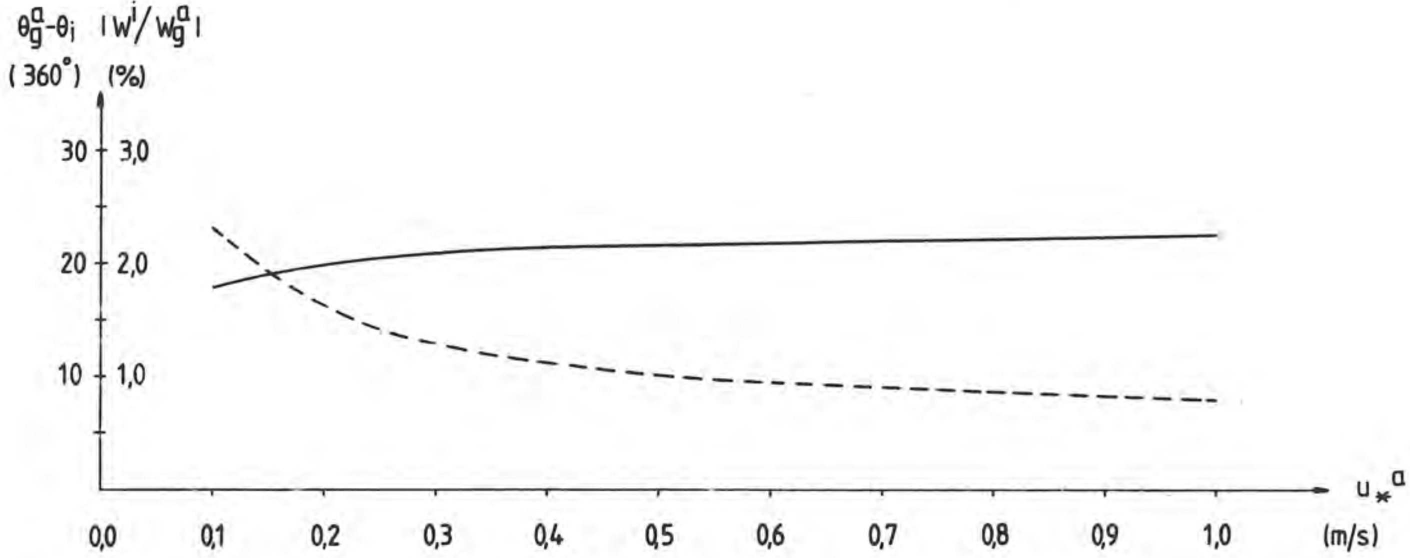


Figure 5. Geostrophic wind factor (fully drawn line) and geostrophic deflecting angle (dashed line) as a function of air friction velocity (U_*^a).

Although air friction velocity varies over this rather large interval geostrophic wind factor and geostrophic deflecting angle could be calculated with an accuracy of (2.0 ± 0.2) % respectively (14.5 ± 8.8) degrees. This is a surprisingly high accuracy if one considers that the only driving force in the stagnant sea case is air stress. The reason for this high accuracy is that a changing air friction velocity is counteracted by a changing water friction velocity. The air friction velocity is however not independent of ice surface roughness. For a given wind speed the logarithmic wind law shows that if roughness height become larger, air friction velocity become larger. This has an interesting implication for free ice drift. From figures 4 and 5 can be seen that an increasing roughness height work in the opposite way to an increasing air friction velocity. This means that if air friction velocity should be treated as a function of roughness height, variations in free ice drift for different air friction velocities should be reduced. It seems therefore justified to draw the conclusion that for an air friction velocity which varies in the interval (0.1 - 1) meter per second the accuracy is at least (2.0 ± 0.2) % and (14.5 ± 8.8) degrees in geostrophic wind factor respectively geostrophic deflecting angle. This indicates that variations in air friction velocity do not influence free ice drift in any drastic way.

3.33 Ice mass

Ice mass (m^i) is determined by ice concentration (N), mean ice thickness (h^i) and mean ice density (ρ^i).

$$m^i = \rho^i N h^i$$

In general ice drift models treat mean ice density as a constant but ice concentration and mean ice thickness as variables. Equations (13) are now solved with parameters according to standard values but with variable ice mass. The solutions are illustrated in figure 6. The figure shows decreasing geostrophic wind factor and increasing geostrophic deflecting angle for increasing ice mass. This indicates that geostrophic wind factor should be higher in the Gulf of Bothnia than in the Arctic and that geostrophic deflecting angle should be smaller in the Gulf of Bothnia than in the Arctic. This was also reported by Leppäranta (1979).

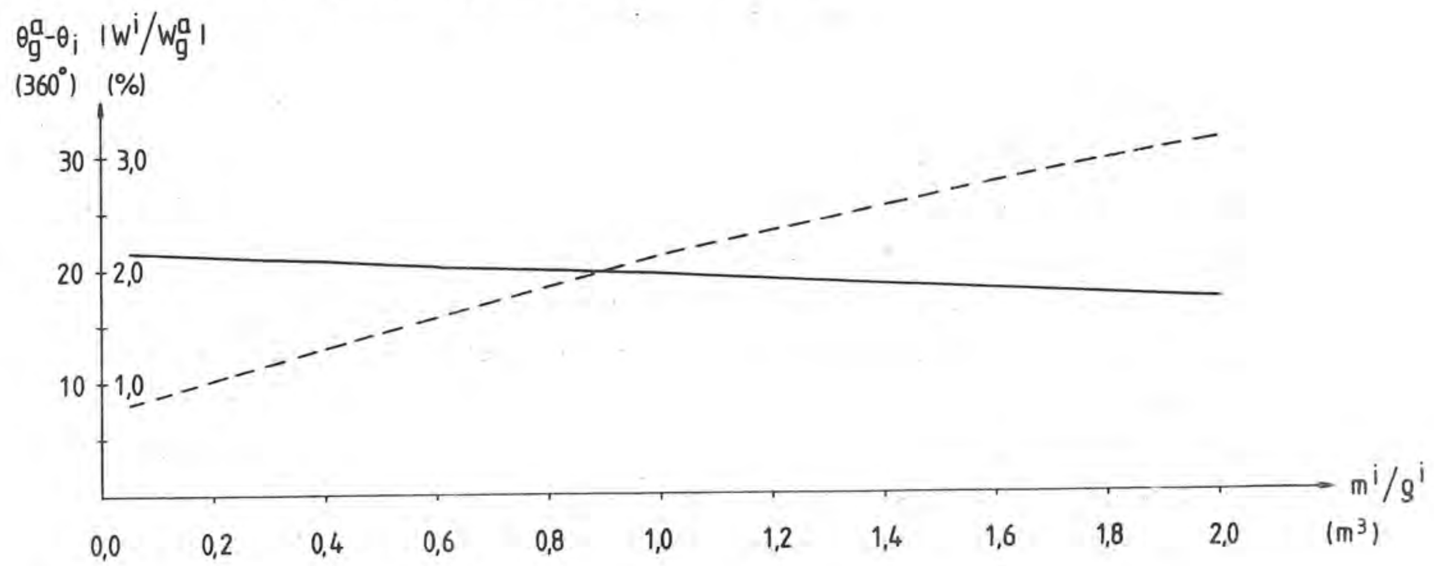


Figure 6. Geostrophic wind factor (fully drawn line) and geostrophic deflecting angle (dashed line) as a function of ice volume (m^i/ρ^i).

As ice mass is an important input data in all forecasting models it is interesting to notice that if the ice mass is determined with an accuracy of $\pm$ 50% in the interval given by figure 6, geostrophic wind factor and geostrophic deflecting angle could be calculated with an accuracy of at least $\pm$0.2% and $\pm$ 10.4 degrees respectively. This is comparable with errors introduced because of constant roughness height and constant air friction velocity.

3.34 Bottom depth

The bottom depth enters the problem in the water stress coefficients because of the requirement that current must be zero at the bottom, see Appendix A2. How the water stress coefficients depend on bottom depth is solved for variable normalized bottom depth (H/D_E^W), where the Ekman depth in the water (D_E^W) is defined according to;

$$D_E^W = \pi \cdot \sqrt{\frac{2\kappa U_*^W |h^W|}{f}}$$

All other parameters are according to standard values.

The solution is shown in figure 7. The water stress coefficients are of the same order of magnitude but almost opposite directed for small depth. When normalized depth becomes larger the C^W-coefficient turns to a depth independent value, which is the same value as if a deep sea was assumed. The D^W-coefficient turns to zero.

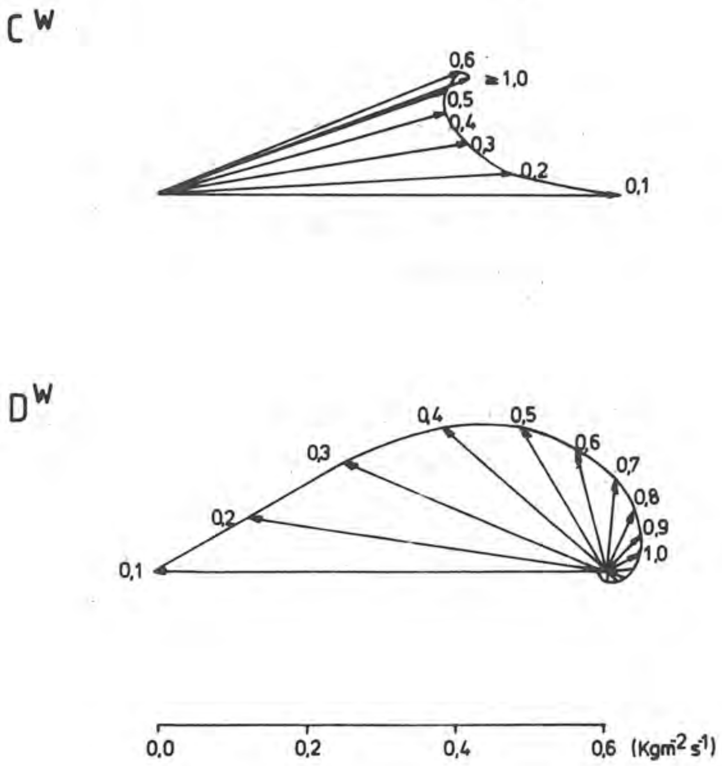


Figure 7. Water stress coefficients as functions of normalized depth. Numbers at arrow tips are normalized bottom depth.

In the stagnant sea case the only water stress coefficient entering is C^W, see equation (3), which means that:

a) the free ice drift in the stagnant case ought to have a maximum at a normalized depth little less than 0.5 because here the C^W-coefficient is minimum.
b) the bottom dependence ought to be more pronounced in the current case because then D^W enters the water stress formula.

How sensitive free ice drift is in the stagnant sea case for a normalized bottom depth which varies is shown in figure 8. The largest changes in geostrophic wind factor and deflecting angle are for small depths, at normalized depths greater than 1.0 free ice drift is independent of depth. A maximum free ice drift is also seen at a normalized bottom depth little less than 0.5, which is due to the water stress minimum mentioned above. As ice probably never is free floating in waters with normalized bottom depths less than 0.3, which for an Ekman depth of 30 meters corresponds to a water depth of

9 meters, the sensitivity in this shallow region could be ignored. In the stagnant sea case the accuracy for normalized bottom depth variations from 0.3 is for geostrophic wind factor and geostrophic deflecting angle (2.1 $\pm$ 0.1) % respectively (10.4 $\pm$ 5.3) degrees. Free ice drift in a stagnant sea is therefore not so sensitive to bottom depth variations compared to variations in roughness height or air friction velocity.

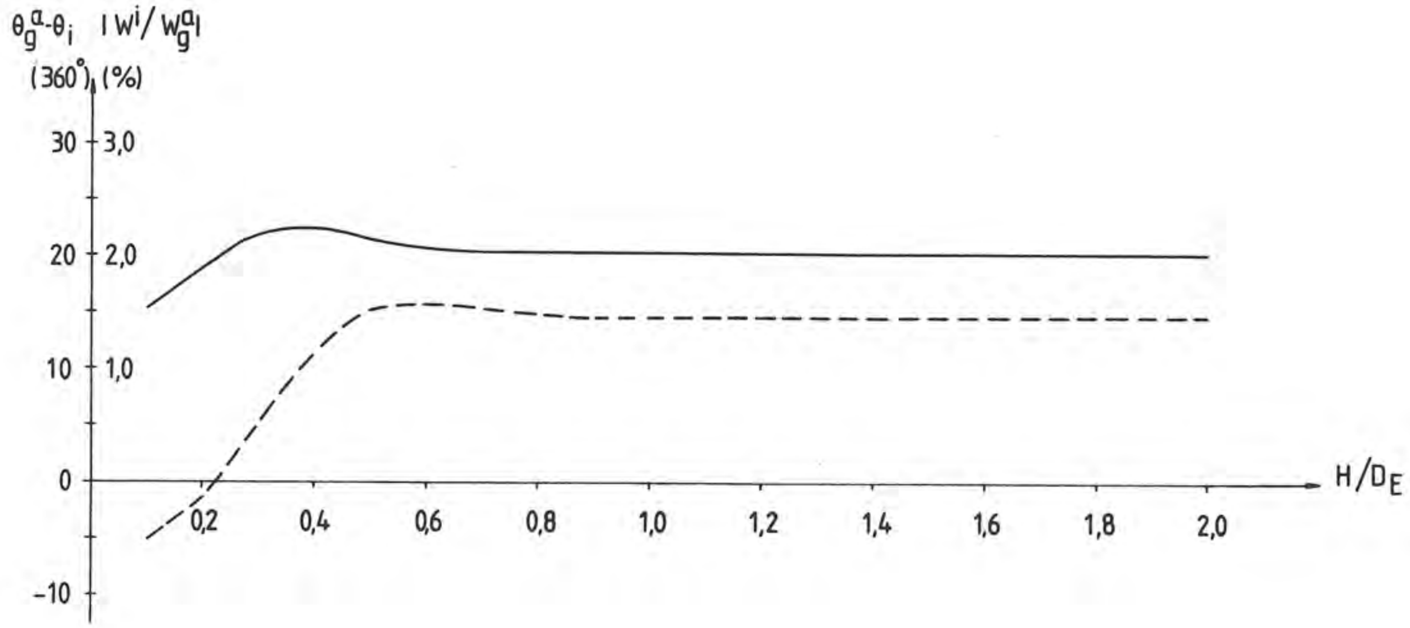


Figure 8. Geostrophic wind factor (fully drawn line) and geostrophic deflecting angle (dashed line) as a function of the normalized bottom depth (H/D_E).

3.4 Current case

Ice drift models generally treat currents in a very simple way, for example when Hibler III (1979) calculated ice drift in the Arctic Ocean he used fixed geostrophic currents, Udin and Ullerstig (1976) assumed negligible geostrophic currents in the Gulf of Bothnia and Doronin (1970) did the same for the Kara Sea. According to McPhee (1977) the effects of geostrophic ocean currents, on short time scales as days, are probably small near the center of the Beaufort Gyre in the Arctic Ocean. However in constrictions like the Bering strait currents are of great importance for ice drift through the strait, Pritchard et al (1979).

Current and density observations in the Gulf of Bothnia during wintertime are rather sparse. Measurements in the Bay of Bothnia, Omstedt and Sahlberg (1977), have shown that the currents under steady or nearly steady drifting ice are directed to the right of the ice drift with current speeds of the same magnitude in the whole water column. Temperature and salinity measurements have shown a well mixed layer down to about 40 meters under the ice. Under this well mixed layer both temperature and salinity formed a weak stratification which however not seemed to influence the current profiles. Currents during wintertime in the Gulf of Bothnia seems therefore to be mainly ice and wind driven. This mechanical forcing sets up horizontal pressure gradients, which cause currents. If one assume homogeneous water during wintertime, the geostrophic current speed could be estimated with following formula:

$$|W_g{}^w| \sim \frac{g\Delta S}{fL}$$

where $|W_g{}^w|$ is the geostrophic current speed, ΔS is the difference in the water level over some characteristic length L. The problem is however to determine the sea surface slope $(\Delta S/L)$ in the sea.

A general idea is however that water level slopes are larger near the coast and larger in constrictions than in the open sea. It seems therefore adequate, in a sensitive analysis, to distinguish between areas where the horizontal pressure gradients are small as in the basins where typical geostrophic current values ought to be 10^{-2} m/s and areas where the horizontal pressure gradients are larger as in the Quarks and probably as in the coastal areas outside fast ice where typical geostrophic current values ought to be 10^{-1} m/s. If the currents not are negligible, the free ice drift equations become:

$$\begin{cases} \left|\dfrac{W^i}{W_g{}^a}\right| = \dfrac{|C^a+(C^w+D^w+im^i f)W_g{}^w/W_g{}^a|}{|C^w+im^i f|} \\[2ex] \Theta_g{}^a-\Theta^i = \arg(C^w+im^i f)-\arg(C^a+(C^w+D^w+im^i f)W_g{}^w/W_g{}^a) \end{cases} \qquad (14)$$

$$
\begin{cases}
C^a = \dfrac{\rho^a\sqrt{ifK^a}}{1+h^a\ln(h^a/Z_o{}^a)\sqrt{if/K^a}} \\[2ex]
C^w = \dfrac{\rho^w\sqrt{ifK^w}}{\tanh(\sqrt{if/K^w}(|H|-|h^w|))+|h^w|\ln|h^w/Z_o{}^w|\sqrt{if/K^w}} \\[2ex]
D^w = -\dfrac{\rho^w\sqrt{ifK^w}}{\sinh(\sqrt{if/K^w}(|H|-|h^w|))+|h^w|\ln|h^w/Z_o{}^w|\sqrt{if/K^w}\cosh(\sqrt{if/K^w}(|H|-|h^w|))}
\end{cases}
$$

In the current case equations (14) are solved for different current factors and for different bottom depths. All other parameters are according to the standard values.

3.41 Small current factors

If the geostrophic current factor is small, but not negligible as in chapter 3.3, free ice drift had to be solved with equations (14). This is done with a current factor given by 10^{-3} and with a current deflecting angle changing from zero to 360 degrees. The solution is shown in figure 9. The current factor corresponds to a geostrophic current speed of 10^{-2} m/s when the geostrophic wind is 10 m/s and for a fixed wind direction the deflecting angle change illustrates a current with different directions. From figure 9 one can see that the wind factor is largest when the geostrophic winds and currents are approximately in the same direction and smallest when they are opposite to each other. The wind deflecting angle is largest when the current direction is about 90 degrees to the right of the ice drift and smallest when the current direction is 90 degrees to the left of the ice drift. The total variation in the wind factor and wind deflecting angle is (2.0 ± 0.2) % and (14.5 ± 3.1) degrees respectively. This shows that neglecting even small geostrophic current introduce errors in the free ice drift which are comparable with errors due to neglecting variations in ice roughness or air friction velocity, see chapter 3.3.

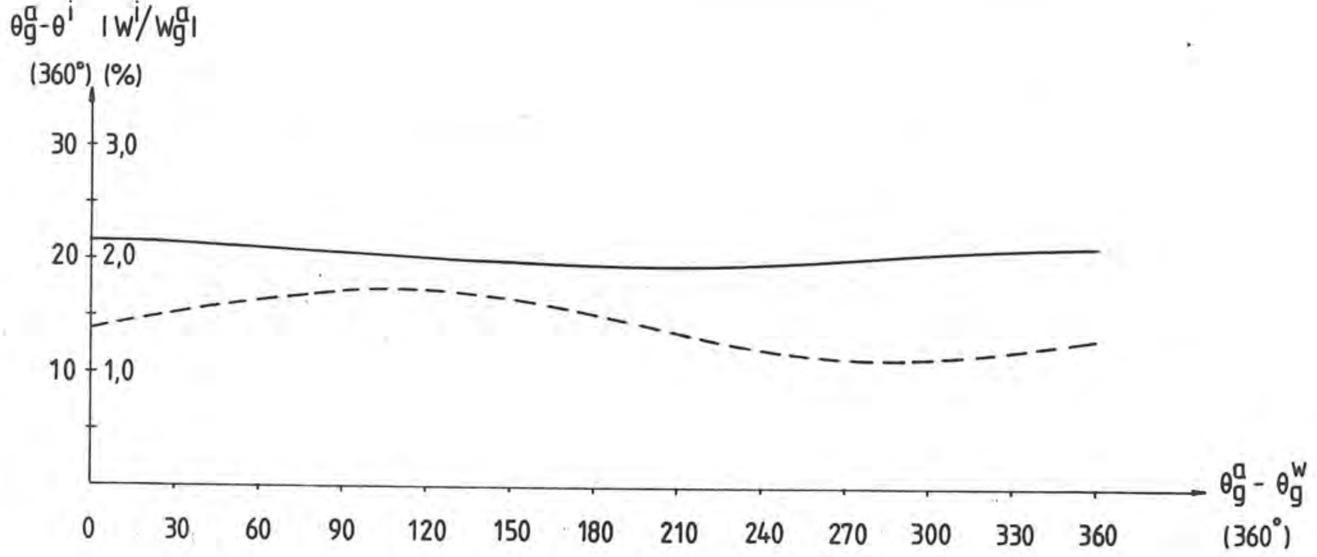


Figure 9. Geostrophic wind factor (fully drawn line) and geostrophic deflecting angle (dashed line) as a function of current deflecting angle ($\theta_g{}^a - \theta_g{}^w$) in the case when the current factor $|W_g{}^w/W_g{}^a|$ is 10^{-3}.

3.42 Larger current factors

For a geostrophic current factor given by 10^{-2}, which for a geostrophic wind speed of 10 m/s corresponds to a current speed of 10^{-1} m/s, the solution for different deflecting angles is shown in figure 10. From this figure it can be seen that the variations in the wind factor and the wind deflecting angle are larger than previous chapter. The wind factor is largest when the geostrophic winds and currents are about in the same direction and smallest when they are opposite to each other. The wind deflecting angle is largest when the current direction is 90 degrees to the right of the ice drift and smallest when the current direction is 90 degrees to the left of the ice drift. The total variation in the wind factor and wind deflecting angle is (2.0 ± 1.1) % and (14.5 ± 32.4) degrees respectively. These variations are much larger than those which were presented in earlier chapters. One could therefore regard the current factor as the parameter which, according to this theory, influence the free ice drift mostly.

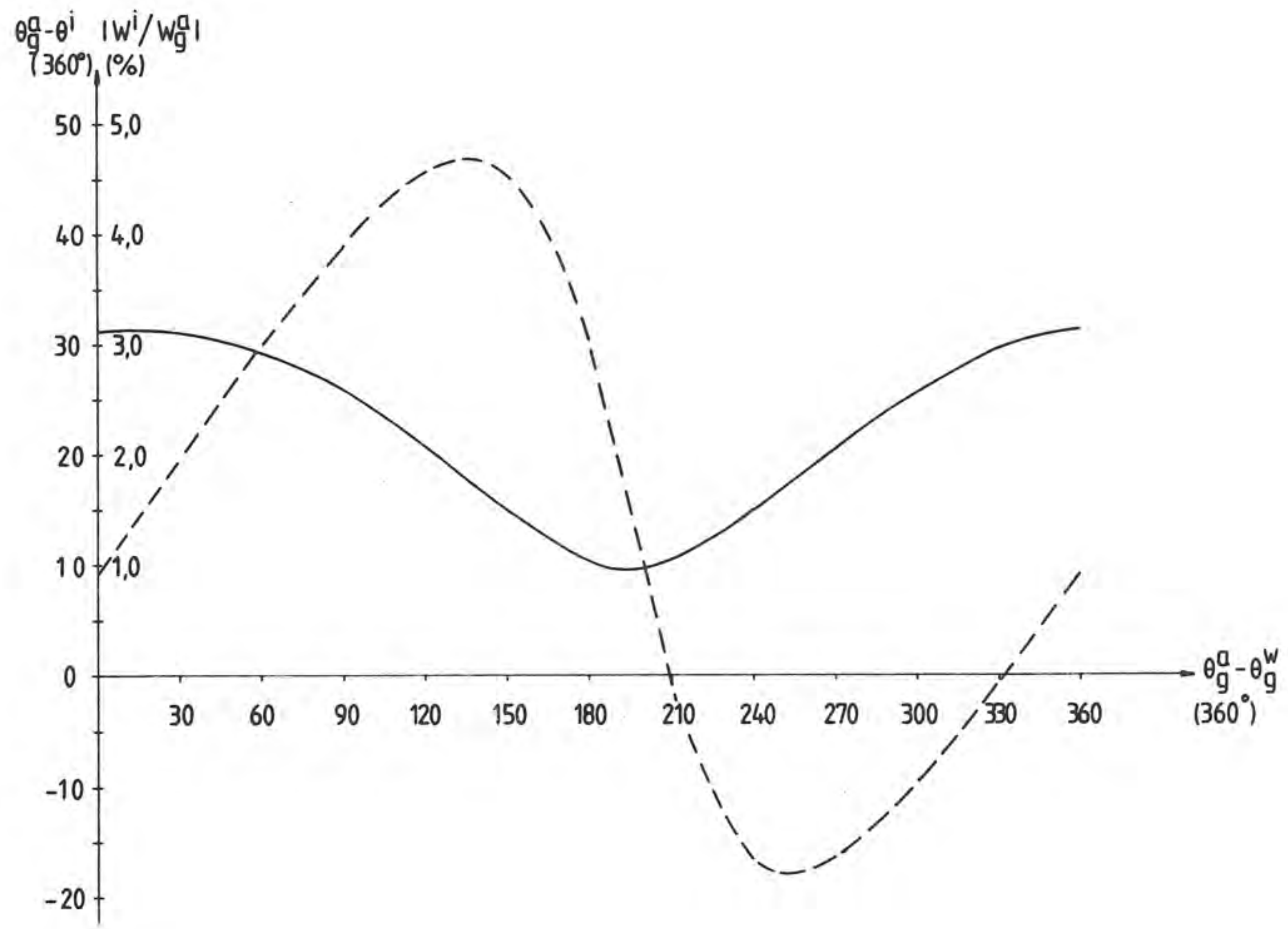
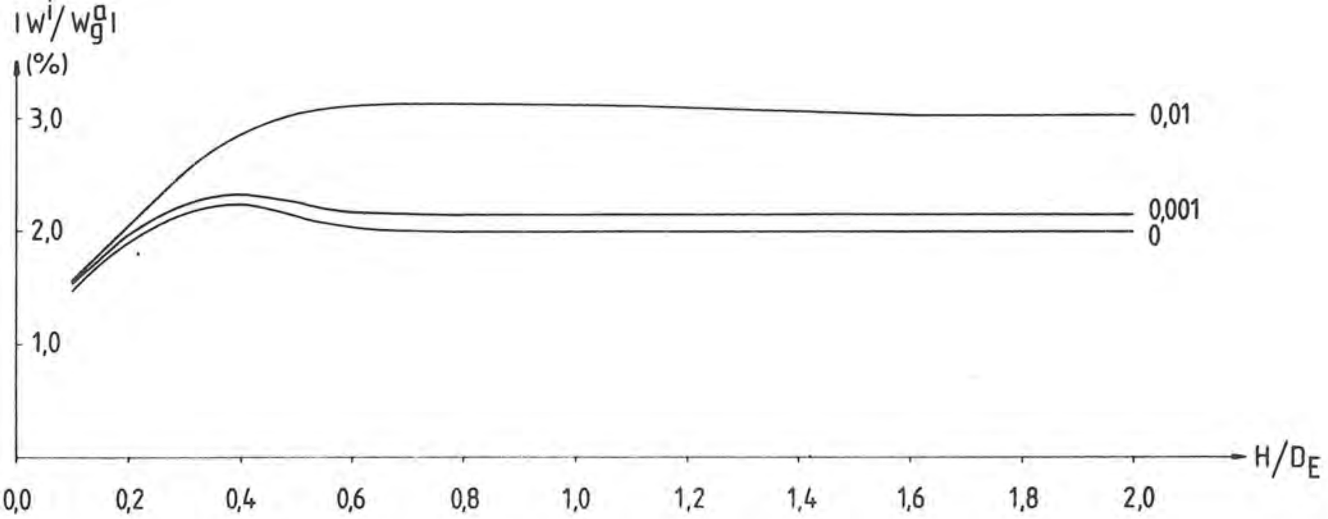


Figure 10. Geostrophic wind factor (fully drawn line) and geostrophic deflecting angle (dashed line) as a function of current deflecting angle ($\theta_g^a-\theta_g^w$) in the case when the current factor $|W_g^w/W_g^a|$ is 10^{-2}.

3.43 Bottom depth

As was earlier pointed out, see chapter 3.34, the free ice drift ought to be more bottom dependent in the current case than in the stagnant sea case. In figure 11 solutions of equations (14) are illustrated for variable bottom depths and with three different geostrophic current factors. It can be seen that the interval over which depth influence the free ice drift became larger when the current factor increase and that the total variation in the wind drift factor and the wind deflecting angle also increase. If one adopt the same idea as in chapter 3.34, that the ice is not free floating when the normalized bottom depth is less than 0.3, the wind factor and the wind deflecting angle still vary between (2.5 - 3.2) % and (-8.0 - +10.2) degrees respectively, when the current factor is 10^{-2}. It seems therefore that an ice drift model which does not neglect currents had to consider bottom depth in the water stress coefficients.



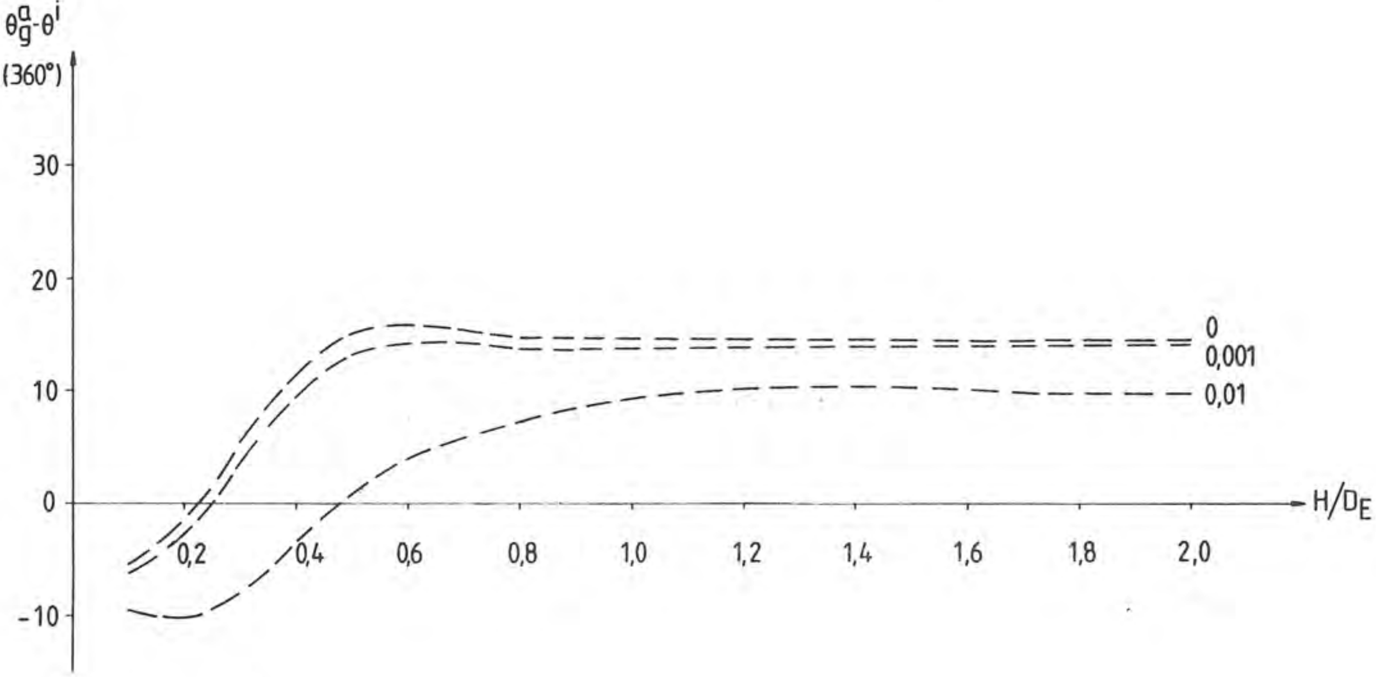


Figure 11. Geostrophic wind factor (fully drawn line) and geostrophic deflecting angle (dashed line) as a function of the normalized bottom depth (H/D_E) in the case when the current factor $|W_g{}^w/W_g{}^a|$ are 10^{-2}, 10^{-3} and zero.

4. SUMMARY AND CONCLUSIONS

In sea ice dynamic modelling work many approximations and parameter choices have to be made. As the parameters vary over large intervals there are several possible approaches. In this paper a sensitivity analysis of steady, free floating ice has been made. The equations are derived for a shallow sea using a logarithmic - Ekman stress approach. This make it also possible to study how important currents and bottom depths are for the ice drift.

The analysis first treat a case which assumes constant stress parameters in a deep sea with negligible currents. This case is used for calibrating the stress parameters in such a way that free ice drift is in accordance with observations and hopefully chosen in a consistent way. The geostrophic wind factor and the geostrophic deflecting angle become 2.0% and 14.5 degrees respectively.

Secondly the free ice drift is analysed for different parameter values but with negligible currents. In this case it is shown that the solutions are less sensitive to different bottom depths than to variations in ice roughness parameters and friction velocities. If constant roughness parameters are assumed the geostrophic wind factor and the deflecting angle could be calculated with an accuracy of (2.0 ± 0.2) % and (14.5 ± 2.5) degrees respectively. If constant friction velocities are assumed the corresponding values are (2.0 ± 0.2) % and (14.5 ± 8.8) degrees respectively. This case also shows that free ice drift is not particular sensitive to variations in ice mass, however the wind factor decreases and the deflecting angle increases when ice mass increases. This shows that the wind factor should be higher in the Gulf of Bothnia than in the Arctic and that the deflecting angle should be less in the Gulf of Bothnia than in the Arctic.

Thirdly the free ice drift is studied when currents are important. In this case it is shown that the free ice drift equation is sensitive even for small current factors. If small current factors (10^{-3}) are neglected the wind factor and the deflecting angle could be calculated with an accuracy of (2.0 ± 0.2) % and (14.5 ± 3.1) degrees respectively. For larger current factors the accuracy become worse. The analysis also shows that bottom depth variations in this case influence the free ice drift noticable, particular for large currents.

In this analysis the accuracy of the wind has not been treated. Quality checks have however clearly shown that the reliability of ice drift forecasts primarily depends on a good forecast of wind. Besides wind the linear free ice drift equations are particular sensitive to currents at least when the currents are large which is often the case in constricted areas as the Quarks and probably in coastal areas outside the fast ice. In this rather shallow areas bottom depth also should be considered in the water stress coefficients.

Acknowledgements

I want to thank my colleagues at the Swedish Meteorological and Hydrological Institute for many valuable discussions, criticism and patience during the work, and to Monika Johansson for typing the manuscript.

The work is a part of the Swedish-Finnish Winter Navigation Research Programme, and has been financed by the Swedish Administration of Shipping and Navigation.

Appendix A: Derivation of the external ice drift forces

A.1 The wind stress

The treatment of the atmospherical boundary layer follow Udin and Ullerstig (1976) with some modifications. The atmosphere is assumed to be neutral stratified and non-accelerating. The turbulent exchange coefficient (K^a) is assumed to be just vertical dependent and increases linearly from the ground up to a height (h^a), called the logarithmical height. Above this level the turbulent exchange coefficient remains constant. The expression then become;

$$K^a = \begin{cases} \kappa\, U_*^{\,a}\, Z & Z_o^{\,a} \le Z \le h^a \\ \kappa\, U_*^{\,a}\, h^a & Z > h^a \end{cases}$$

where $Z_o^{\,a}$ is the roughness height on the upper ice surface, $U_*^{\,a}$ the friction velocity in the air and κ von Karman´s constant. With above assumption the logarithmic and the Ekman equations are;

$$\frac{\partial}{\partial Z}\left[K^a \frac{\partial W^a}{\partial Z}\right] = 0 \qquad Z_o \le Z \le h^a$$

$$\text{if}\left[W^a - W_g^{\,a}\right] = K^a \frac{\partial^2 W^a}{\partial Z^2} \qquad Z > h^a$$

W^a is the wind, $W_g^{\,a}$ the geostrophic wind, f the coriolis parameter and i the complex number defined by $i = \sqrt{-1}$.

The solutions of the logarithmic and the Ekman equations are matched together by assuming continuity in the wind velocity and its derivatives at the level h^a.

The wind stress on the ice surface is after that derivated from following definition;

$$\tau^a = \rho^a K^a \frac{\partial W^a}{\partial Z} \qquad \text{at } Z = Z_o^{\,a}$$

which gives the wind stress formula

$$\tau^a = C^a W_g^{\,a} \tag{2}$$

where

$$C^a = \frac{\rho^a \sqrt{ifK^a}}{1+h^a \ln(h^a/Z_o{}^a)\sqrt{if/K^a}} \qquad ; \; K^a = \kappa U_*{}^a h^a$$

it can be seen that C^a is a complex constant which depends on roughness height ($Z_o{}^a$), friction velocity ($U_*{}^a$) and logarithmical height (h^a).

A2. The water stress

In the sea homogeneous stability and non-accelerating currents are assumed. The turbulent exchange coefficient (K^w) is assumed to be just vertical dependent and increases linearly from the surface under the ice down to a depth ($-h^w$). At deeper layers the turbulent exchange coefficient remains constant down to the bottom depth ($-H$). With the depth positive upwards the expression become;

$$K^w = \begin{cases} \kappa\, U_*{}^w\, |Z| & |Z_o{}^w| \leq |Z| \leq |h^w| \\ \kappa\, U_*{}^w\, |h^w| & |h^w| < |Z| \leq |H| \end{cases}$$

$|Z_o{}^w|$ is the absolute value of the roughness depth on the lower ice surface, ($U_*{}^w$) the friction velocity in the water and (κ) von Karman's constant. With above assumption the logarithmic and the Ekman equations are;

$$\frac{\partial}{\partial Z}\left(K^w \frac{\partial W^w}{\partial Z}\right) = 0 \qquad |Z_o{}^w| \leq |Z| \leq |h^w|$$

$$\text{if } (W^w - W_g{}^w) = K^w \frac{\partial^2 W^w}{\partial Z^2} \qquad |h^w| < |Z| \leq |H|$$

where the geostrophic current ($W_g{}^w$) is defined in analogy with the geostrophic wind velocity. The boundary conditions are that the current velocity at the roughness depth ($-Z_o{}^w$) is the same as the ice velocity and that the current velocity is zero at the bottom ($-H$).

The solutions of the logarithmic and Ekman equations are matched together by assuming continuity in the current velocity and its derivatives at the depth $(-h^W)$. The water stress on the lower ice surface is after that derivated from following definition

$$\tau^W = \rho^W K^W \frac{\partial W^W}{\partial Z} \qquad \text{at} \quad Z = -Z_o^{\ W}$$

which gives the water stress formula

$$\tau^W = C^W (W_g^{\ W} - W_i) + D^W W_g^{\ W} \tag{3}$$

where $(\ddot{C}^W)$ and $(\ddot{D}^W)$ are complex constants which depend on roughness depth $(-Z_o^{\ W})$, friction velocity $(U_*^{\ W})$, logarithmical depth $(-h^W)$ and bottom depth $(-H)$.

With all depth variables $(Z_o^{\ W}, h^W, H)$ positive, the explicit expressions for the water stress coefficients are:

$$C^W = \frac{\rho^W \sqrt{ifK^W}}{\tanh(\sqrt{if/K^W}(H-h^W)) + h^W \ln(h^W/Z_o^{\ W})\sqrt{if/K^W}}$$

$$D^W = -\frac{\rho^W \sqrt{ifK^W}}{\sinh(\sqrt{if/K^W}(H-h^W)) + h^W \ln(h^W/Z_o)\sqrt{if/K^W}\cosh(\sqrt{if/K^W}(H-h^W))}$$

$$K^W = \kappa U_*^{\ W} h^W$$

In the case of a deep sea, the water stress coefficients become simplier , D^W goes to zero and C^W could be written as;

$$C_\infty^{\ W} = \frac{\rho^W \sqrt{ifK^W}}{1 + h^W \ln(h^W/Z_o^{\ W})\sqrt{if/K^W}}$$

where $C_\infty^{\ W}$ is the water stress coefficient for a deep sea. This coefficient is comparable with the air stress coefficient in Appendix A1.

In the case of negligible logarithmical depth and a deep sea, the water stress coefficient becomes even more simple;

$$C_\infty^{\ W} = \rho^W \sqrt{ifK^W} = \rho^W \sqrt{fK^W}(\cos 45^\circ + i \sin 45^\circ)$$

This form of stress coefficient could also easily be derivated from the classic Ekman theory, which states that the wind driven surface current velocity in a deep sea deviate by 45^{o} to the right of the wind stress in the Northern Hemisphere.

A.3 The coriolis force and the gravitational force

The coriolis force acts to the right of the ice drift and depends just on ice velocity and ice mass.

$$C = -i\ m^{i}\ f\ W^{i} \tag{4}$$

The gravitational force is due to the sea surface tilting which force the ice to move to lower levels.

$$G = -m^{i}\ g\ \nabla S$$

If the sea is non-accelerating, hydrostatic, homogeneous and the pressure is constant at the ice water interface one could interpretend the sea surface slope as a frictional free velocity (W_g^{w}) defined in the same way as it was done when the water stress formula was derivated. The gravitational force could then be written as;

$$G = i\ m^{i}\ f\ W_g^{w} \tag{5}$$

where (W_g^{w}) is the geostrophic current field. It is caused by a balance between the sea surface tilt and the coriolis force in the sea. The frictional part of the current field is due to a balance between the frictional force and the coriolis force in the water. This part of the current field is treated in the derivation of the water stress.

LITERATURE REFERENCES

Doronin, Y.P., 1970: On a method of calculating the compactness and drift of ice floes, AIDJEX Bulletin, NO 3, PP22-39, University of Washington, Seattle, Washington 98105.

Hibler III, W.D., 1979: A dynamic thermodynamic sea ice model. Journal of Physical Oceanography, Volume 9, No 4, July 1979, 815-846.

Joffre, S.M., 1978: Studies of the winter-time boundary layer over the Baltic Sea based on pilot balloon soundings. Merentutkimuslaitoksen Julkaisu/ Havsforskningsinstitutets skrift, No 243, Helsinki 1978, Helsingfors, Finland.

Johannessen, O.M., 1970: Note on some vertical profiles below ice floes in the Gulf of St. Lawrence and near the North Pole. Journal of Geophysical Research, Vol. 75, No 15, May 20, 1970.

Leppäranta, M., 1979: On the drift and deformation of sea ice fields in the Bothnia Bay. Finnish Board for Navigation and Swedish Administration of Shipping and Navigation. In press. Helsinki, Finland.

Ling, C.H., and Untersteiner, N., 1974: On the calculation of the roughness parameters of sea ice. AIDJEX Bulletin, NO 23, University of Washington, Seattle, Washington 98105.

McPhee, M.G., 1977: An analysis of pack ice drift in Summer, Preprint, volume I, a symposium on Sea Ice processes and models, Sept. 1977, AIDJEX, University of Washington, Seattle, Washington 98105.

Omstedt, A., and Sahlberg, J., 1977: Some results from a joint Swedish-Finnish sea ice experiment, March 1977. Swedish Administration of Shipping and Navigation and Finnish Board for Navigation, No 26, Norrköping, Sweden.

Pickett, R.L., 1977: The observed winter circulation of Lake Ontario. Journal of Physical Oceanography, Vol 7, Jan. 1977, 152-156.

Pritchard, R.S., Reimer, R.W., and Coon, M.D., 1979: Ice flow through straits. Flow Research Company, Kent, Washington 98031. USA.

Tennekes, H., 1973: The logarithmic wind profile. Journal of the Atmospheric Sciences, Vol 30, March 1973, 234-238.

Udin, I. and Omstedt, A., 1976: SEA ICE -75: Dynamical report, Swedish Administration of Shipping and Navigation and Finnish Board of Navigation, Rep. No 16:8, Norrköping, Sweden.

Udin, I., and Ullerstig, A., 1976: A numerical model for forecasting the ice motion in the Bay and Sea of Bothnia, Swedish Administration of Shipping and Navigation and Finnish Board of Navigation, Rep. No 18, Norrköping, Sweden.

Untersteiner, N., and Badgley, F.I., 1965: The roughness parameters of Sea Ice, Journal of Geophysical Research, Vol. 70, No 18, Sept. 1965, 4573-4577.

Valli, A., and Leppäranta, M., 1975: Calculation of ice drift in the Bothnian Bay and the Quark. Finnish Board of Navigation and Swedish Administration of Shipping and Navigation, Rep. No 13. Helsinki, Finland.

SMHI Rapporter, METEOROLOGI OCH KLIMATOLOGI (RMK)

Nr 1 Thompson, T, Udin, I och Omstedt, A:
Sea surface temperatures in waters surrounding Sweden (1974)

Nr 2 Bodin, S:
Development on an unsteady atmospheric boundary layer model (1974)

Nr 3 Moen, L:
A multi-level quasi-geostrophic model for short range weather predictions (1975)

Nr 4 Holmström, I:
Optimization of atmospheric models (1976)

Nr 5 Collins, W G:
A parameterization model for calculation of vertical fluxes of momentum due to terrain induced gravity waves (1976)

Nr 6 Nyberg, A:
On transport of sulphur over the North Atlantic (1976)

Nr 7 Lundqvist, J-E och Udin, I:
Ice accretion on ships with special emphasis on Baltic conditions (1977)

Nr 8 Eriksson, B:
Den dagliga och årliga variationen av temperatur, fuktighet och vindhastighet vid några orter i Sverige (1977)

Nr 9 Holmström, I och Stokes, J:
Statistical forecasting of sea level changes in the Baltic (1978)

Nr 10 Omstedt, A och Sahlberg, J:
Some results from a joint Swedish-Finnish Sea Ice Experiment, March, 1977 (1978)

Nr 11 Haag, T:
Byggnadsindustrins väderberoende, seminarieuppsats i företagsekonomi, B-nivå (1978)

Nr 12 Eriksson, B:
Vegetationsperioden i Sverige beräknad från temperatur-observationer (1978)

Nr 13 Bodin, S:
En numerisk prognosmodell för det atmosfäriska gränsskiktet grundad på den turbulenta energiekvationen (1979)

Nr 14 Eriksson, B:
Temperaturfluktuationer under senaste 100 åren (1979)

Nr 15 Udin, I och Mattisson, I:
Havsis- och snöinformation ur datorbearbetade satellitdata - en metodstudie (1979)

Nr 16 Eriksson, B:
Statistisk analys av nederbördsdata. Del I. Arealnederbörd (1979)

Nr 17 Eriksson, B:
Statistisk analys av nederbördsdata. Del II. Frekvensanalys av månadsnederbörd (1980)

Nr 18 Eriksson, B:
Årsmedelvärden (1931-60) av nederbörd, avdunstning och avrinning (1980)

Nr 19 Omstedt, A:
A sensitivity analysis of steady, free floating ice (1980)