MODIFYING A JET MODEL FOR
COOLING WATER OUTLETS

By B Vasseur

SMHI Rapporter
HYDROLOGI OCH OCEANOGRAFI
Nr RHO 17 (1979)

SMHI

Sveriges meteorologiska och hydrologiska institut

Dnr 512/323

MODIFYING A JET MODEL FOR COOLING WATER OUTLETS

By B Vasseur

SMHI Rapporter
HYDROLOGI OCH OCEANOGRAFI
Nr RHO 17 (1979)

MODIFIERING AV EN JET MODELL FÖR KYLVATTENUTSLÄPP

ESSN 0347-7827
SVERIGES METEOROLOGISKA OCH HYDROLOGISKA INSTITUT
Norrköping 1979

ACKNOWLEDGEMENTS

The author wishes to express his gratitude to Mona Englund, who did a great work in preparing the manuscript for publication, and to the translatress Constance Lindqvist.

ABSTRACT

A modification of Prych's mathematical model (1972) for predicting the spreading of cooling water from a surface outlet is presented. The main modifications are a change in the calculation of the initial values after the zone of flow establishment, more realistic density calculations and the introduction of the fact that spreading of the jet's heat content takes place faster than the spreading of its velocity.

In the modified model the calculated plumes are noticeably narrower and thicker than in Prych's model. Furthermore, the jet's deflection by the ambient current is stronger and excess temperature on the surface diminishes faster.

SAMMANFATTNING

En modifiering av Prychs matematiska modell (1972) presenteras. Modellen används för att beräkna kylvattens spridning från ytutsläpp. De huvudsakliga förändringarna i modellen består i dels ändrade beräkningar av när övertemperaturen och jetströmmen kan antas ha uppnått en approximativ normalfördelning, del mer realistiska beräkningar av densiteten och dels införandet av det faktum att jetstrålens värmeinnehåll sprids snabbare än dess rörelsemängd.

Plymerna blir märkbart smalare och mer djupgående vid beräkningar med den modifierade modellen än med Prychs modell. Dessutom avböjer jetstrålen snabbare av recipientströmmen och övertemperaturen vid ytan minskar. Detta är mer i överensstämmelse med uppmätta förhållanden.

CONTENTS

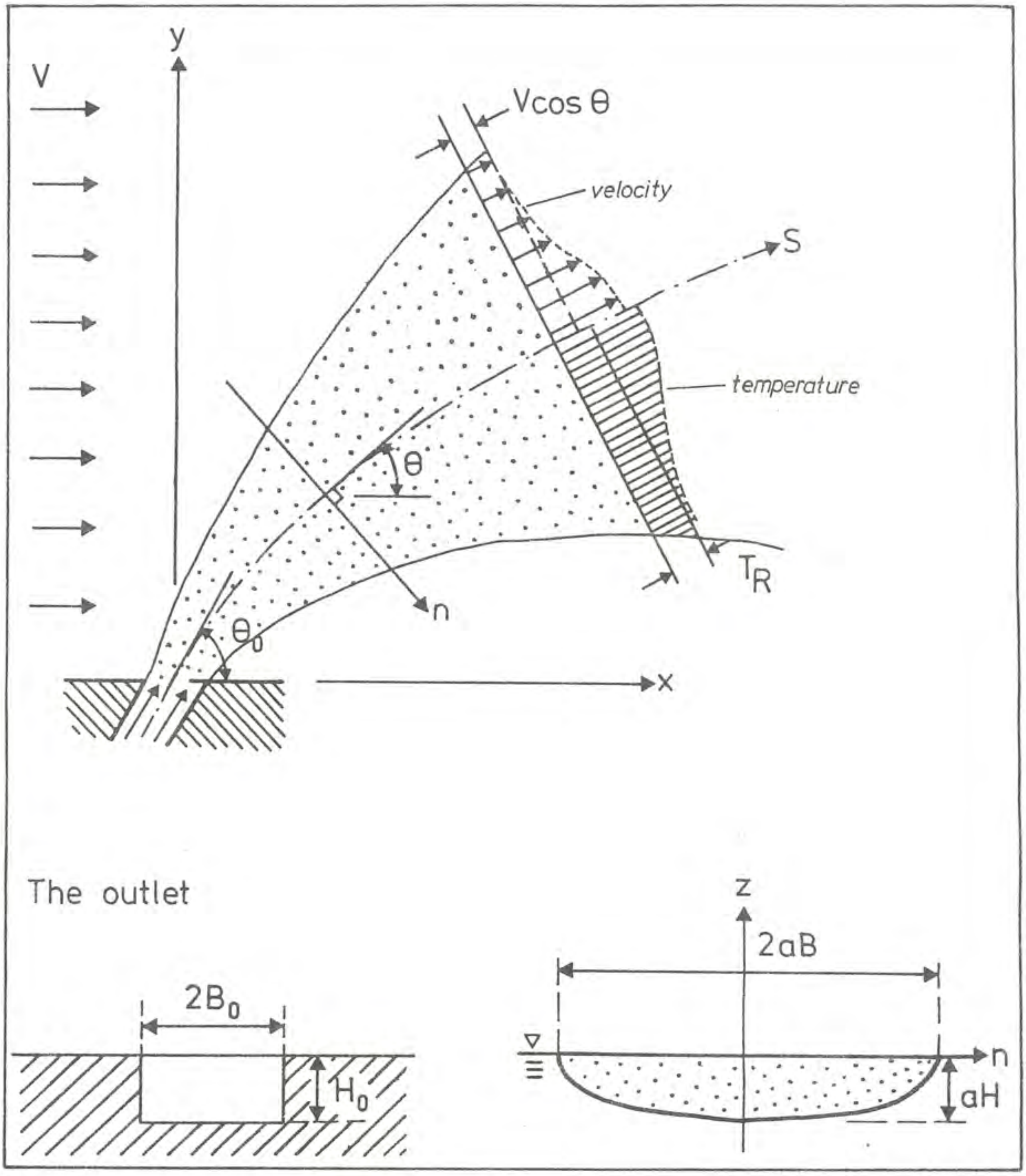
INTRODUCTION

In order to theoretically describe and calculate the spreading of cooling water from surface outlets, the Section of Oceanography at the Swedish Meteorological and Hydrological Institute has used an integral model developed by Prych, (1972).

In the model Gaussian distributions are assumed for the velocity and temperature distributions in the jet after the zone of flow establishment. In addition, conservation equations for volume, heat and momentum in two horizontal directions, two geometrical equations and an equation for the jet´s lateral spreading have been used. (See Prych, 1972).

An integral model, because of its mathematical composition, cannot fully describe a cooling-water outlet, but in order to make Prych´s model as good as possible, some modifications have been made in this study.

> The calculation of the initial values after the zone of flow establishment has been changed according to better empirical relations.
>
> Density calculations are made by using an equation that takes both temperature and salinity into consideration and that follows the surveyed conditions very closely.
>
> The spreading of the jet´s heat content takes place faster than spreading of its velocity.



Discharge velocity = u_0

temperature = $T_0 = T_R + \Delta T_0$

salinity = $S_0 = S_R + \Delta S_0$

aspect ratio = $A = \frac{2B_0}{H_0}$

Jet region:

$$\frac{n^2}{(aB)^2} + \frac{z^2}{(aH)^2} \leq 1$$

$$z \leq 0$$

Figure 1. Definition sketch for coordinate systems, outlet and jet region.

I. ZONE OF FLOW ESTABLISHEMENT (ZFE)

Description of the ZFE

When the cooling-water is let out into the recipient, it is well mixed. The temperature is, therefore, homogeneous. Due to heavy turbulence, the current velocity is almost the same throughout the whole outlet (see the example in figure I.1).

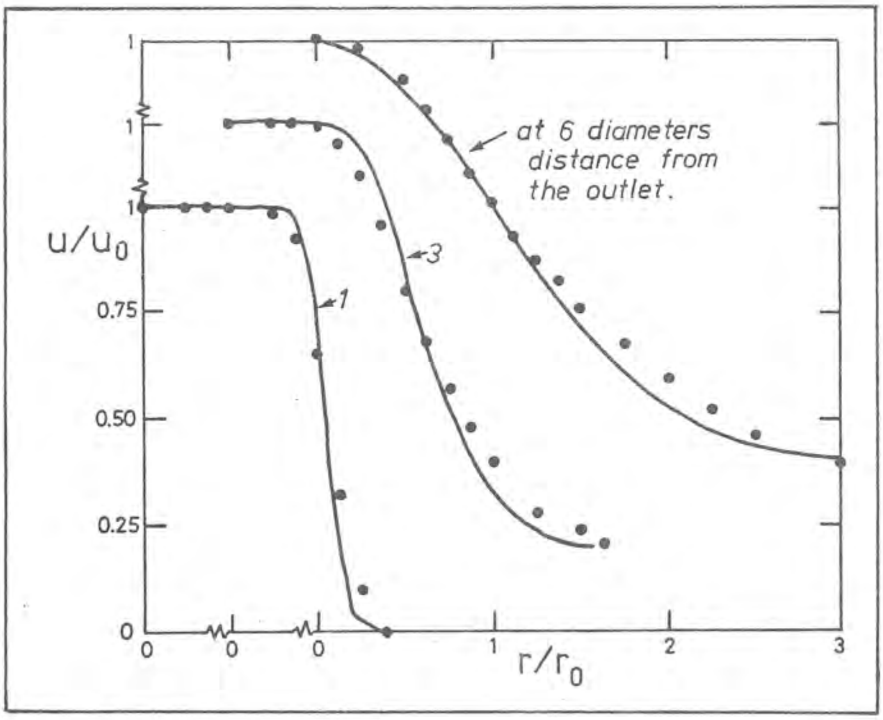


Figure I.1 Current profiles for a round jet with $F_0 = \infty$, after Sami, Carmody and Rouse, (1967).

In the boundary between the cooling-water and the recipient's water a shear layer forms that grows inwards towards the middle of the jet, see figure I.2. When this has reached the jet's center-line and the homogeneous core areas have disappeared, the shear layers have produced temperature and velocity fields which are Gaussian distributed. This is the theoretical end of zone of flow establishment (ZFE). In theory, the maximum temperature and velocity in the middle of the jet are thus constant throughout the entire ZFE, after which they decrease. For jets with $F_0 = \infty$ it has been shown theoretically that

the decreasement takes place in proportion to the distance raised to the power of one constant. In such cases the initial phase can be established by Albertson´s etc. method (see figure 1.2) where current or temperature are plotted down by distance on logarithm paper and two straight lines approximate the extension of the quantity of dots. The ZFE ends where these two lines intersect.

Unfortunately, decreasement doesn´t take place in a surface discharge according to this simple connection where F_0 is finite. An example of this can be seen in figure I.3.

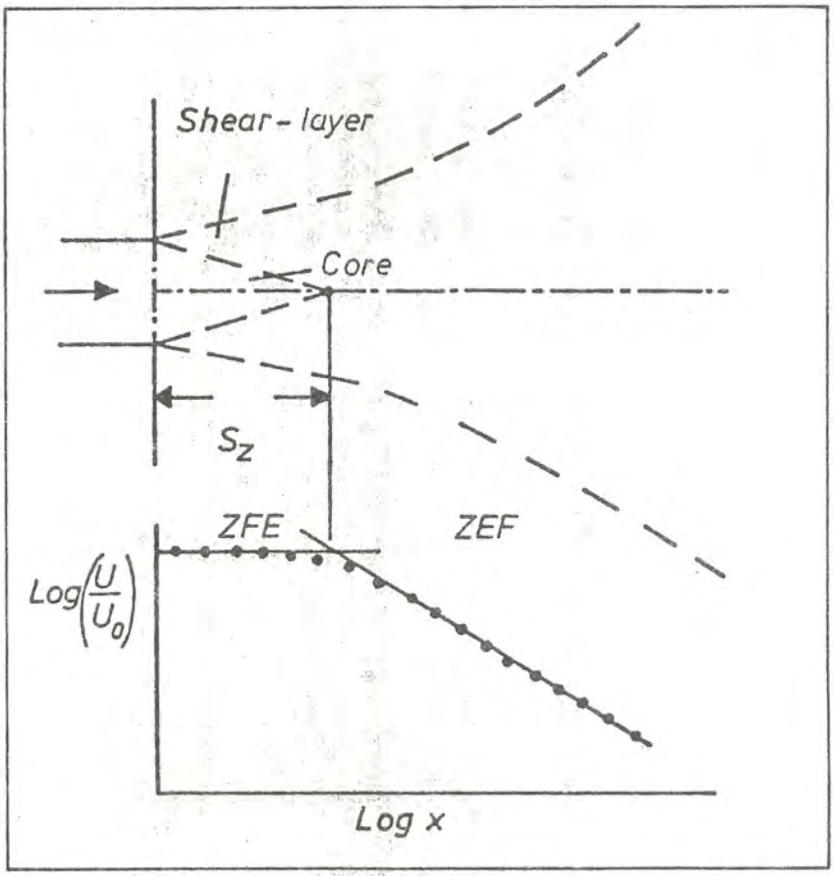


Figure I.2 Definition of the zone of flow establishment, (ZFE), after Albertson, Dai, Jensen and Rouse, (1950).

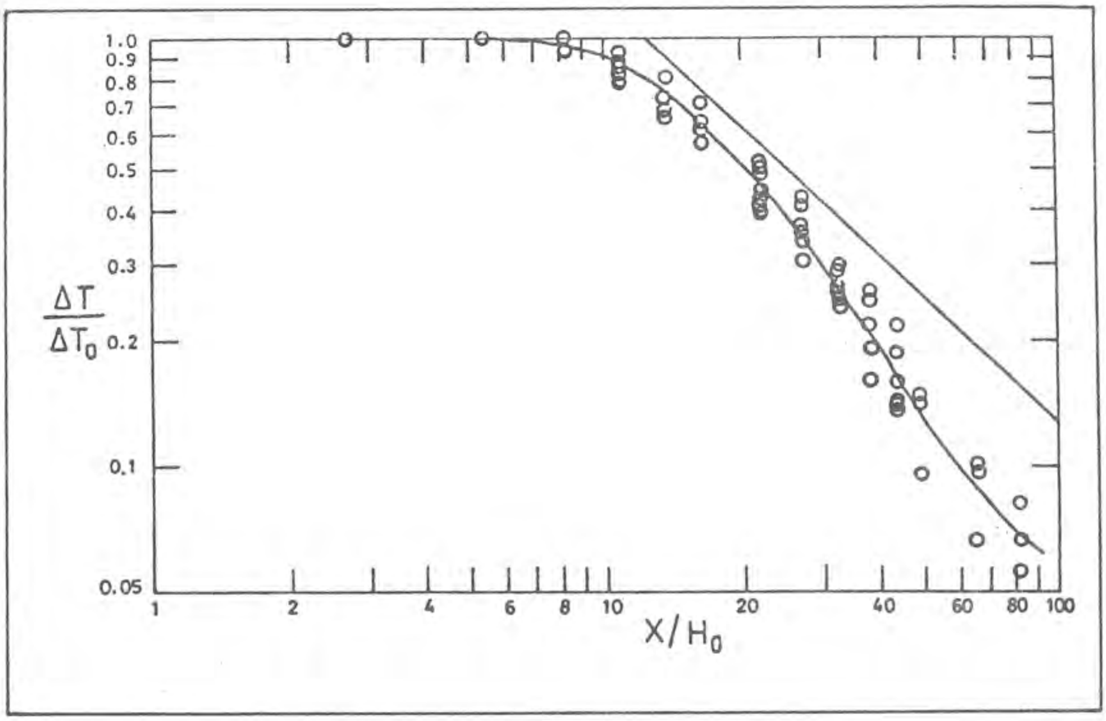


Figure I.3 The centerlines excess temperature for F_0 = 7.5 - 19.7, after Stefan, Bergstedt and Mrosla, (1975).

The limit of the core region can even be caused by lateral or vertical diffusion or both. Stefan, Bergsted and Mrosla state, supported by their experiments, that for Froude numbers lower than about 3 the lateral shear layer doesn´t reach the center because of the pronounced lateral buoyancy-induced spread of the jet. In this case the size of the initial area is established by the vertical shear.

The water entrained in the jet gathers momentum in the periphery of the jet and carries it towards the jet axis. The inward-going lump of fluid absorbs heat more slowly than it does momentum. Heat absorption in the plume takes place, therefore, further in than does the absorption of momentum. This mechanism, together with the density-driven width increasement, results in a wider (less steep gradient) thermal plume than does the momentum-plume.

Consequently, the excess temperature and current velocity of a jet discharge into a stagnant recipient in the zone of established flow can be written as follows:

$$\Delta T(s,n,z) = \Delta T(s)\exp(-(\frac{n}{\lambda_H B(s)})^2)\exp(-(\frac{z}{\lambda_V H(s)})^2)$$

$$u(s,n,z) = u(s)\exp(-(\frac{n}{B(s)})^2)\exp(-(\frac{z}{H(s)})^2),$$

where λ_H and λ_V are respectively the horizontal and vertical spreading factor for scalar parameters.

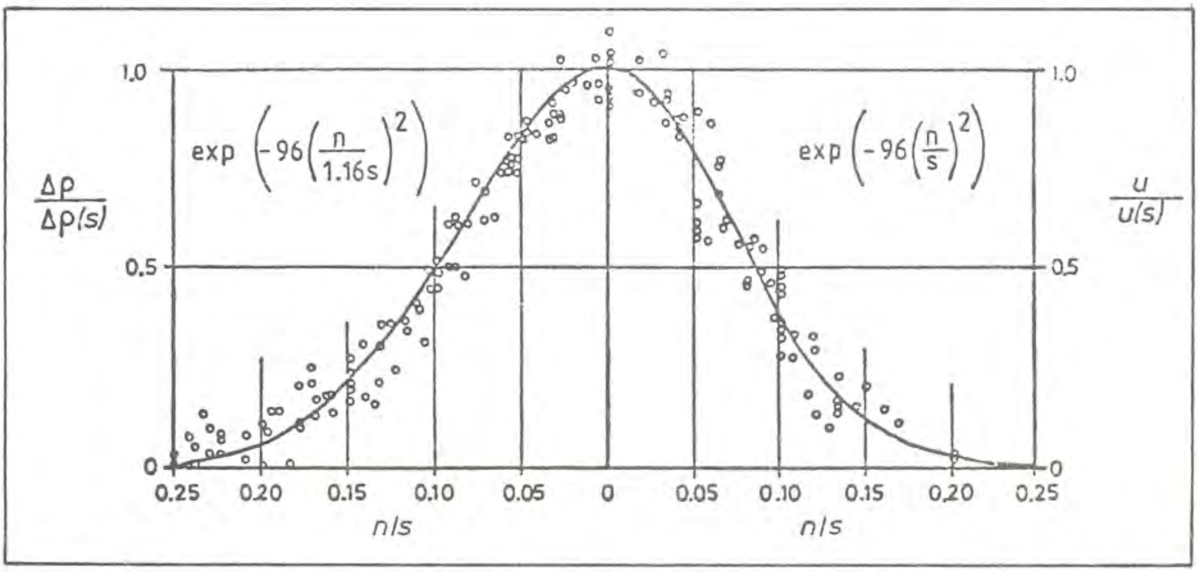


Figure I.4 Distribution of density and velocity in a round jet, after Rouse, Yih and Humphreys, (1952).

This implies that the core of excess temperature is shorter than the core of the jet´s velocity.

Rouse etc., (1952), investigated the diffusion coefficient for a round vertical jet with F_0 = 4.45 and found the value of 1.16 (see figure I.4). This is the most frequently used value found in literature (see for example, Abraham, (1963); Fan and Brooks, (1969)), even if there are spokesmen for other values. Stefan

and Vaidyaraman, (1972), used $\lambda_H = \lambda_V = 1.05$; Albertson etc., (1950), found $\lambda = 1.12$, for a round jet with $F_0 = \infty$, and Lal and Rajaratnam, (1975), found $\lambda_H = 1.86$ and $\lambda_V = 1.12$, to name some examples. The last value mentioned for λ_H seems, however, somewhat improbable.

For $\lambda_H = \lambda_V = 1.16$ the excess temperature at the end of the ZFE becomes approximately $0.87\Delta T_0$ if one define the ZFE by the extension of the velocity core.

The Length of the ZFE

In their modification of Prych's model, Shirazi and Davis, (1974), carried out laboratory experiments to ascertain how the length of the ZFE was dependent upon A (the outlets' width-height relationship), and F_0. They found the following relation:

$$s_z = 5.4\, H_0 \left(\frac{A^2}{F_0} \right)^{1/3} \qquad (I.1)$$

It can be established directly that this expression has a limited applicability. $F_0 \to \infty$ yields incorrectly that $s_z = 0$, and $A = \infty$ yields an infinite ZFE.

According to the above, the ZFE is partly dependent on the horizontal shear and partly on the vertical shear. If the width of the outlet is considerably larger than its height, i.e. A is large, it is the vertical shear that decides the extension of the ZFE. That $s_z \to \infty$ when $A \to \infty$ implies that the equation completely disregards the vertical shear.

Stefan etc., (1975), investigated how the jet's maximum temperature diminished, see figure I.5. Their adaptation of the surveyed values yielded:

$$s_z = H_0 \left(16.0 - 12.8 \frac{\Delta T}{\Delta T_0}\right)^{1.2} A^{\left(0.85 - 0.44 \frac{\Delta T}{\Delta T_0}\right)} \exp(-0.9R) \left(1 + \frac{0.5F_0 - 1.5}{\exp(0.4F_0)}\right) \qquad (I.2)$$

where $R = V/u_0$ and V is the recipient's current.

The formula is complicated and has many empirical constants. The dependency of R is, for example, very uncertain as their surveyed values show that the constatnt is a mean of values between 0.06 and 2.5.

Even in this formula s_z is unreasonably large for very large values for A.

One can see that the formula predicts the length of the ZFE well where A = 1.0 within these experimental boundaries, but this isn´t very surprising since Stefan etc. used these data exclusively (where A = 1.0) in order to determine the extension´s dependency on F_0. This connection isn´t applicable to other data sets with other A values. Figure I.5 shows where A = 2.4, a complex dependency with a minimum for the ZFE extension when F_0 = about 7. This minimum isn´t reflected at all in the empirical equation but the curve shows instead a maximum at about $F_0 = 5$. Stefan etc. explained the discrepancy as being caused by a strong secondary movement which causes an upwelling along the center line, relatively close to the outlet.

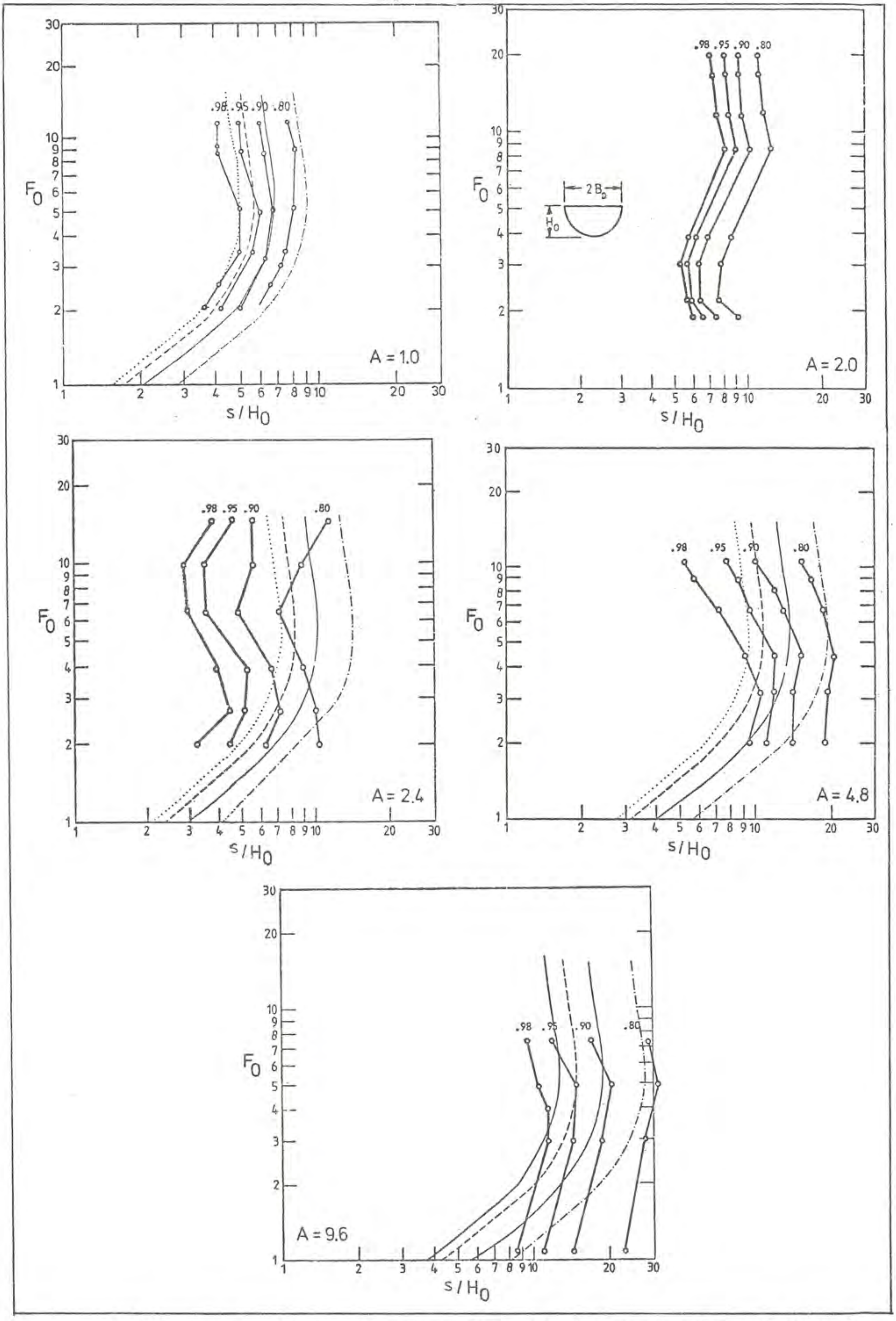


Figure I.5 The length of the ZFE with different values of F_0 and A. Comparison of equation I.2 and surveyed distances between the outlet and the position of the center line where $\Delta T/\Delta T_0$ = 0.98, 0.95, 0.90 and 0.80 respectively when R = 0.

In a later analysis of the same data, Stefan and Shanmugham, (1977), found the following relation:

$$s = H_0(17 + \frac{31}{F_0})(A')^{(1.4+2.1/F_0)} \quad \text{for } \Delta T = 0.90\Delta T_0 \qquad (I.3)$$

and

$$s = H_0(30 + \frac{24}{F_0})(A')^{(1.5+1.4/F_0)} \quad \text{for } \Delta T = 0.80\Delta T_0$$

where A´ is the relation between the outlet´s width and the rest of the outlet´s circumference ($A' = \frac{A}{A+2}$ for rectangular outlets).

Prych´s calculation of the length of ZFE is based on a rule of thumb which says that s_z = 4 x the diameter of the outlet.

Experiments carried out by, among others, Albertson et al., (1950), have, however, shown that s_z = 6.2 x the diameter of the outlet. If a calculation for a rectangular outlet is made for it´s corresponding circle outlet one gets:

$$s_z = 9.89\sqrt{B_0 H_0} \qquad (I.4)$$

Figure I.6 shows Stefan and Shanmugham´s equation, Shirazi and Davis' equation, and the equation (I.4) compared with Stefan´s et al. values. The adaption of Stefan and Shanmugham´s equation for $\Delta T = 0.90\Delta T_0$ to data is good except for A = 1.0. However, it can be established that the dependency on the Froude number is not in accordance with experimental data.

Shirazi and Davis´ equation shows an incorrect dependency on the Froude number, but its dependency on the aspect ratio has the right trend for moderate values. The equation (I.4) on the other hand, agrees well with the values for $\Delta T = 0.90\Delta T_0$, which, as per above, is approximately at the end of ZFE.

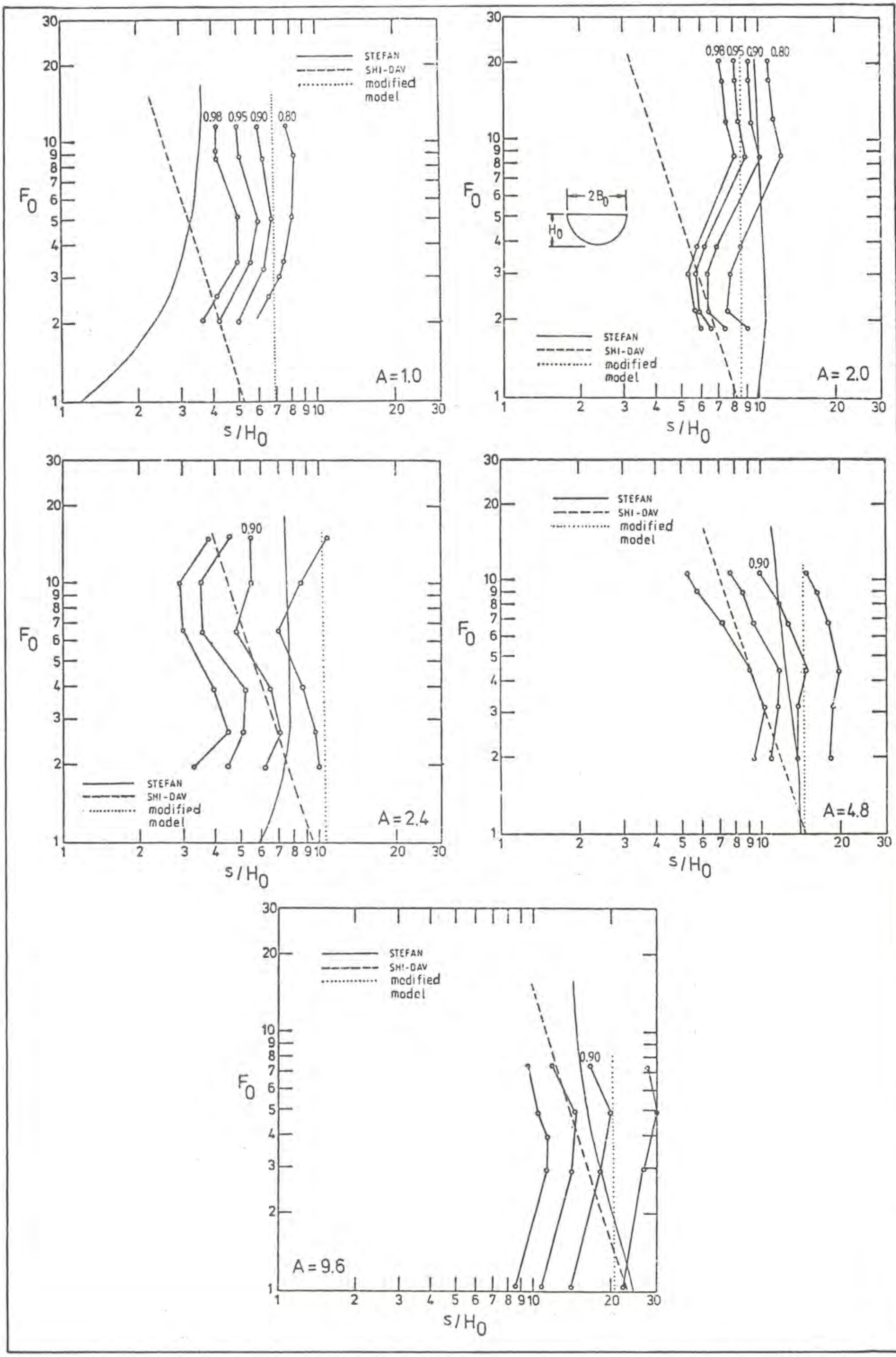


Figure I.6 The length of the ZFE with different values of F_0 and A. Comparison between the discharge and the point on the centerline where $\Delta T/\Delta T_0$ = 0.98, 0.95, 0.90 and 0.80 and equations I.1, I.3 and I.4.

On the support of Gordier's data, Fan, (1967), established the extension's dependency on R, for a round jet, at $F_0 = \infty$. The relation was:

$$s_z = 6.2\ D_0 \exp(-3.32R)$$

where D_0 = the diameter of the outlet.
If the exponential assumption is inserted in (I.4) one gets:

$$s_z = 9.89 \sqrt{B_0\ H_0}\ \exp(-3.32R) \qquad (I.5)$$

Unfortunately, the equation has the same disadvantage as, among others, the Shirazi - Davis equation. The length of the ZFE becomes unreasonably large where

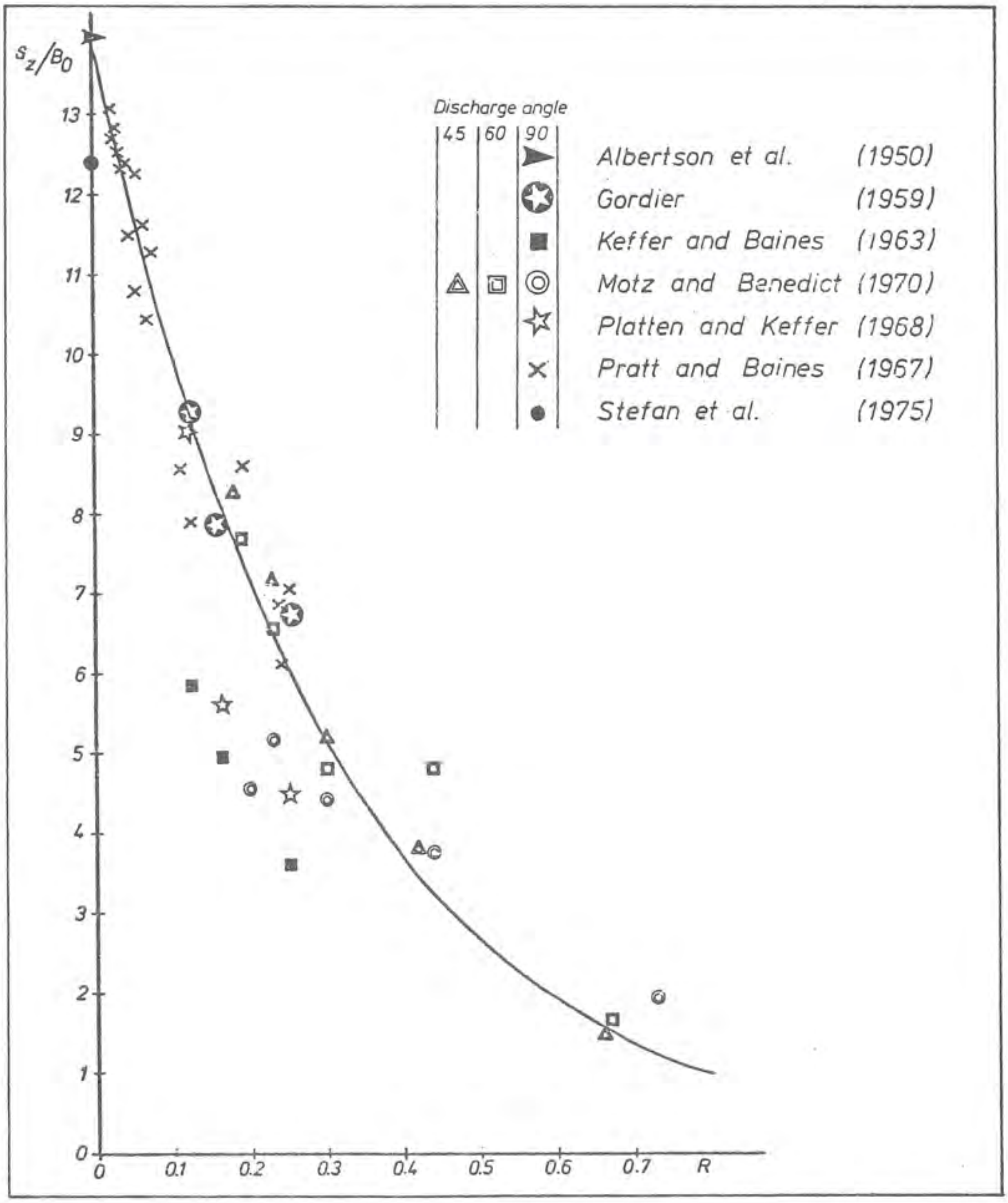


Figure I.7 The length of the ZFE. Comparison of equation (I.5) and experimental data.

there are very high values for A. In practice, however, this isn´t any great limitation since the width of the cooling-water outlet is often of the same magnitude as its depth.

In figure I.7, equation (I.5) is compared with experimental data.

Lateral Spreading within the ZFE.

Lateral spreading depends on the outlet's Froude number. The lower the number the higher the lateral spreading.

An experiment with tracer particles on the surface made by Stefan, Bergstedt and Mrosla, (1975), resulted in

$$\Phi = \frac{225 + 10\ F_0}{2.25 + F_0}$$

Stefan and Shanmugham, (1977), changed this relation to:

$$\Phi = 10 + 60A^{-0.02}\ F_0^{-0.5}$$

based on the same experiment of data. In the experiment A was varied between 1 and 10.3. This means, according to the equation, a change of 4.5 percent at most. Since the standard deviation in the experiment, after Stefan and Shanmugham, is $\pm\ 5^{\circ}$, the dependency of A is very uncertain and ought to be excluded. The equation then becomes:

$$\Phi = 10 + 60\ F_0^{-0.5}$$

Keffer and Baines, (1963), studied the spreading-angle for a round jet with $F_0 = \infty$. They found that the spreading angle was independent of R (see figure I.8). Their values show that the spreading angle within the ZFE is approximately 11° for B i.e. approximately 15.5° for $\sqrt{2}$ B.

The most well known experimental values originate from surveys made by Albertson, Dai, Jensen and Rouse, (1950). They found the spreading angle to be 18° for a round jet with $F_0 = \infty$ when R = 0. This value is,

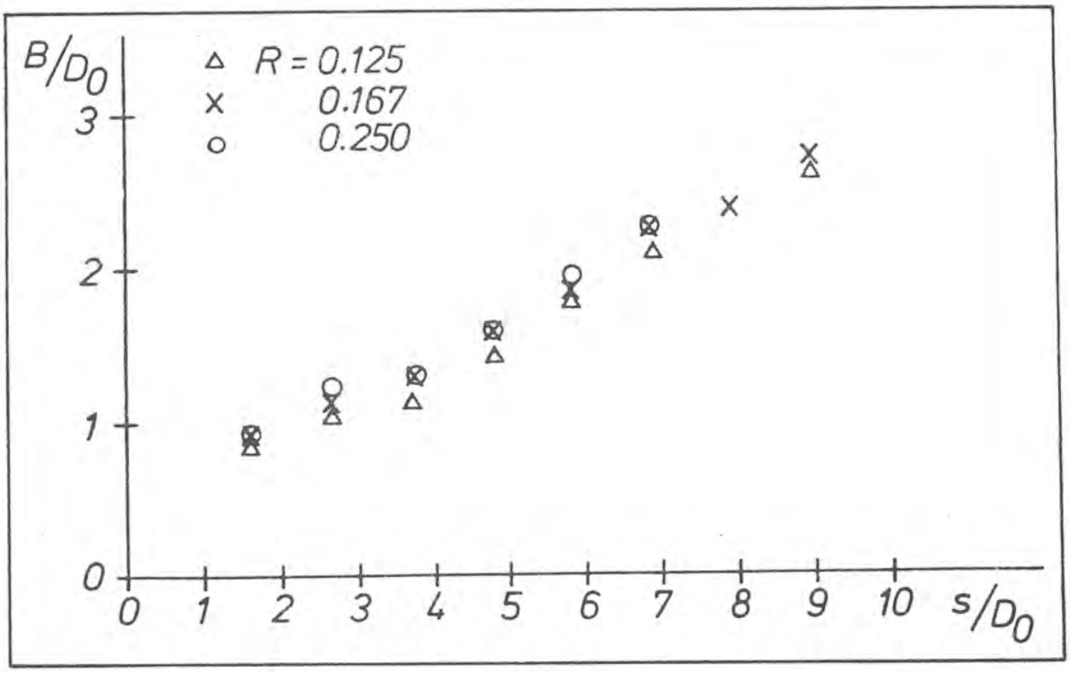


Figure I.8 Lateral spreading at different values of R according to Keffer and Baines, (1963).

however, somewhat large since the spreading angle increases after the ZFE and their values are taken well downstream from this.

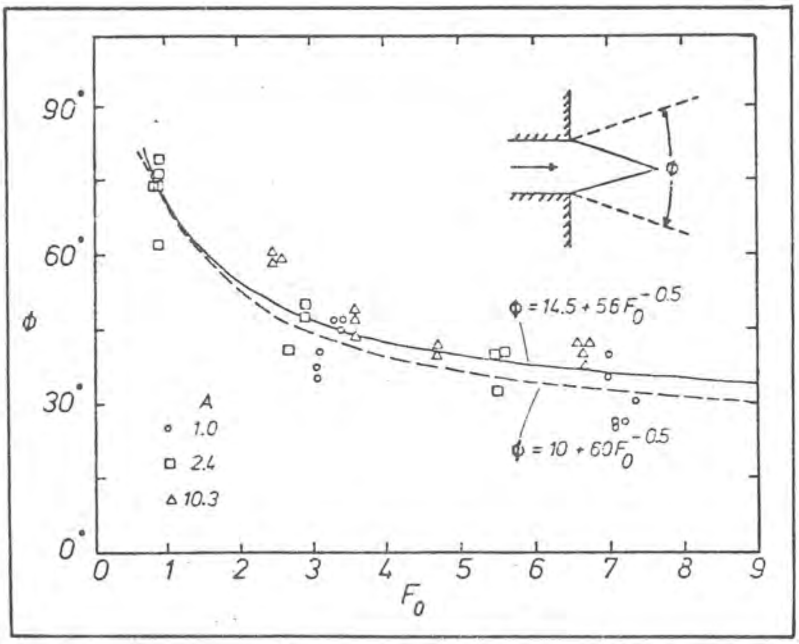


Figure I.9 Horizontal spreading angle. Measurements taken by Stefan, Bergstedt and Mrosla, (1975).

Where $F_0 = \infty$ and $R = 0$ the equation for conservation of momentum is reduced to:

$$\frac{\pi}{4} B_z H_z u_0^2 = 2 B_0 H_0 u_0^2.$$

If $B_0 = H_0$ then $B_z = H_z$ ought to be true, which yields:

$$B_z = \sqrt{\frac{8}{\pi}} B_0 \qquad \text{(I.6)}$$

It can also be expressed:

$$B_z = \frac{1}{\sqrt{2}} (B_0 + s_z \text{ tg } \frac{\Phi}{2}) \qquad \text{(I.7)}$$

In order for (I.6) and (I.7) to be identical, the spreading angle must be 14.5°, which agrees fairly well with Keffer and Baines´ values. For this reason, Stefan and Shanmugham´s equation is revised to:

$$\Phi = 14.5 + 56 F_0^{-0.5}$$

If one neglects the experimental data with $F_0 = \infty$ that are not accounted for in their figure, then the adaption will be about equal to their equation (see figure I.9).

One gets:

$$B_z = \frac{1}{\sqrt{2}} (B_0 + s_z \text{ tg } (7.3 + 28 F_0^{-0.5}))$$

which is used in the model.

Prych´s establishing of B_z is more complicated. First, he calculated how the spreading angle would be if the jet was assumed to have a rectangular cross-sectional surface and entrainment was ignored. The spreading angle is then only dependent on the density difference between the jet and the surrounding water.

He finds:

$$\frac{dB}{ds} = \frac{1}{\sqrt{\dfrac{u_0^2}{g\dfrac{\Delta\rho_0}{\rho_R}H} - 1}}$$

Since entrainment has been ignored, then

$$B' H' = B_0 H_0$$

where H' and B' are height and half of the width respectively of the assumed rectangular cross-sectional area. This yields

$$B' = \frac{B_0}{F_0^2} \left(\left(\frac{2}{3} \frac{s_z}{B_0} F_0^2 + (F_0^2 - 1)^{3/2} + 1 \right) \right.$$

Next B_z and H_z are calculated with the assumption that

$$\frac{B_z}{H_z} = \frac{B'}{H'}$$

and by using the equation for conservation of momentum.

In this way, great importance is attached to the density effect and width increasement becomes very large for moderate values for F_0. It is interesting to find that, among others, Koh and Fan, (1970), and also Motz and Benedict, (1970), have been of contrary opinion and assumed that the density effect is negligible within the ZFE. The actual truth, according to the experiments done, should be found somewhere in between.

Deflection of the Jet during the ZFE.

If friction is ignored within the ZFE the conservation of the jet's momentum in terms of x- and y- direction yields:

$$(M_z + P_z)\cos\Theta_z = (M_0 + P_0)\cos\Theta_0 + VQ_z \qquad (I.8)$$

$$(M_z + P_z)\sin\Theta_z = (M_0 + P_0)\sin\Theta_0 \qquad (I.9)$$

where

$$M_z = Q_z \left(\frac{u_0}{2}\cos(\Theta_0 - \Theta_z) + V\cos\Theta_z\right)$$

$$P_z = \frac{\sqrt{\pi}}{2} g \frac{\Delta\rho_z}{\rho_R} B_z H_z^2 \lambda_H \lambda_V^2$$

$$M_0 = Q_0 u_0$$

$$P_0 = g \frac{\Delta\rho_0}{\rho_R} B_0 H_0^2$$

$$Q_z = \pi B_z H_z \left(\frac{u_0}{2}\cos(\Theta_0 - \Theta_z) + V\cos\Theta_z\right)$$

(I.8) and (I.9) gives

$$tg\Theta_z = \frac{\left(1 + \frac{1}{2F_0^2}\right)\sin\Theta_0}{\left(1 + \frac{1}{2F_0^2}\right)\cos\Theta_0 + R\frac{\pi B_z H_z}{4 B_0 H_0}(\cos(\Theta_0 - \Theta_z) + 2R\cos\Theta_z)} \qquad (I.10)$$

If friction is ignored in order to conform with the above, Prych obtains

$$tg\,\Theta_z = \frac{\left(1 + \frac{1}{2F_0^2}\right)\sin\Theta_0}{\left(1 + \frac{1}{2F_0^2}\right)\cos\Theta_0 + R} \qquad (I.11)$$

Figure I.10 shows that agreement between Motz and Benedict's, (1970), data and (I.10) is good, but that (I.11) yields too little deflection.

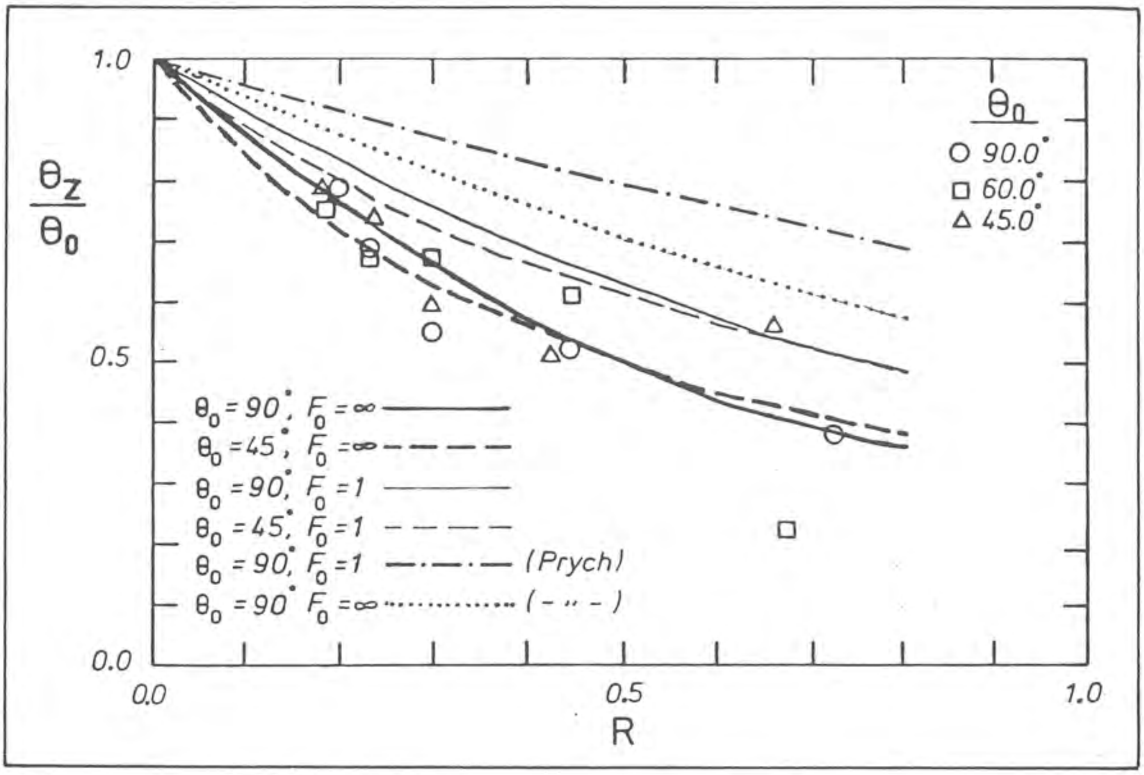


Figure I.10 The deflection of the jet by the crossflow after the ZFE. A comparison between equations (I.10), (I.11) and data. From Motz and Benedict, (1970).

The coordinates for the end of ZFE are:

$$x_z = s_z \cos\left(\frac{\Theta_0 + \Theta_z}{2}\right)$$

$$Y_z = s_z \sin\left(\frac{\Theta_0 + \Theta_z}{2}\right)$$

The Jet's Temperature and Depth Extension at the End of the ZFE.

Heat transport to the atmosphere within the ZFE is negligible. The heat balance for the jet can then be

$$\Delta T_0 Q_0 = \frac{\pi}{2} \Delta T_z \; \lambda_H \; \lambda_V \; B_z \; H_z \left(\frac{u_0}{S_\lambda} \cos(\Theta_0 - \Theta_z) + V\cos \Theta_z\right)$$

where $S_\lambda = \sqrt{(1 + \lambda_H^2) \quad (1 + \lambda_V^2)}$

which yields

$$\frac{\Delta T_z}{\Delta T_0} = \frac{4 \; S_\lambda \; B_0 \; H_0}{\pi \; \lambda_H \; \lambda_V \; B_z \; H_z (\cos(\Theta_0 - \Theta_z) + S_\lambda \; R \cos \Theta_z)}$$

If $\lambda_H = \lambda_V = 1.16$, $F_0 = \infty$ and $R = 0$ then $\Delta T_z = 0.87 \; \Delta T_0$ is obtained, which agrees well with the experimental data. Prych, on the other hand, assumed that $\Delta T_z = \Delta T_0$.

From (I.8) and (I.9) the following can also be obtained

$$H_z = \frac{2B_0 H_0 \; \left(1 + \dfrac{1}{2F_0^2}\right) \cos \; (\Theta_0 - \Theta_z)}{\dfrac{\pi}{4} B_z \cos(\Theta_0 - \Theta_z) (\cos(\Theta_0 - \Theta_z) + 2R\cos\Theta_z) + \dfrac{2\lambda_V \; S_\lambda \; B_0}{\sqrt{\pi} \; F_0^2 (\cos(\Theta_0 - \Theta_z) + S_\lambda R\cos\Theta_z)}}$$

if one assumes

$$\frac{\Delta \rho_z}{\Delta \rho_0} = \frac{\Delta T_z}{\Delta T_0}$$

Since ΔT_z is approximately $0.90 \; \Delta T_0$ and the absolute temperature is high, this assumption only yields a slight deviation for $\Delta \rho_z$ from the true value.

In Prych's model, one must solve an equation of fifth degree in order to solve H_z. Moreover, his method yields too narrow plumes after ZFE.

II ZONE OF ESTABLISHED FLOW - ZEF

Equation of State

In his model, Prych assumes that there exists a linear relation between density and temperature. Since excess temperature and, consequently, even the temperature interval are most often of the magnitude of 10 °C, it is a rather rough assumption, particularly for low ambient temperatures. Another drawback is that Prych assumes that salinity is constant, i.e., doesn´t allow for different salinities in the cooling-water and in those layers in the recipient which are affected by the outlet.

Moreover, the linear coefficient of correlation must be established before every plume calculation and be introduced as input-data to the model. This coefficient is dependent upon salinity and temperature in the recipient and in the outlet. A natural improvement of the model is, thus, to introduce salinity as a new variable and to calculate density in more realistic terms. Such a one, that gives good values for the relatively low salinities that occur around the Swedish coast and that yields realistic density maxima at low temperatures but still demands rather little computer time, has been developed by Fredrich and Levitus, (1972), and modified by Wilmot, (1976).

$$\rho = 1+ 10^{-3} \cdot \Big[-0.072169 + T_5(0.049762 + T_5(-0.0075911 +0.000035187\ T_5)) + S(0.80560 + T_5(-0.0030063 + 0.000037297\ T_5))\Big] \quad \text{when } T_5 = T - 0.5$$

Distribution Profiles for Current, Temperature and Salinity.

Both theoretical observations and laboratory experiments have shown that these parameters are approximately Gaussian distributed for small R according to:

$$u(s,n,z) = u(s)\exp\left(-\frac{n^2}{B^2(s)}\right)\exp\left(-\frac{z^2}{H^2(s)}\right) + V\cos\Theta(s)$$

$$T(s,n,z) = T_R + \Delta T(s)\exp\left(-\frac{n^2}{\lambda_H^2 B^2(s)}\right)\exp\left(-\frac{z^2}{\lambda_V^2 H^2(s)}\right)$$

$$S(s,n,z) = S_R + \Delta S(s)\ \exp\left(-\frac{n^2}{\lambda_H^2 B^2_{(s)}}\right)\exp\left(-\frac{z^2}{\lambda_V^2 H^2_{(s)}}\right)$$

where

$u(s)$	= jet velocity relative to recipient current in the jet axis
$\Delta T(s)$	= excess temperature in the jet axis
$\Delta S(s)$	= excess salinity in the jet axis
V	= recipient current
T_R	= recipient temperature
S_R	= recipient salinity
λ_H, λ_V	= spreading factors for scalar parameters in horizontal and vertical directions.

Since the jet flow can be expressed as

$Q(s) = \int\int u(s,n,z)\,dndz$

and $V\cos\Theta(s)$ is independent of n and z the cross-sectional area must be finite. A realistic area for the above distribution is a semi-ellipse according to:

$$\left(\frac{n}{aB(s)}\right)^2 + \left(\frac{z}{aH(s)}\right)^2 \leq 1 \qquad z \leq 0$$

where a = constant.

Prych puts $a = \sqrt{2}$ with the fact that the equations

are maximally reduced with this value as the only reason. The value implies that excess temperature along the boundary is 13.5 % of the excess temperature at the jet axis which can seem to be a somewhat too high value. Experiments have, none the less, shown that $a = \sqrt{2}$ is a realistic value, and it is used from now on in this study.

Volume Flux, Momentum Flux and Excess Kinematic Heat Flux.

The volume flux:

$$Q(s) = \pi B(s)H(s) \left[\frac{u(s)}{2} (\text{erf}(\sqrt{2}))^2 + V\cos\theta(s)\right]$$

The momentum flux:

$$M(s) = \pi B(s)H(s) \left[\frac{u^2(s)}{4} (\text{erf}(2))^2 + u(s)V\cos\theta(s)(\text{erf}(\sqrt{2}))^2 + V\cos^2\theta(s)\right]$$

The excess kinematic heat flux:

$$J(s) = \frac{\pi}{2} \Delta T(s)B(s)H(s)\lambda_H\lambda_V \left[u(s)\text{erf}\left(\sqrt{2(1+\frac{1}{\lambda_H^2})}\right)\text{erf}\left(\sqrt{2(1+\frac{1}{\lambda_V^2})}\right) + V\cos\theta(s)\ \text{erf}\left(\frac{\sqrt{2}}{\lambda_H}\right)\ \text{erf}\left(\frac{\sqrt{2}}{\lambda_V}\right)\right]$$

Prych approximates all values of the error-function to 1.0. This can be delicate to work with. For example $(\text{erf}(\sqrt{2}))^2 = 0.91$ and if $\lambda_H = \lambda_V = 1.16$ $\text{erf}\left(\frac{\sqrt{2}}{\lambda_H}\right)\ \text{erf}\left(\frac{\sqrt{2}}{\lambda_V}\right) = 0.84$. But since the model contains more essential errors that overshadow these, and, furthermore, the expressions are much simpler with this approximation it has, therefore, been used in this version of Prych´s model.

The above expression is thus:

$$Q(s) = \pi B(s)\ H(s) \left[\frac{u(s)}{2} + V\cos\theta\ (s)\right]$$

$$M(s) = \pi B(s)\ H(s) \left[\frac{u(s)}{2} + V\cos\theta\ (s)\right]^2$$

$$J(s) = \frac{\pi}{2} \Delta T(s)B(s)H(s)\lambda_H\lambda_V \left[\frac{u(s)}{S_\lambda} + v\cos\theta\ (s)\right]$$

Prych establishes the continuity of the volume flux with

$$\frac{dQ}{ds} = \left(\frac{dQ}{ds}\right)_{J,H} + \left(\frac{dQ}{ds}\right)_{J,V} + \left(\frac{dQ}{ds}\right)_{R,H} + \left(\frac{dQ}{ds}\right)_{R,V}$$

The horizontal and vertical jet entrainment are given by

$$\left(\frac{dQ}{ds}\right)_{J,H} = \sqrt{\pi}\, E\ H(s) \sqrt{u^2(s) + V^2 \sin^2 \Theta(s)}$$

$$\left(\frac{dQ}{ds}\right)_{J,V} = 2E \int_0^{\sqrt{2}B(s)} f(Ri) \sqrt{u^2(s) \exp\left(\frac{-2n^2}{B^2(s)}\right) + V^2 \sin^2\Theta(s)}\ dn$$

and the horizontal and vertical ambient entrainment are assumed to be

$$\left(\frac{dQ}{ds}\right)_{R,H} = 3.5\ \pi\ \varepsilon_H \frac{H(s)}{B(s)}$$

$$\left(\frac{dQ}{ds}\right)_{R,V} = 3.5\ \pi\ \varepsilon_V \frac{B(s)}{H(s)}\ f(Ri)$$

where

$$f(ri) = 0 \qquad \text{for } Ri > 0.8$$

$$= \frac{\exp(-5Ri) - \exp(-4)}{1 - \exp(-4)} \qquad \text{for } Ri < 0.8$$

$$Ri = g\, \frac{\Delta\rho(s)}{\rho_R}\ \frac{2\ H(s)}{\left(u^2(s) \exp\left(-\frac{2n^2}{B^2(s)}\right) + V^2 \sin^2\Theta(s)\right)}$$

Prych recommends the use of $\varepsilon_H = \varepsilon_V = 0$, i.e. that the recipient´s turbulence be ignored. Odgaard, (1975), suggests that it be disregarded to begin with, but that it be taken into account when the jet velocity is heavily reduced. Therefore, he multiplies the expression by $1 - (u(s)/u_0)^{1/4}$. My opinion is essentially the same, but it isn´t meaningful to continue the calculations when the jet velocity is much smalller than the recipient´s current. Therefore,

I think that $\varepsilon_H = \varepsilon_V = 0$ be written in accordance with Prych´s suggestion, and that cooling-water is treated as a passive trace element when the jet characteristics have disappeared. The density effect is by then small, and often other factors such as wind stress are more important.

In the equation for the Richardson number, Ri, and in all the equations below, Wilmot´s calculation for density is, of course, used.

The heat conservation equation is

$$\frac{dJ}{ds} = -k\lambda_H \sqrt{\pi}\,\Delta T(s)B(s)$$

where k = coefficient for heat loss to the atmosphere.

This yields with the above expression for J(s)

$$\frac{d\Delta T}{ds} = -\Delta T(s)\left[\frac{2}{\pi B(s)H(s)(u(s)+V\cos\Theta(s)\ S_\lambda)}\frac{dQ}{ds}\right.$$

$$+\frac{(2-S_\lambda)V\sin\Theta(s)}{(u(s)+V\cos\Theta(s)\ S_\lambda)}\frac{d\Theta}{ds}+\frac{1}{B(s)}\left(1-\frac{2Q(s)}{\pi B(s)H(s)(u(s)+V\cos\Theta(s)S_\lambda)}\right)\frac{dB}{ds}$$

$$+\frac{1}{H(s)}\left(1-\frac{2Q(s)}{\pi B(s)H(s)(u(s)+V\cos\Theta(s)S_\lambda)}\right)\frac{dH}{ds}$$

$$\left.+\frac{2k\,S_\lambda}{\sqrt{\pi}\lambda_V H(s)(u(s)+V\cos\Theta(s)S_\lambda)}\right]$$

A comparable expression is obtained for the salinity difference where $\Delta T(s)$ is replaced by $\Delta S(s)$ and when the last term in the above equation, which calculates heat loss to the atmosphere, is removed.

The conservation equations in x- and y-directions for momentum flux are:

$$\frac{d}{ds}(M\cos\Theta) = -\frac{d}{ds}(P\cos\Theta) + S_{Fx} + F_D\sin\Theta(s) + \frac{d}{ds}(VQ)$$

$$\frac{d}{ds}(M\sin\Theta) = -\frac{d}{ds}(P\sin\Theta) + S_{Fy} - F_D\cos\Theta(s)$$

where

$$P(s) = \int_{-\infty}^{0}\int_{-\infty}^{\infty}\int_{-\infty}^{z} g\,\frac{\Delta\rho(s,n,z')}{\rho_R}\,dz'\,dn\,dz$$

$$S_{Fx} = 0.02C_F\left(\frac{\nu}{H(s)}\right)^{1/4}\int_{-\sqrt{2}B(s)}^{\sqrt{2}B(s)} \Delta u^{3/4}\left(V\sin^2\Theta(s) - u(s)\cos\Theta(s)\exp\left(\frac{-n^2}{B^2(s)}\right)\right)dn$$

$$S_{Fy} = -0.02C_F\sin\Theta(s)\left(\frac{\nu}{H(s)}\right)^{1/4}\int_{-\sqrt{2}B(s)}^{\sqrt{2}B(s)} \Delta u^{3/4}\left(V\cos\Theta(s) + u(s)\exp\left(-\frac{n^2}{B^2(s)}\right)\right)dn$$

$$F_D = \frac{1}{\sqrt{2}}\,C_D\,H(s)\,V|V|\sin^2\Theta(s)$$

$$\Delta u(s,n) = \left(u^2(s)\exp\left(\frac{-2n^2}{B^2(s)}\right) + V^2\sin^2\Theta(s)\right)^{1/2}$$

ν = kinematic viscosity

C_F = interfacial shear drag coefficient

C_D = form drag coefficient

If the density distribution in P(s) is approximated so that

$$\Delta\rho(s,n,z') = \Delta\rho(s)\exp\left(\frac{-n^2}{\lambda_H^2 B^2(s)}\right)\exp\left(\frac{-(z')^2}{\lambda_V^2 H^2(s)}\right)$$

which yields insignificant errors, then

$$P(s) = \frac{\sqrt{\pi}}{2}\,g\,\frac{\Delta\rho(s)}{\rho_R}\,\lambda_H\,\lambda_V^2\,B(s)\,H^2(s)$$

From above one gets:

$$\frac{d\Theta}{ds} = \frac{-(F_D - S_{Fy}\cos\Theta(s) + S_{Fx}\sin\Theta(s) + V\sin\Theta(s)\,\frac{dQ}{ds})}{M(s) + P(s)}$$

and

$$\frac{dH}{ds} = \Big[(V\cos\Theta(s) - \frac{2Q(s)}{\pi B(s)H(s)})\,\frac{dQ}{ds} - \Big(\frac{\sqrt{\pi}}{2}g\,\frac{\Delta\rho(s)}{\rho_R}\,\lambda_H\lambda_V^2 H^2(s) - \frac{Q^2(s)}{\pi B^2(s)H(s)}\Big)\frac{dB}{ds} - \frac{\sqrt{\pi}}{2}g\,\frac{\lambda_H\lambda_V^2}{\rho_R}\,B(s)H^2(s)\,\frac{d\Delta\rho}{ds} + S_{Fx}\cos\Theta(s) + S_{Fy}\sin\Theta(s)\Big]\cdot\Big[\sqrt{\pi}\,g\,\frac{\Delta\rho(s)}{\rho_R}\,\lambda_H\lambda_V^2\,B(s)H(s) - \frac{Q^2(s)}{\pi B(s)H^2(s)}\Big]^{-1}$$

(II.1)

Prych splits lateral spreading into a density dependent and a density independent term:

$$\frac{dB}{ds} = \left(\frac{dB}{ds}\right)_b + \left(\frac{dB}{ds}\right)_{nb}$$

The independent term is calculated from the assumption below:

$$\frac{\left(\frac{dB}{ds}\right)_{nb}}{\frac{dH}{ds}} = \frac{\left(\frac{dQ}{ds}\right)_H}{\left(\frac{dQ}{ds}\right)_V} \cdot \frac{B(s)}{H(s)}$$

where $\Delta\rho = 0$ is used in (II.1) as a consequence of the independence

$$\frac{dH}{ds} = \Big[\Big(V\cos\Theta(s) - \frac{2Q(s)}{\pi B(s)H(s)}\Big)\frac{dQ}{ds} + \frac{Q^2(s)}{\pi B^2(s)H(s)}\,\frac{dB}{ds} + S_{Fx}\cos\Theta(s) + S_{Fy}\sin\Theta(s)\Big]\cdot\Big[\frac{-Q^2(s)}{\pi B(s)H^2(s)}\Big]^{-1}$$

The density dependent term is calculated on the assumption that the boundary moves at a right angle to itself as does the celerity of a density front. A vector analysis yields thus:

$$\left(\frac{d(\sqrt{2}\,B)}{ds}\right)_b = \frac{c}{\sqrt{\bar{u}^2 - c^2}} = \frac{1}{\sqrt{F^2 - 1}}$$

where $F = \dfrac{\bar{u}}{\sqrt{g\,\dfrac{\overline{\Delta\rho}}{\rho}\,\bar{H}}}$ is the gross densimetric Froude number, $\bar{u}$, $\bar{H}$, and $\overline{\Delta\rho}$ is respectively the jet´s average value of the velocity, depth and density difference to the ambient water and c is the celerity of a density front.

Shirazi and Davis, (1974), on the other side, assumed that the density-dependent spreading is mainly balanced by the shear stress. According to an argument based on physical reasoning and to an analysis made by Koh and Fan, (1970), they changed the dependence to

$$\left(\frac{d(\sqrt{2}\,B)}{ds}\right)_b = \frac{1}{\sqrt{\frac{B}{H}\,F^2 - 1}}$$

Dunn, Policastro and Paddock, (1975), discussed the introduced factor of B/H and came to the conclusion, that the reasoning made by Shirazi and Davis is plausible but hardly rigorous and that the form of the insertion must be considered as chosen rather arbitrarily. This insertion gave rise to narrower but deeper plumes. Experiments made by, among others, Stolzenbach and Harleman, (1971), and comparisons with field data show that Prych´s model often predicts plumes that are too wide and that the Shirazi and Davis plumes have a tendency to become too narrow (see Dunn, Policastro and Paddock, (1975)).

In this model Prych´s approach is used, because in my opinion shear stress is only important in that part of the far field where the model does not work satisfactorily anyhow.

It can be established that when $F \rightarrow 1.0$ the lateral spreading becomes absurdly large and why the model only works satisfactorily for Froude numbers clearly larger than 1. But if we introduce Shirazi and Davis´ density-dependent spreading we would have worse problems with F since H would be greater. Furthermore, lateral spreading probably can´t be regarded in this simple way as a summation of a density dependent and an independent term.

The appearance of the trajectory is obtained from:

$$\frac{dx}{ds} = \cos\Theta(s)$$

$$\frac{dy}{ds} = \sin\Theta(s)$$

Since density is a function of temperature and salinity, the above equations can be solved numerically.

III A COMPARISON BETWEEN PRYCH´S MODEL AND THE MODIFIED MODEL.

The plumes, calculated by Prych´s model, have often been too wide or too narrow. In the modified model, the fundamental equations, except for ZFE, have not been changed much. In spite of this, the calculated plumes are noticably narrower and thicker. Furthermore, the jet´s deflection by the surrounding recipient current is stronger and excess temperature on the surface diminishes faster. Part of the explanation for this is the change in the calculations for ZFE and the change in the spreading ratio between heat and momentum. Examples of this are given in figures III.1-2, which show the excess temperature in the plume´s near field in two different cases.

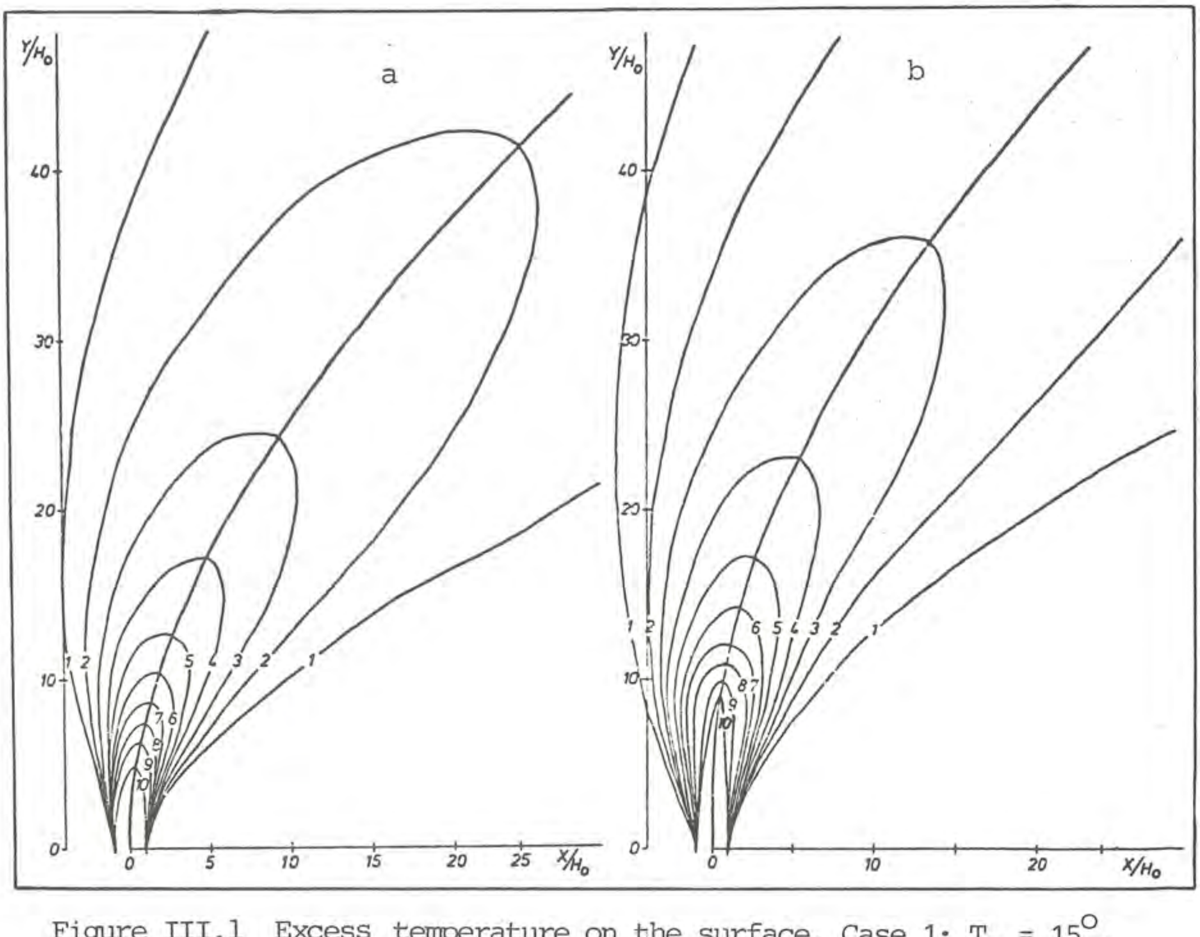


Figure III.1 Excess temperature on the surface. Case 1; $T_R = 15^o$, $S_R = 0‰$, A = 2.0, R = 0.125 and $F_0 = 6.30$.
a = modified model
b = Prych´s model

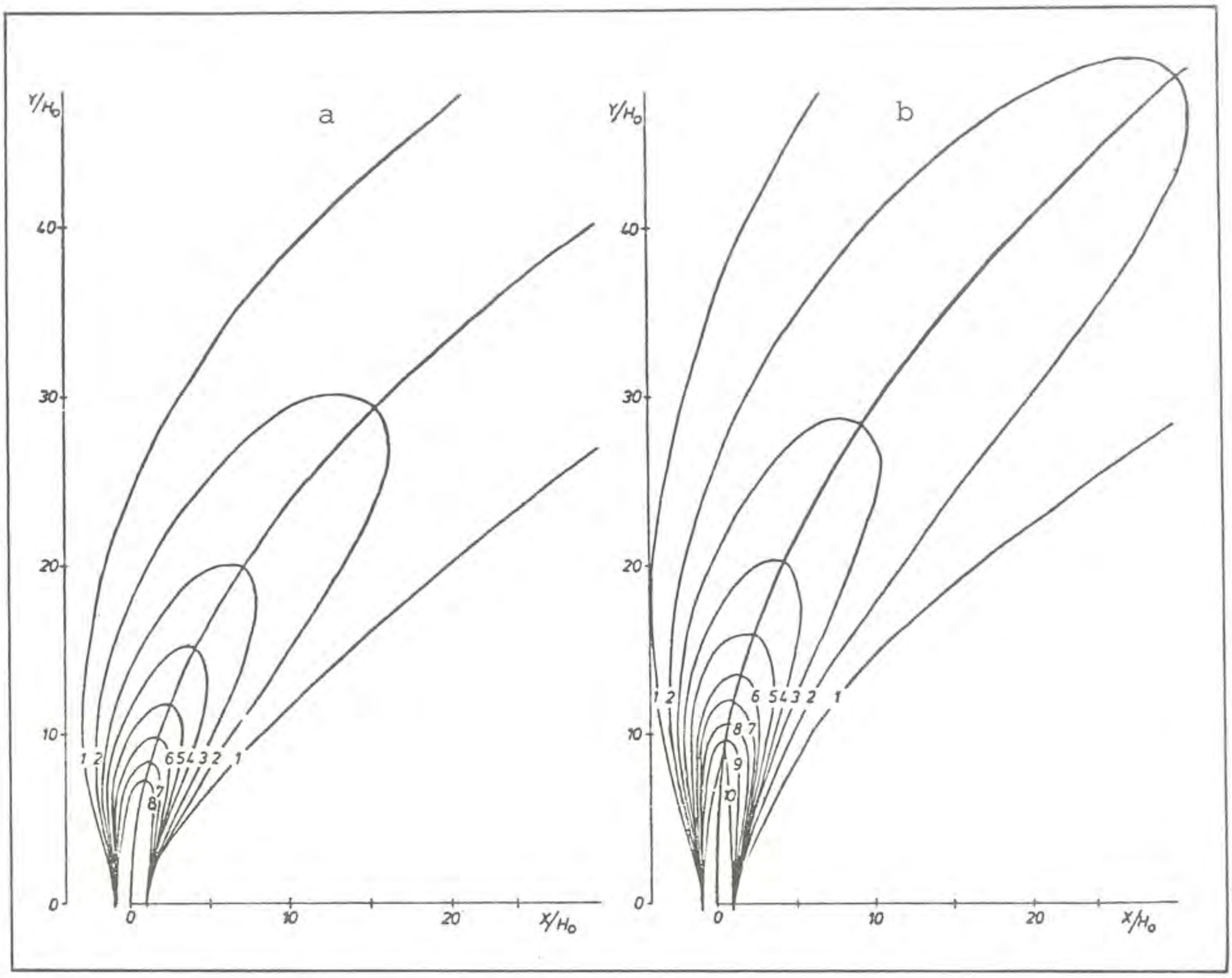


Figure III.2 Excess temperature on the surface. Case II; T_R = 5 ^oC, S_R = 0 ‰, A = 2.0, R = 0.125 and F_0 = 9.85.
a = Modified model
b = Prych´s model

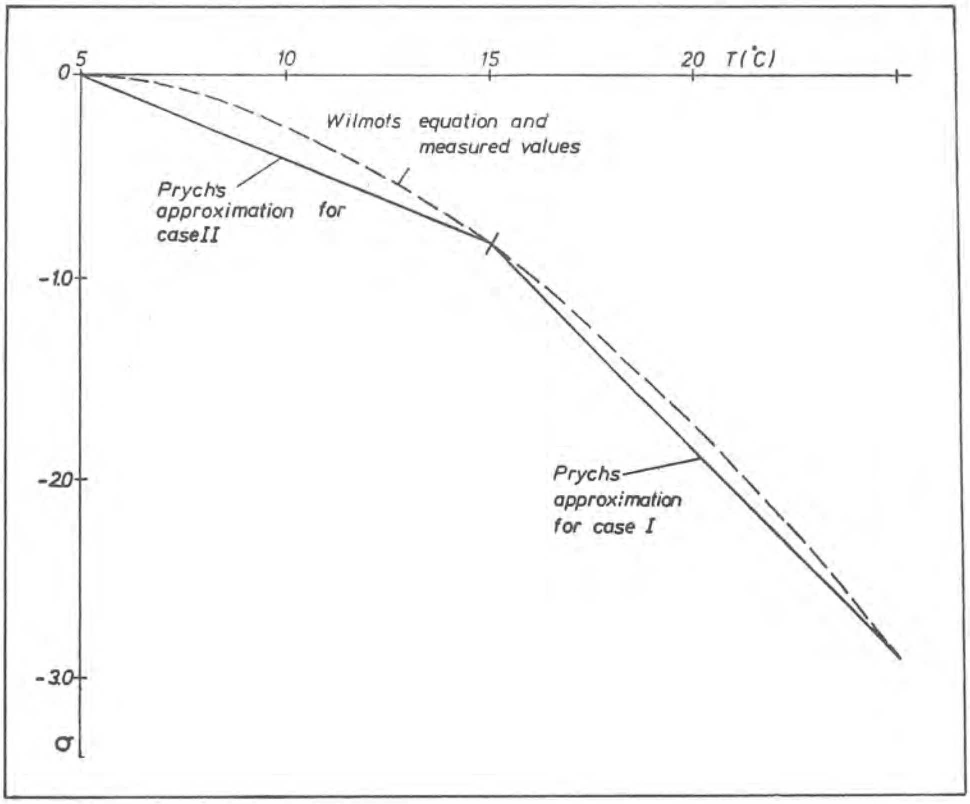


Figure III.3 Examples of Prych´s model and the modified model´s density approximation.

The other part of the explanation is the improved density calculation. Wilmot´s equation of state is, as can be seen in figure III.3, very close to reality. On the other hand, Prych´s approximation is significantly poorer, expecially at low temperatures.

The effect of the more realistic density calculations can be made clear by Figures III.4-5. In the first example the ambient temperature is 15 °C, which means that Prych´s approximation yields a rather good picture of the dependency that density has on temperature, while the approximation in the second example when T_R = 5 °C is poorer. The figures show that deviation between the two models increases when the ambient temperature decreases. Particularly, it can be established that the width decreases and the depth increases, which is desirable. Furthermore, the plume is deflected faster by the ambient current.

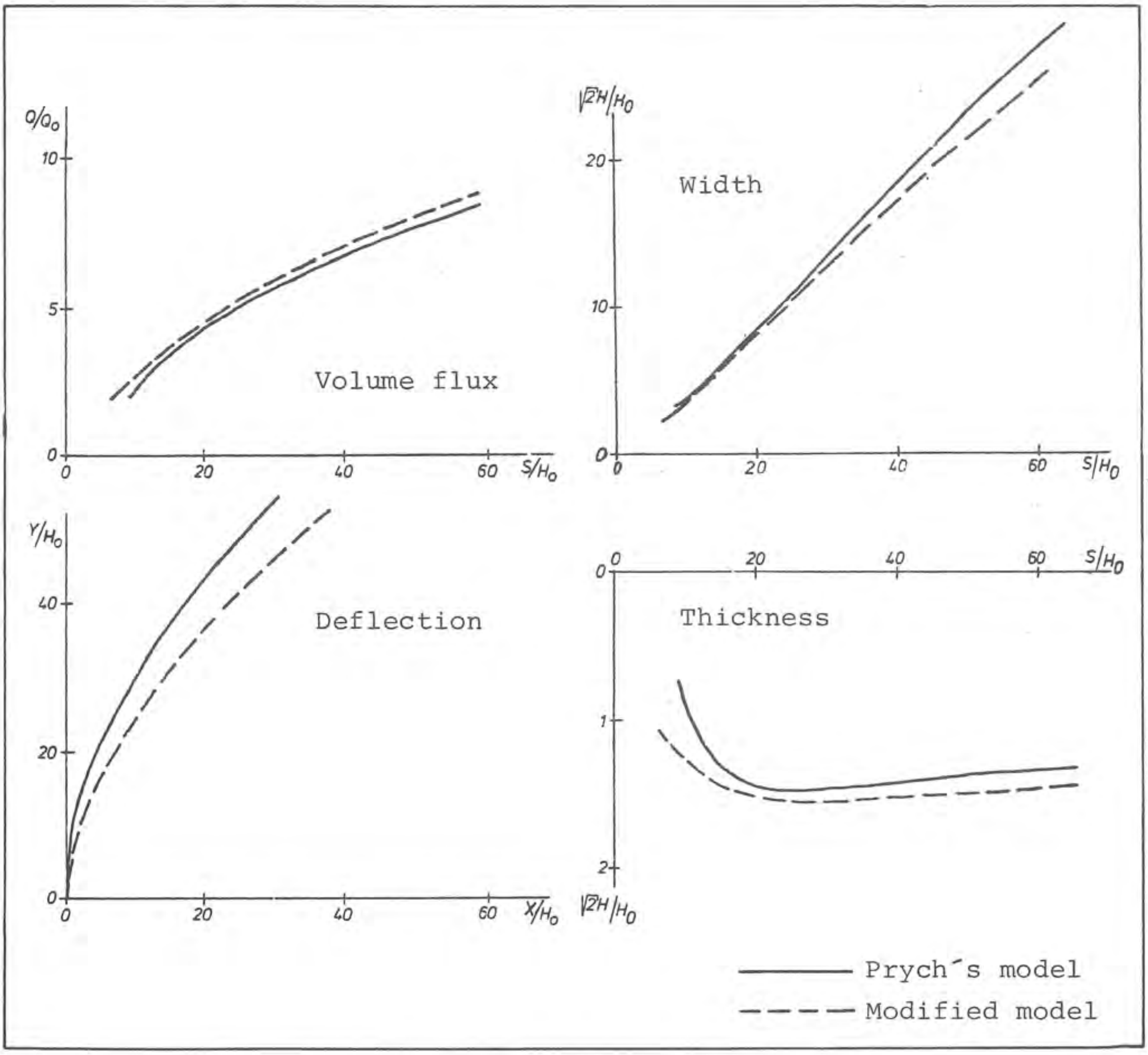


Figure III.4 Case I. T_R = 15 °C, S_R = 0‰, A = 2.0, R = 0.125 and F_0 = 6.30.

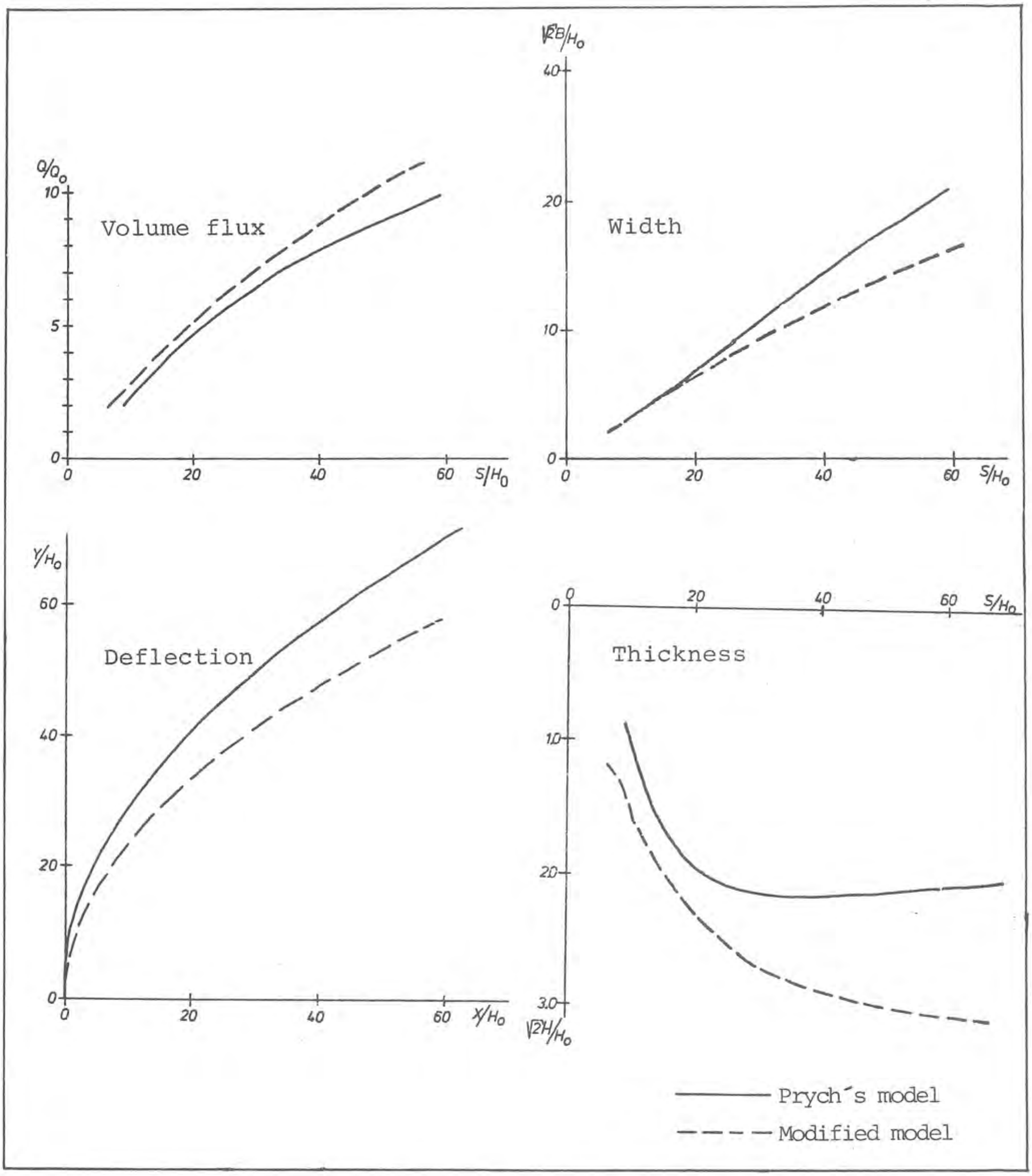


Figure III.5 Case II. $T_R = 5\ ^oC$, $S_R = 0‰$, A = 2.0, R = 0.125 and $F_0 = 9.85$.

IV COMPARISON BETWEEN CALCULATED AND SURVEYED COOLING-WATER PLUMES.

In an attempt to verify Prych's model, Weil, (1974), compared calculated and surveyed cooling-water plumes off the coast of the Oskarshamn nuclear power plant. The discharge takes place in Hamnefjärden, see figure IV.1, in the direction nearly directly towards the opposite shore. Weil began his calculations at the mouth of Hamnefjärden, where the surveyed Froude number was very near 1.0 in all cases.

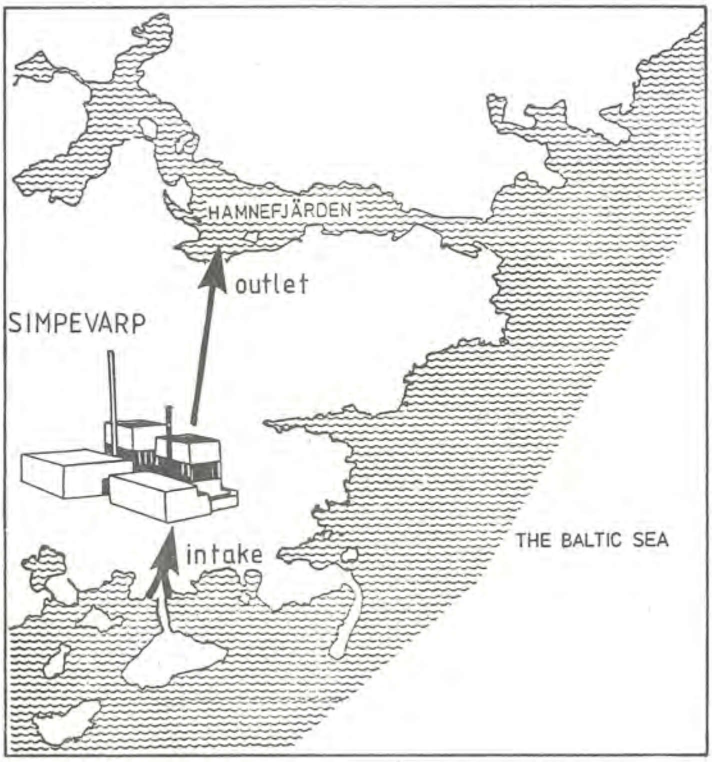


Figure IV.1. The position of the intake and outlet at Oskarshamn nuclear power plant.

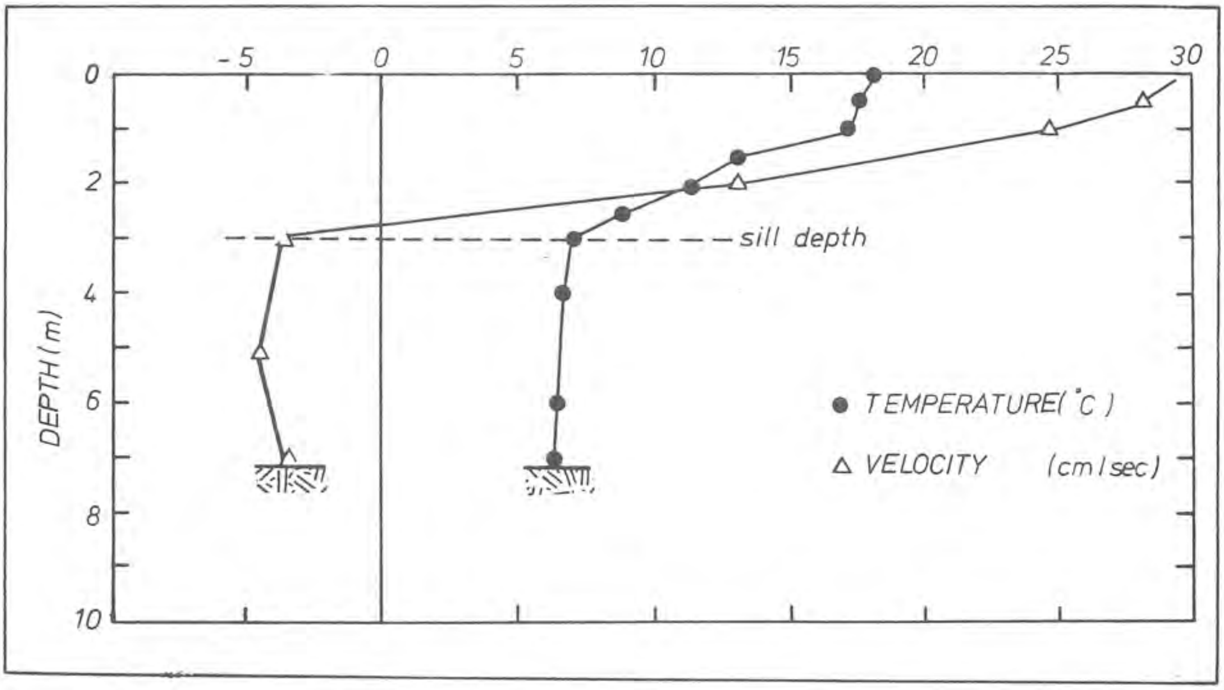


Figure IV.2 Temperature and current profile in the mouth of Hamnefjärden.

When the Froude number is so low, the model gives rather poor results, due to, among other things, the fact that the density dependency of lateral spreading remains as

$$\left(\frac{dB}{ds}\right)_b = \frac{1}{\sqrt{2(F^2 - 1)}}$$

which forms immoderately wide plumes. When the Froude number is so low, the plume hasn´t even any pronounced jet characteristics, and instead is easily pressed towards the shore as a passive part of the recipient current.

In spite of these bad conditions, Weil has shown that predictions with Prych´s model of plume axis temperatures and trajectory are reasonably good. The calculated plumes are, on the other hand, too wide.

No other proper verifications for Swedish cooling-water outlets have been carried out.

V LIST OF SYMBOLS

A	aspect ratio
A'	relation between the outlet's width and the rest of the outlet's circumference
a	dimensionless coefficient
B	local characteristic width of jet
B'	half width of jet at end of Prych's initial zone analysis
B_0	half width of outlet
C_D	form drag coefficient
C_F	interfacial shear drag coefficient
c	celerity of a density front
D	diameter
E	entrainment coefficient for a two-dimensional homogeneous density jet flow
F	gross densimetric Froude number
F_D	kinematic form drag per unit length of jet
F_0	densimetric Froude number at outlet, $u_0 / \sqrt{g \frac{\Delta\rho_0}{\rho_R} H_0}$
f	multiplier to reduce vertical entrainment because of vertical density gradient
g	acceleration due to gravity
H	local characteristic thickness of jet
H'	depth of jet at end of Prych's initial zone analysis
H_0	depth of outlet
J	local kinematic heat flux in jet
k	atmospheric heat transfer coefficient
M	local kinematic s-momentum flux in jet
n	curvilinear coordinate, horizontal and perpendicular to s
P	kinematic pressure force on jet cross section
Q	local volume flux in jet
R	ratio of ambient current to discharge velocity
Ri	Richardson number
r	radius
S	salinity
ΔS	salinity in excess of ambient salinity
S_{Fx}, S_{Fy}	kinematic shear forces in x- and y-directions per unit length of jet

S_λ dimensionless coefficient, $\sqrt{(1+\lambda_H^2)(1+\lambda_V^2)}$

s curvilinear coordinate along jet trajectory

s_z distance from outlet to end of ZFE

T water temperature

ΔT water temperature in excess of ambient water temperature

u local excess jet velocity on jet trajectory

$\bar{u}$ mean velocity in s-direction in jet

Δu vector difference between the velocity at the surface and the velocity of the ambient fluid

V ambient current velocity

x rectilinear coordinate parallel to ambient current

y rectilinear coordinate, horizontal and perpendicular to x

z coordinate in vertical direction

z' dummy variable

ε ambient turbulent diffusion coefficient

Θ angle between positive s- and x- directions

λ spreading factor for scalar parameters

ν kinematic viscosity

ρ fluid density

$\Delta\rho$ local density disparity (from ambient)

Φ spreading angle

Subscripts

0 refers to discharge

R refers to ambient

J refers to jet

H refers to horizontal

V refers to vertical

b refers to buoyant spreading

nb refers to non-buoyant spreading

z refers to conditions at end of ZFE

VI REFERENCES

Abraham, G.(1963) "Jet Diffusion in Stagnant Ambient Fluid", Delft Hydraulics Laboratory, Publ. No 29.

Albertson, M.L., Dai, Y.B., Jensen, R.A. och Rouse, H., (1950) "Diffusion of Submerged Jets", Transactions American Society of Civil Engineers, Vol. 115, pp. 639-664.

Dunn, W.E., Policastro, A.J. och Paddock,R.A., (1975) "Surface Thermal Plumes: Evaluation of Mathematical Models for the Near and Complete Field". Argonne National Laboratory, Argonne, Illinois.

Fan, L.-N. (1967) "Turbulent Buoyant Jets into Stratified or Flowing Ambient Fluids". Report No. KH-R-15, W.M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology.

Fan, L.-N. och Brooks, N.H. (1969) "Numerical Solutions of Turbulent Buoyant Jet Problems", Report KH-R-18, W.M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology.

Fredrich, H. och Levitus, S. (1972) "An Approximation to the Equation of State for Sea Water, Suitable for Numerical Ocean Models", Journal of Physical Oceanography, Vol.2, No. 4.

Gordier, R.L. (1959) "Studies on Fluid Jets Discharging Normally into Moving Liquid", St. Anthony Falls Hyd. Lab. Technical Paper 28, Series B, University of Minnesota.

Hirst, E. (1972) "Zone of Flow Establishment for Round Buoyant Jets", Water Resources Research, Vol.8, No.5, pp. 1234-1246.

Keffer, J.F. och Baines, W.D. (1963) "The Round Turbulent Jet in Crosswind", Journal of Fluid Mechanics, Vol. 15, No. 4, pp. 481-496.

Koh, R.C.Y. och Fan, L.-N. (1970) "Mathematical Models for the Prediction of Temperature Distributions Resulting from the Discharge of Heated Water into Large Bodies of Water", U.S. EPA, Water Pollution Control Research Series, Report 1613 DWO 10/70.

Lal, P.B.B. och Rajaratnam, N. (1975) "An Experimental Study of Heated Surface Discharges into Quiescent Ambients", University of Alberta, Department of Civil Engineering, Canada.

Motz, L.H. och Benedict, B.A. (1970) "Heated Surface Jet Discharged into a Flowing Ambient Stream", Department of Environmental and Water Resources Engineering, Vanderbilt University, Nashville, Tenn. Report No.4.

Odgaard, J. (1975) "Parameter Sensitivity in the Estimation of Excess Temperatures in Natural Water Bodies", IAHR Proceedings, Vol. 3, pp. 191-198.

Platten, J.L. och Keffer, J.F. (1968) "Entrainment in Deflected Axisymmetric Jets at Various Angles to the Stream", University of Toronto, Department of Mechanical Engineering, Technical Publication Series, Rep. UTME-TP-6808.

Pratte, B.D. - Baines, W.D. (1967)"Profiles of the Round Turbulent Jet in a Cross Flow", Journal of the Hydraulics Division, ASCE, Vol. 95, No. HY6, pp. 53-64.

Prych, E.A. (1972) "A Warm Effluent Analyzed as a Buoyant Surface Jet", Sveriges Meteorologiska och Hydrologiska Institut, Serie Hydrologi. Nr.21, Stockholm.

Rouse, H., Yih, C.S. och Humphreys, H.W. (1952) "Gravitational Convection from a Boundary Source", Tellus, Vol.4, No. 3, pp. 201-210.

Sami,S., Carmody, T. och Rouse,H. (1967) "Jet Diffusion in the Region of Flow Establishment", Journal of Fluid Mechanics, Vol. 27, No.2, pp. 231-252.

Shirazi, M.A. och Davis,L. (1974) "Workbook of Thermal Plume Prediction, Volume 2, Surface Discharge", National Environmental Research Center, Office of Research and Development. U.S. EPA, Corvallis, Oregon.

Stefan,H., Bergstedt,L. och Mrosla,E. (1975) "Flow Establishment and Initial Entrainment of Heated Water Surface Jets", St. Anthony Falls Hydraulic Laboratory, University of Minnesota.

Stefan,H., Hayakawa,N. och Schiebe, F. (1971) "Surface Discharge of Heated Water Surface Jets", IAHR Proceedings, Vol. pp. 333-341.

Stefan, H. och Vaidyaraman,P. (1972) "Jet Type Model for the Three-Dimensional Thermal Plume in a Crosscurrent and under Wind", Water Resources Research, Vol. 8, No. 4, pp. 998-1014.

Stolzenbach, K. och Harleman, D.R.F. (1971) "An Analytical and Experimental Investigation of Surface Discharges of Heated Water", Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics, Department of Civil Engineering, MIT, Cambridge, Mass., Report No. 135.

Weil, J. (1974) "Verification of Heated Water Jet Numerical Model", Sveriges Meteorologiska och Hydrologiska Institut, Rapporter Hydrologi och Oceanografi. Nr RHO 1.

Wilmot, W. (1976) "A Numerical Model of the Effects of Reactor Cooling Water on Fjord Circulation", Sveriges Meteorologiska och Hydrologiska Institut, Rapporter Hydrologi och Oceanografi, Nr. RHO 6.

Notiser och preliminära rapporter

Serie HYDROLOGI

Nr 1 Sundberg-Falkenmark M
Om isbärighet. Stockholm 1963

Nr 2 Forsman, A
Snösmältning och avrinning. Stockholm 1963

Nr 3 Karström, U
Infrarödteknik i hydrologisk tillämpning: Värmebilder som hjälpmedel i recipientundersökningar. Stockholm 1966

Nr 4 Moberg, A
Svenska sjöisars isläggnings- och islossningstidpunkter 1911/12-1960/61. Del 1. Redovisning av observationsmaterial. Stockholm 1967

Nr 5 Ehlin, U & Nyberg, L
Hydrografiska undersökningar i Nordmalingsfjärden. Stockholm 1968

Nr 6 Milanov, T
Avkylningsproblem i recipienter vid utsläpp av kylvatten. Stockholm 1969

Nr 7 Ehlin, U & Zachrisson, G
Spridningen i Vänerns nordvästra del av suspenderat meteriel från skredet i Norsälven i april 1969. Stockholm 1969

Nr 8 Ehlert, K
Mälarens hydrologi och inverkan på denna av alternativa vattenavledningar från Mälaren. Stockholm 1970

Nr 9 Ehlin, U & Carlsson, B
Hydrologiska observationer i Vänern 1959-1968 jämte sammanfattande synpunkter. Stockholm 1970

Nr 10 Ehlin, U & Carlsson, B
Hydrologiska observationern i Vänern 17-21 mars 1969. Stockholm 1970

Nr 11 Milanov, T
Termisk spridning av kylvattenutsläpp från Karlshamnsverket. Stockholm 1971

Nr 12 Persson, M
Hydrologiska undersökningar i Lappträskets representativa område. Rapport I. Stockholm 1971

Nr 13 Persson, M
Hydrologiska undersökningar i Lappträskets representativa område. Rapport II. Snömätningar med snörör och snökuddar. Stockholm 1971

Nr 14 Hedin, L
Hydrologiska undersökningar i Velens representativa område. Beskrivning av området, utförda mätningar samt preliminära resultat. Rapport I. Stockholm 1971

Nr 15 Forsman, A & Milanov, T
Hydrologiska undersökningar i Velens representativa område. Markvattenstudier i Velenområdet. Rapport II. Stockholm 1971

Nr 16 Hedin, L
Hydrologiska undersökningar i Kassjöåns representativa område. Nederbördens höjdberoende samt kortfattad beskrivning av området. Rapport I. Stockholm 1971

Nr 17 Bergström, S & Ehlert, K
Stochastic Streamflow Syntheses at the Velen representative Basin. Stockholm 1971

Nr 18 Bergström, S
Snösmältningen i Lappträskets representativa område som funktion av lufttemperaturen. Stockholm 1972

Nr 19 Holmström, H
Test of two atuomatic water quality monitors under field conditions. Stockholm 1972

Nr 20 Wennerberg, G
Yttemperaturkartering med strålningstermometer från flygplan över Vänern under 1971. Stockholm 1972

Nr 21 Prych, A
A warm water effluent analyzed as a bouyant surface jet. Stockholm 1972

Nr 22 Bergström, S
Utveckling och tillämpning av en digital avrinningsmodell. Stockholm 1972

Nr 23 Melander, O
Beskrivning till jordartskarta över Lappträskets representativa område. Stockholm 1972

Nr 24 Persson, M
Hydrologiska undersökningar i Lappträskets representativa område. Rapport III. Avdunstning och vattenomsättning. Stockholm 1972

Nr 25 Häggström, M
Hydrologiska undersökningar i Velens representativa område. Rapport III. Undersökning av torrperioderna under IHD-åren fram t o m 1971. Stockholm 1972

Nr 26 Bergström, S
The application of a simple rainfall-runoff model to a catchment with incomplete data coverage. Stockholm 1972

Nr 27 Wändahl, T & Bergstrand, E
Oceanografiska förhållanden i svenska kustvatten. Stockholm 1973

Nr 28 Ehlin, U
Kylvattenutsläpp i sjöar och hav. Stockholm 1973

Nr 29 Andersson, U-M & Waldenström, A
Mark- och grundvattenstudier i Kassjöåns representativa område. Stockholm 1973

Nr 30 Milanov, T
Hydrologiska undersökningar i Kassjöåns representativa område. Markvattenstudier i Kassjöåns område. Rapport II. Stockholm 1973

SMHI Rapporter

HYDROLOGI OCH OCEANOGRAFI

Nr RHO 1 Weil, J G
Verification of heated water jet numerical model.
Stockholm 1974

Nr RHO 2 Svensson, J
Calculation of poison concentrations from a hypothetical accident off the Swedish coast.
Stockholm 1974

Nr RHO 3 Vasseur, B
Temperaturförhållanden i svenska kustvatten.
Stockholm 1975

Nr RHO 4 Svensson, J
Beräkning av effektiv vattentransport genom Sunninge sund.
Stockholm 1975

Nr RHO 5 Bergström, S & Jönsson, S
The application of the HBV runoff model to the Filefjell research basin.
Norrköping 1976

Nr RHO 6 Wilmot, W
A numerical model of the effects of reactor cooling water on fjord circulation.
Norrköping 1976

Nr RHO 7 Bergström, S
Development and Appl. of a Conceptual Runoff Model.
Norrköping 1976

Nr RHO 8 Svensson, J
Seminars at SMHI 1976-03-29—04-01 on Numerical Models of the Spreading of Cooling-water.
Norrköping 1976

Nr RHO 9 Simons, J & Funkquist, L & Svensson, J
Application of a numerical model to Lake Vänern.
Norrköping 1977

Nr RHO 10 Svensson, J
A statistical study for automatic calibration of a conceptual runoff model.
Norrköping 1977

Nr RHO 11 Bork, I
Model studies of dispersion of pollutants in Lake Vänern.
Norrköping 1977

Nr RHO 12 Fremling, S
Sjöisars beroende av väder och vind, snö och vatten.
Norrköping 1977

SMHI Rapporter

HYDROLOGI OCH OCEANOGRAFI

Nr RHO 13	Fremling, S Sjöisars bärighet vid trafik. Norrköping 1977
Nr RHO 14	Bork, I Preliminary model studies of sinking plumes. Norrköping 1978
Nr RHO 15	Svensson, J & Wilmot, W A numerical model of the circulation in Öresund. Evaluation of the effect of a tunnel between Helsingborg and Helsingör. Norrköping 1978
Nr RHO 16	Funkquist, L En inledande studie i Vätterns dynamik. Norrköping 1979
Nr RHO 17	Vasseur, B Modifying a jet model for cooling water outlets. Norrköping 1979