STATISTICAL ANALYSIS OF SNOW SURVEY DATA

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1. Introduction

Spatial variation of snow cover in Sweden has been very little studied up til now. This is quite remarkable as snow plays a predominant rôle in runoff formation in the whole of Sweden. In operational practice today the snow cover is calculated as accumulated precipitation for periods with temperatures below zero. Errors of snow measurements from precipitation gauges are very large and that is why the accumulated precipitation gives only rough estimates of the actual snow cover. Snowmelt periods during winter make it necessary to include a snowmelt procedure into calculations, which introduces a further source of uncertainty. Snowdepths are measured in connection with precipitation measurements but are not completed by information about water content, so important for hydrologists. The present practice can give acceptable results in certain regions with stable climate conditions and forests i e in the woodlands of Norrland. For other parts of Sweden i e for the mountain region and the south of Sweden the present practice is doubtful.

The program of water balance studies in small research basins in Sweden includes snow surveys. The present program, which is a continuation of the IHD-program for representative basins, was started in 1978. To give the necessary background for snow surveys in small research basins, all snow surveys that have been done within the IHD-program, the present program and also others (see chapter 2) were analyzed statistically. The result of this statistical study is presented here. Conclusions are drawn of this statistical analysis to give the necessary guide for snow survey in small research basins.

The observational material used is naturally very non-homogeneous. The goals for data collection have been different and different institutions have been involved. A statistical treatment should be adapted to the data quality. Application of statistical methods was limited by the character of observational data and only parts of it would be considered for each statistical treatment.

Distribution functions of the snow cover were studied for all observational data. The space distribution, being characterized by the correlation function, was calculated for snow courses of a certain length with equidistant observations. In a few cases the observational data also permitted to study the influence of landscape on snow cover distribution by means of analysis of variances. The results of the statistical analyses are given in chapter 3.

Standard errors in the estimation of areal means of the snow cover from snow surveys give a fundamental information about how to perform snow surveys. In chapter 4 standard errors are analyzed taking into consideration a space dependence of the snow cover.

Chapter 5, finally, summarizes the experiences, drawn from the statistical analysis and give advices how to perform snow surveys.

2. Observations

The observational data studied in this paper, snow depths and water equivalents, were taken from several snow surveys performed in small research basins. Most of the measurements were done in basins belonging to the IHD-program. In addition to the normal surveys, with a distance varying from 25 to 100 m between the measuring points, some more dence ooservational series of snow depth (distance between measuring points: 1 m) were specially made for this study.

The different sites of the snow surveys are described below, in figure 2.1 and table 3.1.

Upper Lule älv
The region is a high mountain area. All sites belong to the Suorva basin. The Lake of Suorva is used as a hydroelectric reservoir. The Swedish State Power Board has performed snow surveys in this area every year since 1944 to decide the inflow to the reservoir. These snow courses are situated in the woodland at an elevation of about 900 m a s l and in the bare mountain area above the upper limit of the forest (elevation about 1000 m a s l). In 1973 80 observation points were added (elevation about 400 m a s l).

Lappträsket (Solmyren, Vuoddasbäcken)
Most of the data from Lappträsket were taken from snow surveys in the subbasins Solmyren and Vuoddasbäcken. The region is representative for the swamp and forest area of northern Sweden. One main characteristic of topography is the presence of several mountain ridges running about parallel through the area.

Kassjöån (Norrsjön, Lilla Tivsjön)
Norrsjön and Lilla Tivsjön are two of the subbasins in the main basin Kassjöån. The area is representative for the forested areas of north central Sweden. The forests are to a quite large extent used for timber cutting which means that there are many clear-cuttings in this area.

Buskbäcken
The basin is representative for the hilly lands of the southern forest region. Before the area was chosen as a representative basin for hydrological purposes, it was used for research work concerning effects of forest fertilization and clearcutting on ground and water.

Tärnsjö
The research basin is situated on the east side of a large esker running north-south. The surface water divide of the basin mainly coincides with the crest of this esker. As in most of the basins vegetation is dominated by coniferous trees.

Stormyra
The basin is representative for the east coastal region of the southern part of Sweden. The landscape within the basin is hilly and there are no lakes. The percentage of bare rocks is high.

Velen, Nolsjön
Velen and Nolsjön (a subbasin of Velen) are situated in the forested areas between the two big lakes Vänern and Vättern. Velen is characterized by a long valley running north-south through almost the entire basin. The surrounding landscape is hilly.

Skärkind
The area is representative for the central plains of Götaland. Cultivated land (65%) dominates the basin. The open fields of the basin are here and there broken by small woodland areas.

Hulubäcken
The area is representative for the southern Swedish peatlands. The whole of this basin consists of peatland. An exception though for the lowest part of the basin where a small coniferous woodland area is to be found.

Table 2.1

Research basins

Basin	Area (km^2)	Elevation (m a s l)	Observation period (snow surveys)
Upper Lule älv	-	400-1000	1944 -
Lappträsket	1004	50- 485	1968 - 1977
Solmyren	27.5	326- 466	1968 -
Vuoddasbäcken	42	187- 439	1968
Kassjöån	165	225- 530	1968 - 1977
Norrsjön	15.3	391- 530	1968 -
Lilla Tivsjön	12.8	246- 460	1968 -
Tärnsjö	15.7	55- 110	1978 -
Buskbäcken	1.8	280- 350	1978 -
Velen	45	120- 175	1966 -
Nolsjön	18	126- 175	1966 -
Stormyra	3.9	10- 77	1979 -
Skärkind	7.6	45- 87	1978 -
Hulubäcken	3.7	322- 348	1979 -

Figure 2.1

Location of studied research basins

3. Spatial variation of the snow pack

Spatial variation of the snow pack is affected by meteorological and physiographic factors as well as their interaction. The free atmospheric processes give rise to a variability of precipitation fields. This variability is of different scales. Meteorologists define the following three scales (Eagleson, 1970):Microscale, Masoscale and Synoptic scale (Macroscale). In the microscale the convection cells predominate. The agglomeration of cells define the mesoscale. Front systems finally are said to be of the synoptic scale. The precipitation fields shows variability in all these scales. The variability between different points in one and the same area is of the same order as the variability between one territory and the other.

The main influence of the landscape physiography is orographic effects. Factors of importance are topography, surface roughness and distribution of land and water. The spatial variation of the snow pack is an integrated effect over several precipitation events. The variability at a certain place caused by the free atmospheric processes, as commented above, is to great extent smoothed out when averaging over time. In a mesoscale the orographic effects thus play the predominant rôle in the space variation of the snow pack.

The weather between storm events also affects the snow cover. Wind drift causes scour and redistribution Physiographic factors such as topography and vegetation play an important rôle in how the snow pack is redistributed by wind drift.

Gray et al (1978) give the following description of snow pack variability in the three different scales:

(1) Macroscale, or regional variability, includes areas from 1 to 1 x 10^6 km^2, depending on location, elevation, orography, etc. On this scale, dynamic meteorological effects such as formation of standing waves, directional flow of wind streams around barriers, and lake effects are important.

(2) Mesoscales, or local (within a region) variability have characteristic linear distances of 10^2 to 10^3 m. On this scale, wind and avalanches can cause redistribution. Deposit and accumulation can be related to terrain variables such as elevation, slope and aspect, and to vegetal cover variables such as canopy and crop density, tree species or crop type, and height and completeness of cover.

(3) Microscale variability encompasses major differences within distances of 10 to 10^2 m. On this scale, accumulation patterns are the result of numerous interactions. Surface roughness and its effect on transport phenomena are of primary importance.

One can get the general pattern of the snow pack distribution from maps over average snow depths (Pershagen, 1969). These maps are based on observations of snow depths at precipitation stations.

The following zones can be identified according to average snow depths and the average number of days in a year with snow cover (fig 3.1)

1. lowlands of Götaland and Svealand
2. highlands of Götaland
3. highlands of Svealand
4. coastal zone of Norrland
5. southern woodlands of Norrland
6. northern woodlands of Norrland
7. mountain region

The zones are ordered according to increasing average snow depth and length of snow period. The zonality reflects climatic variations.

The variability within climatic zones, of micro- and mesoscale, can be studied with the help of snow survey data. The significance of differences in the mean of snow surveys performed in different areas can be tested by analyses of variance. The distribution of the snow pack over a homogeneous area is characterized by its distribution function. The main characteristics of the distribution function are the mean and the standard deviation. As an alternative to the standard deviation the coefficient of variation can be used, which is more stable value. The coherence of snow cover characteristics from point to point within a homogeneous area can be described by the spatial correlation function or the structural function.

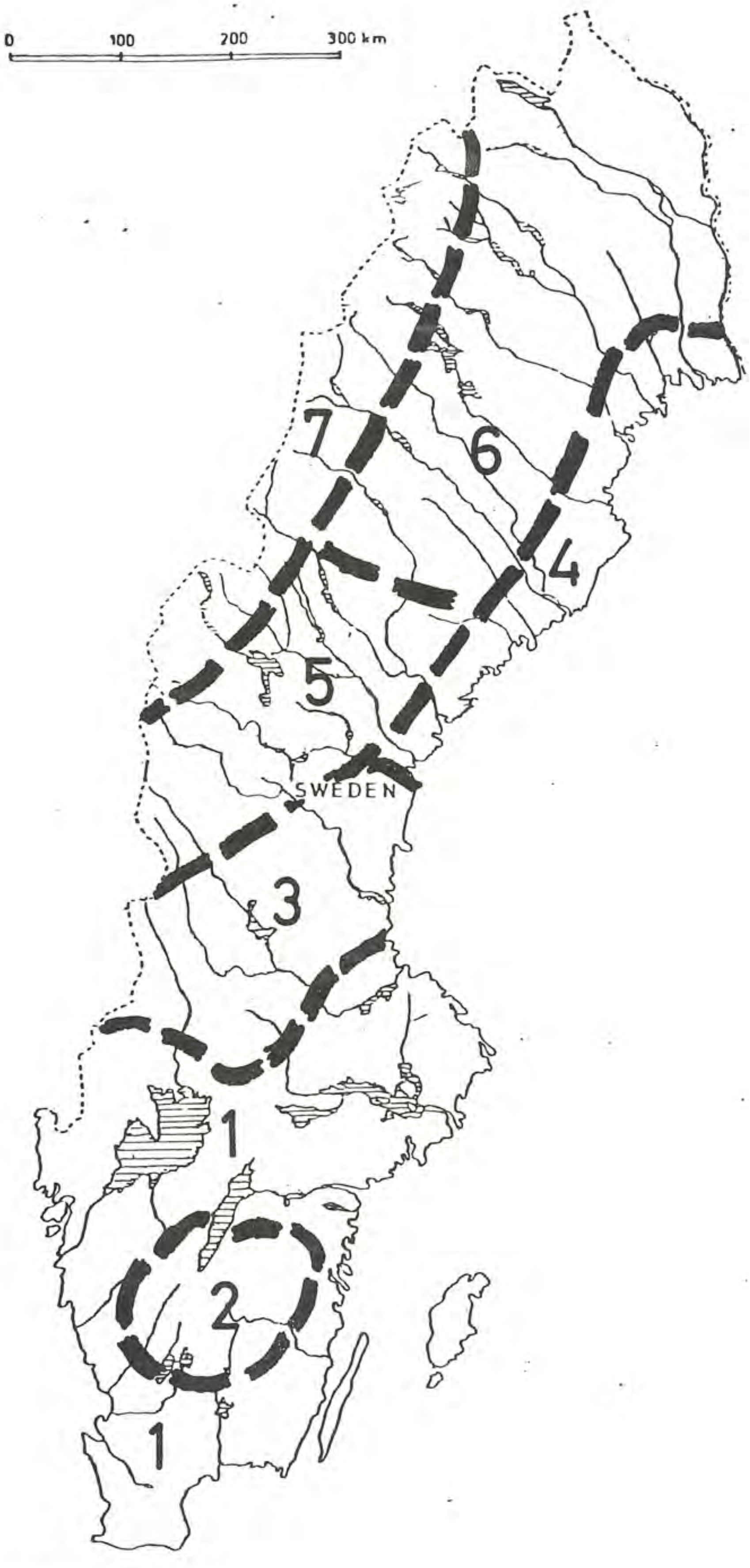


Figure 3.1

Zones reflecting average snow depths and length of snow period.

3.1 Mean values

Snow surveys should be conducted in relatively uniform areas, representative of the surrounding territory. The criteria of the representativity of snow survey data can be the differences in average values of snow cover characteristics for different areas. Comparison of averages of snow cover also describes the homogeneity of a region.

3.1.1 Dependence on physiography

The main efforts in snow studies have been to define the dependence of snow cover with height and to find out differences in snow accumulation between forests and open fields. Assuming that the causes to differences in the snow cover are of orographic origin, the height and distribution of open fields and forests are important factors. They are, however, not the only ones that can be of significance. It is well known, that in coastal regions the maximal accumulation of snow is observed several kilometers inland. Changes in distribution of land and water is thus of importance too. Other sudden roughness variations can cause the same kind of phenomena with delayed effects in space. Wind drift causes redistribution of the snow pack with accumulation on places that are less wind exposed e g forest edges, troughs in the landscape. Differences in the means of water equivalents of the snow pack and the means of snow depths estimated from snow surveys can, thus, be of a very local character. We now assume that a comparison is made within the climatic zones that were sketched in fig. 3.1.

For areas with stable climatic conditions relations can be set up between the mean snow equivalent and height or between the mean accumulation in forests and open fields. Such relationships are of course valid only for the area for which they have been derived. They further on do not show stability from year to year (M Persson, 1971, A Waldenström, 1975)(fig 3.2). The statistical uncertainty that is involved in the determination of the relationships is one cause to this instability. Another reason is that the weather situations that give snow, show an everchanging pattern from year to year and give rise to different accumulation patterns of the snow pack. For an area with significant differences in heights the relation between mean snow equivalent and height can be expected to be the most stable one, as with increasing height the average snow accumulation period increases.

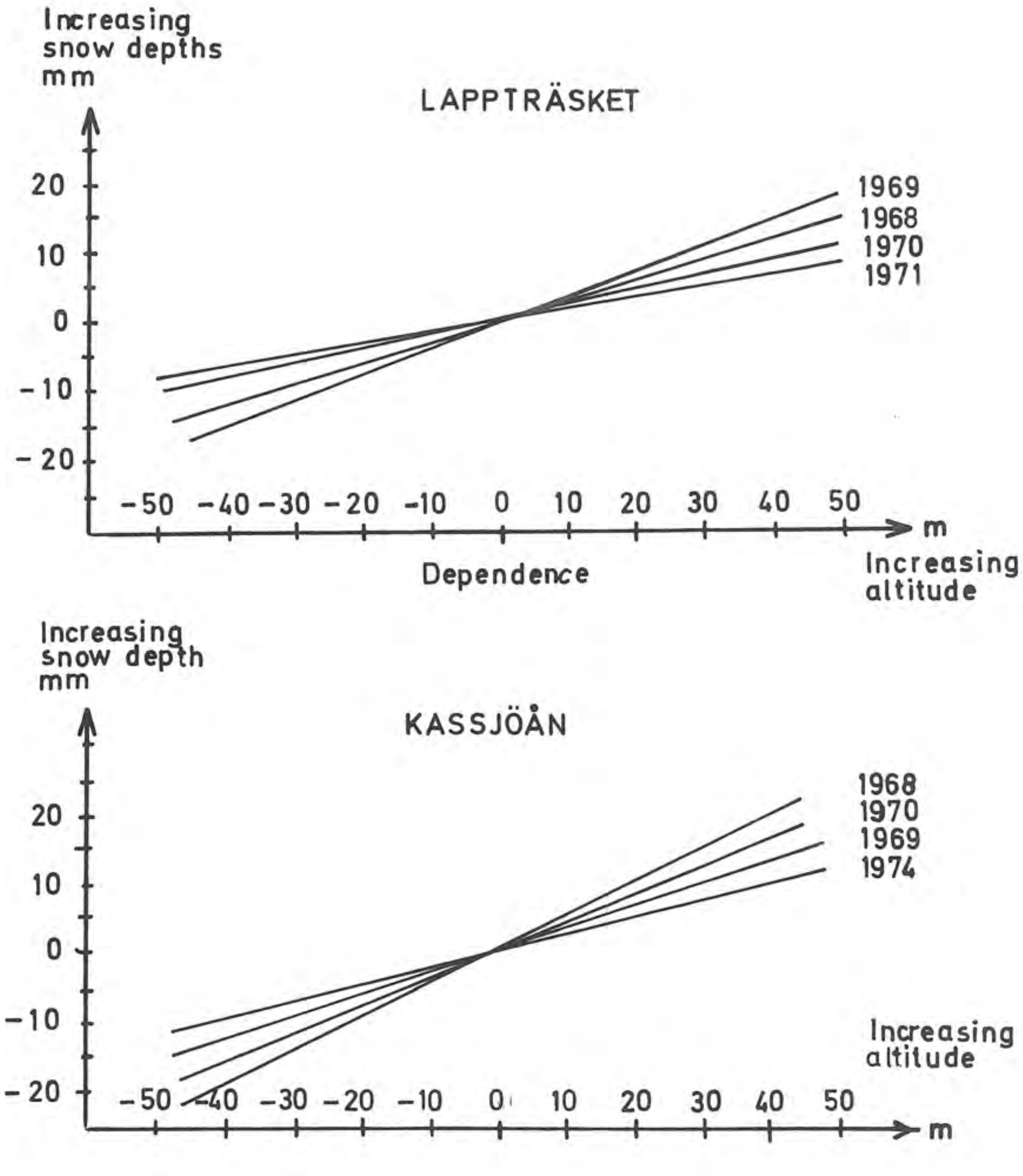


Fig. 3.2

Dependence snow depth and altitude for different years in Lappträsket and Kassjöån.

3.1.2 Stratified sampling

The mean value for an area A is calculated as a weighted mean of the snow surveys made in different homogeneous subareas:

$$m^{*} = \sum_{i}^{k} c_i \bar{x}_i \quad (\Sigma c_i = 1) \qquad (3.1)$$

where m^{*} is the estimated areal mean

$\bar{x}_i$ mean value of the i:th snow survey

and c_i the weight factor

The weights are chosen as $c_i = a_i/A$, where a_i is the area of subarea i.

To estimate the areal mean we thus used stratified sampling. Our strata are homogeneous subareas. Within each subarea we have one or more snow surveys. What we gain by applying stratified sampling is better precision in the estimated mean. This gain in precision will be illustrated below with some examples.

The snow surveys in Skärkind, Vuoddasbäcken and Solmyren in 1977 and 1978 allow us to apply the analysis of variances (AV), which is a method to study differences in the means of subsamples. The variances of the total sample and subsamples are the tools for this analysis. This information about the variances is the only one we need to see whether a stratification is motivated or not.

The Skärkind catchment was divided into three subareas and one survey was performed in each of them. The following AV-table can be set up:

Table 3.1 AV-table for Skärkind data

Source of variation	Sum of squares	Degrees of freedom	Mean squares	F-test quantity
Between subareas	25.61	2	12.80	3.00
Residual	332.93	78	4.26	
Total	358.53	80	4.48	

The F-test quantity is not significant at a five percent level. There is, thus, no reason to believe that there are differences in the means between the three subareas. The estimated means of the water equivalent of the snow pack for the three subareas are respectively 11.9, 12.0 and 11.4 cm. The weighted mean according to fomula (3.1) is 11.8 cm while the mean for the total sample is 11.7 cm.

The precision in the weighted mean estimated by formula (3.1), was gotten from the variance of the mean which is expressed as:

$$V(m^{*}) = \sum_{i=1}^{k} c_i^2 V(\overline{x}_i) = \sum_{i=1}^{k} c_i^2 \sigma_i^2 / n_i \qquad (3.2)$$

where σ^2 is the variance of the i:th snow survey and n_j number of samples. The standard error of the weighted mean (square root of the mean variance $V(m^{*})$) is in the case estimated to 0.231 cm. This value should be compared with the standard error of the total sample, which is 0.235 cm. The gain of stratified sampling is in this case negligible.

The Vuoddasbäcken catchment was subdivided into four areas. Within each subarea four snow surveys were carried out. Twoway AV was applied to see the difference among subareas and among snow surveys within subareas. The following table was calculated:

Table 3.2 Twoway AV-table for Vuoddasbäcken data

Source of variation	Sum of squares	Degrees of freedom	Mean squares	F-test quantity
Among subareas	81.9	3	27.210	45.794
Among snow surveys	2.1	3	0.692	1.160
Residual	100.8	169	0.596	
Total	184.8	175	1.056	

The F-test value among subareas is significant at 0.1% level while the one among snow surveys is not significant at 5% level. We, thus, have no reason to distinguish among different snow surveys within one subarea and can unite them into one sample. We then performed one way AV between subareas for the joint samples and got the following AV-table:

Table 3.3 Oneway AV-table for Vuoddasbäcken data

Source of variation	Sum of squares	Degrees of freedom	Mean squares	F-test quantity
Among subareas	81.9	3	27.310	45.66
Residual	102.9	172	0.598	
Total	184.8	175	1.056	

Table 3.3 is actually derived by adding the second and third row of table 3.2 to get the residuals. Means for the water equivalent of the snow pack for different areas are 8.81, 8.11, 8.55 and 7.01 cm respectively.

A joint test with Scheffe's method (Blom, 1977) gave significance at 5% level of differences in the means among all subareas except for the first and the third one.

The weighted mean for the Vuoddasbäcken catchment is in this case equal to 8.24 while the total mean is calculated to 8.12. The standard error in the two cases are 0.61 and 0.78 respectively. By stratifying we get a reduction of the error by 22%. We have to add 112 independent observations to the 176 we already have to get the error down from 0.78 to 0.61 if we consider one total sample. The difference is indeed of importance in this case.

We calculate the following twoway AV-table for the Solmyra data.

Table 3.4 Twoway AV-table for Solmyra data

Square od variation	Sum of squares	Degrees of freedom	Mean squares	F-test qiantity
Among sub-areas	10.3	3	3.45	3.712
Among snow survey	0.1	1	0.115	0.123
Residual	84.6	91	0.929	
Total	95.0	95		

The F-test quantity among subareas is significant at 5% level while the oneway between snow surveys is not significant at this level. Means for the four subareas are 7.45, 8.15, 8.23, 8.22, respectively.

Application of Scheffe's method for the joint significance test of differences in the means showed that only the first subarea differs significantly. In this case we can consider to unite subareas 2, 3 and 4 into one sample.

3.2 Coefficient of variation, CV

Space variability of the snow cover is characterized by its standard deviation σ. Dividing it by the mean m we get the coefficient of variation $CV = \sigma/m$. CV is expected to be a more stable parameter than σ. In the future we shall only discuss the coefficient of variation CV.

3.2.1 Relations of CV for depth, density and water equivalent

There are actually three parameters to describe the snow pack: snow depth (h), snow density (d) and water equivalent (w). The water equivalent of the snow pack is calculated from the snow depth and the snow density as

$$w = h \cdot d \qquad (3.3)$$

Let us use formula (3.3) to calculate the relationship between CVs for the three parameters h, d and w. From the probability theory we know that the expected value, E, and the variance, V, for a function g of random variables x_i, i=1, ..., n can be approximately written:

$$E\{g(x_1, \ldots, x_n)\} \simeq g(m_1, \ldots, m_n) \qquad (3.4)$$

$$V\{g(x_1, \ldots, x_n)\} \simeq \sum_{i=1}^{n} V(x_i)\left(\frac{\partial g}{\partial m_i}\right)^2 +$$

$$+ 2 \sum_{i<j} C(x_i, x_j) \frac{\partial g}{\partial m_i} \frac{\partial y}{\partial m_j} \qquad (3.5)$$

where C is the covariance and m_i, i=1 ..., n the expected value of x_i. In our case the function g is given by equation (3.3).

We now can determine:

$$E(w) = E(h \cdot d) \simeq \overline{h} \cdot \overline{d} \qquad (3.6)$$

and

$$V(w) = V(h \cdot d) \simeq V(h) \cdot \frac{\partial^2}{\partial h} (hd) + V(d) \frac{\partial^2}{\partial d} (hd) +$$

$$2C(h,d) \frac{\partial}{\partial h} (hd) \frac{\partial}{\partial d} (hd) =$$

$$\mp V(h)d^2 + V(d)h^2 + 2C(h,d)h\ d \qquad (3.7)$$

Division of (3.7) by $\{E(h \cdot d)\}^2 = (\overline{h} \cdot \overline{d})^2$

$$CV_w^2 \simeq CV_h^2 + CV_d^2 + 2\rho_{hd} \cdot CV_h\ CV_d \qquad (3.8)$$

where ρ_{hd} is the correlation coefficient between snow depth h and density d. The analysis of our data gives at hand that as a rule, CV_h is zero to fifty percent larger than CV_d and that ρ_{hd} is between 0.2-0.5. CV_h is, thus, the most dominating factor to explain the variability of the water equivalent of the snow pack but CV_d is not negligible. In fig. 3.3 an illustration of this is given. The peakedness of the water equivalent is increased by the positive correlation between snow depth and snow density. Another observation is that the variation of the density is of a more random character than that of snow depths.

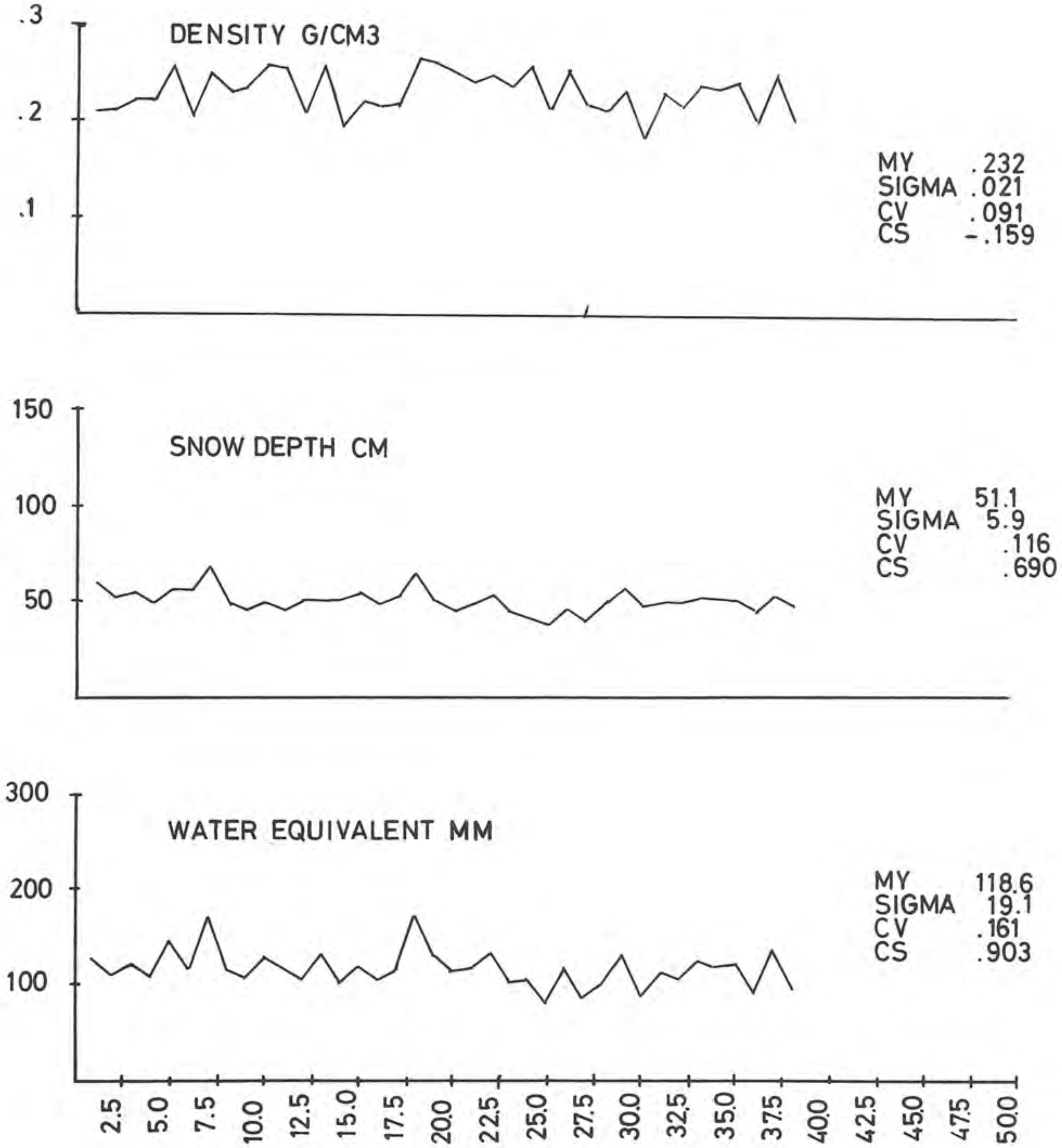


Fig. 3.3

Density, snow depth and water equivalent for one snow survey in Skärkind 1977.

3.2.2 Optimal relation between depth and density measurements

The optimal relation between the number of snow depth and snow density measurements depends on the relation between the coefficients of variation for depth and density. We also must remember that a snow density measurement takes more time and is more expensive than a snow depth measurement. Let us consider these two factors to find the optimal relation between depth and density measurements. We formulate our problem in the following way: For a given time T (or sum of money) let us distribute the measurements so that the standard error of the mean is minimal. Snow depth measurements require one time unit and snow density measurements k time unites. We make n_h observations of depth and n_d of density. It takes t time units to move from one measuring point of snow-depth to another. It is assumed that the density measurement is made near the snow depth observation. The standard error cen be expressed as:

$$\varepsilon(n_h, n_d) = CV_h/\sqrt{n_h} + CV_d/\sqrt{n_d} \qquad (3.9)$$

(we assume that observations are made independently) and the constraint of fixed time:

$$(t+1)\ n_h + n_d \cdot k = T \qquad (3.10)$$

The Lagrangian function is now written down as:

$$L(n_h, n_d, \lambda) = CV_h/\sqrt{n_h} + CV_d/\sqrt{n_d} + \lambda\,|(t+1)\ n_h + n_d\ k - T| \qquad (3.11)$$

The minimum of (3.9) is found by derivating L with respect to n_h, n_d and the Lagrangian multiplier λ and set equal to zero. We determine

$$\frac{\partial L}{\partial n_h} = -\frac{1}{2} \cdot CV_h \cdot n_h^{-1.5} + \lambda(t+1) = 0$$

$$\frac{\partial L}{\partial n_d} = -\frac{1}{2}\ CV_d \cdot n_d^{-1.5} + \lambda \cdot k = 0 \qquad (3.12)$$

$$\frac{\partial L}{\partial \lambda} = (t+1)\ n_h + n_d \cdot k - T = 0$$

Solving the equation system we get the following relation

$$n_h/n_d = \left(\frac{CV_h}{CV_d}\ \frac{k}{t+1}\right)^{0.667} \qquad (3.13)$$

Some examples for different CV_h/CV_d, t and k are given below.

In the table we have excluded the cases when n_h/n_d becomes less than 1.0 as we always assume that the number of depth measurement is greater or equal to the number of density measurements.

A realistic time unit for a snow depth measurement can be 1 minute and a density measurement takes 4-6 minutes. In one minute we move say 25 m. From the table we then see that for distances around 25 m between sample points the snow depth measurements at every second or third point should be completed with density measurements, for 50 m every second and for 100 m both depth and density measurements should be performed at every point.

Table 3.5 Relation n_h/n_d as a function of CV_h/CV_d, t and k (eq. 3.13)

n_h/n_d	t=1 CV_h/CV_d 1.0	t=1 CV_h/CV_d 1.5	t=2 CV_h/CV_d 1.0	t=2 CV_h/CV_d 1.5	t=4 CV_h/CV_d 1.0	t=4 CV_h/CV_d 1.5
k 2	1.0	1.3	-	1.0	-	-
k 4	1.6	2.1	1.2	1.6	-	1.1
k 6	2.1	2.7	1.6	2.1	1.1	1.5

3.2.3 CV of water equivalent

The coefficient of variation CV in space of the water equivalent of the snow pack is a parameter that shows stable values in the lowland forests and open areas while it highly variates in the mountain area (tab 3.6).

Table 3.6 Coefficient of variation (Northern Sweden)

Bare mountain area above the upper limit of the forest	0.30 - 0.70
Mountain area with sparsely grown trees	0.25 - 0.80
Woodland	0.12 - 0.22
Open areas	0.09 - 0.19

The deviation for woodland and open areas are of the order of the expected random errors.

Most observations of the snow cover have been performed during the last few years, which does not allow a statistical analysis of the variations from year to year. The two series from Lule älv representing bare and forested mountain area were the only long series (24 and 27 years, respectively). The statistical parameters for these were calculated and are shown in Table 3.7.

Table 3.7 Statistical parameters of upper Lule älv area (annual values), water equivalent of snow pack

		Bare mountain area	Forested mountain area
Number of years		24	27
Average number of obs		20	40
spatial mean at the end of snow acc period (mid April)	mean (cm)	51.4	19.7
	stand dev (cm)	14.6	6.9
	coeff of var	0.28	0.35
	expected random error	5.63	1.6
	corr coeff	0.41	
spatial coeff of variation at the end of snow acc period (mid April)	mean	0.49	0.51
	stand dev	0.14	0.16
	expected random error	0.09	0.07
	corr coeff	0.40	

The variability of the spatial mean and coefficient of variation is in all cases larger than can be expected from pure randomness. The remaining variability (when the random error is accounted for) is caused by variability in the atmospheric processes from year to year. Noticeable is the relatively small correlation between the two series. The correlation between the spatial coefficients of variation 0.40 is a spurious correlation. The correlation ρ between two quotients x/z and y/w can be expressed as (Benson, 1965)

$$\rho(x/z,y/w) = \frac{\rho_{xy}\eta_x\eta_y-\rho_{xw}\eta_x\eta_w-\rho_{yz}\eta_y\eta_z+\rho_{zw}\eta_z\eta_w}{(\eta_x^2+\eta_z^2-2\rho_{xz}\eta_x\eta_z)^{1/2}\ (\eta_y+\eta_w-2\rho_{yw}\eta_y\eta_w)^{1/2}} \quad (3.14)$$

where η_x is the coefficient of variation of variable x. In our case x and y represent standard deviation for the two different areas and z and w their means. The correlation between z and w is 0.41 as given in table 3.7. The correlation between the standard deviations x and y is 0.02.

The fact that the correlation between the coefficients of variation is 0.40 is thus explained by the correlation in the means and not in the actual measure of variability - the standard deviation. The variability between the two areas is thus not at all correlated. At each area the relation between means and variability was analyzed. The correlation coefficients between means and standard deviation are for the two cases 0.60 and 0.53. We use eq. (3.14) to calculate the correlation between means and coefficient of variation. For this case we set x and w equal to the means, y equal to the standard deviation and z=1. We find the correlation coefficients -0.44 and -0.55, respectively. The relations are plotted in figs. 3.4 and 3.5.

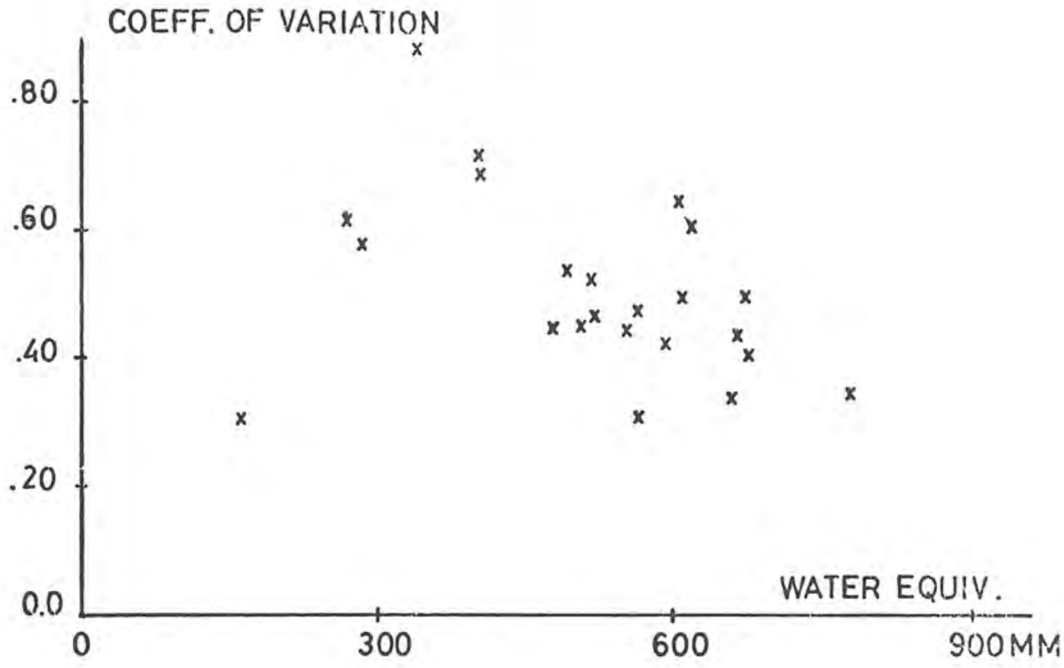


Fig. 3.4

Relation between coeff. of variation and mean water equivalent for Upper Lule älv, mountain area above tree line

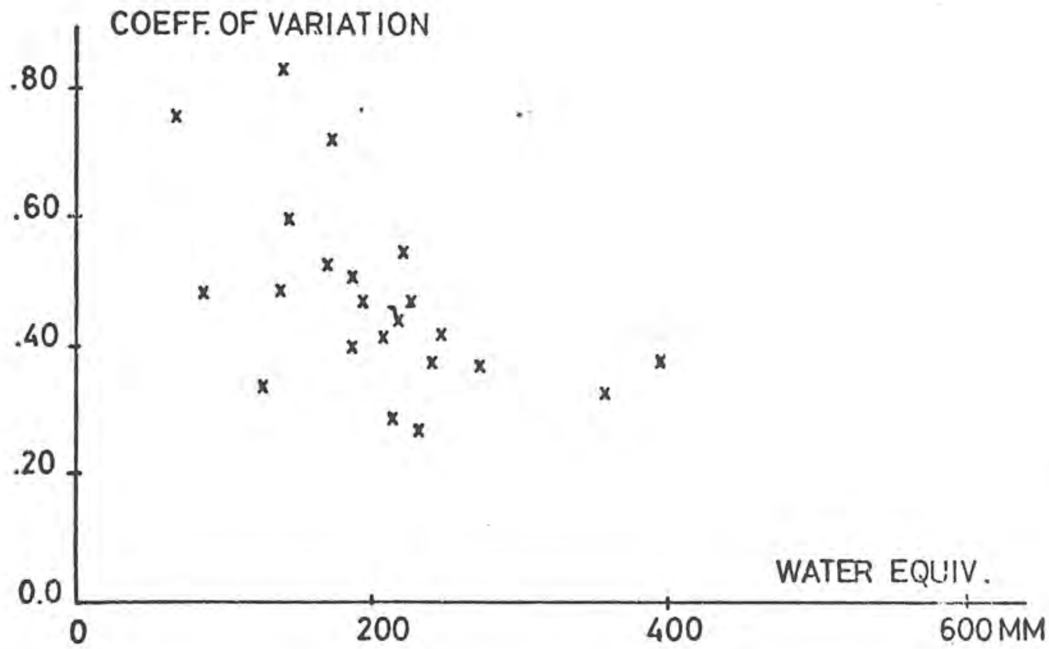


Fig. 3.5

Relation between coeff. of variation and mean water equivalent for Upper Lule älv, mountain area with sparsely grown trees

3.3 Distribution functions

Water equivalent data, totally 105 samples with an average of 50 sample points, were compared to various analytical distribution functions. The normal, log-normal and gumbel distributions were used. Comparisons were performed both visually from plots and by using χ^2-test. Results of the χ^2-test are given in table 3.8.

Table 3.8 Number of rejections of χ^2-test of analytical distribution functions

Significance level, p %	Distribution functions			Expected number of rejection
	Normal	Lognormal	Gumbel	
5	23	28	29	5.25
1	12	18	18	1.05
0.1	6	8	8	0.10

Using the Poisson approximation of the binomial distribution we can calculate the probability of the observed number of rejections when the expected number of rejections is known. In all cases this probability is less than 0.01%. Judging from the χ^2-test none of the tested analytical distributions are thus applicable.

One reason of the complex structure of the distribution of snow survey data is nonhomogeneity in the data. An example can illustrate this:

From 1973 the number of measurements in the upper Lule älv area, woodland,was increased by 80 points. From formerly a good fit to the normal curve we now got the picture shown in figure 3.6. The two data groups were separated and plotted again, see the diagrams, figure 3.7. "Forest-A" represents the area with the additional 80 points, situated at a level of 500 m lower than for "Forest-B". "Forest-A" is also more densely forested. Both the means of water equivalents and the coefficients of variation strongly differ between the two areas.
A cautious conclusion could be that for a homogeneous area the normal distribution can be used. At least it is not worse than other simple analytical distributions judging from visual comparisons of plotted distributions.

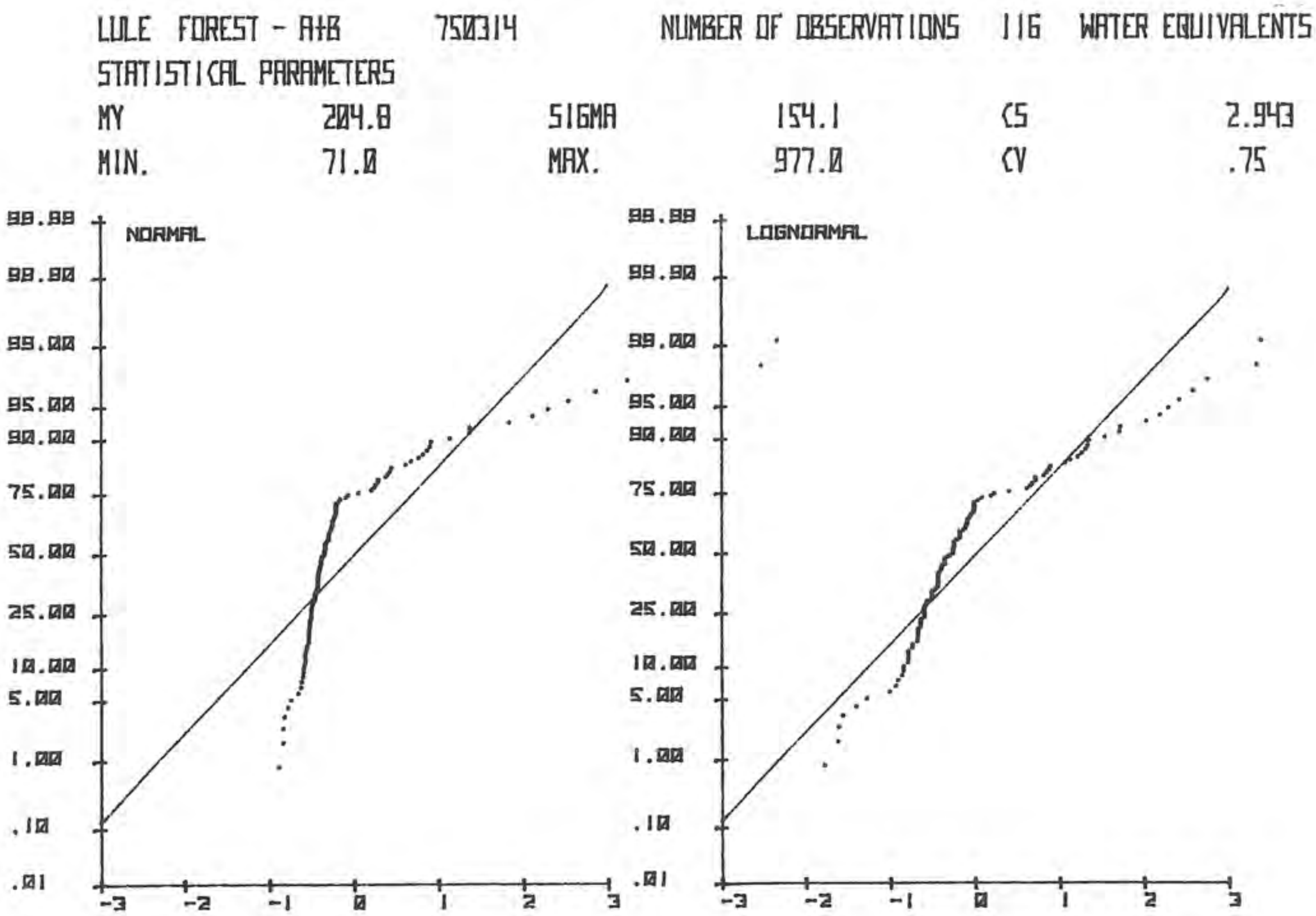


Fig. 3.6

Plots on probability paper of joint samples from Upper Lule älv.

3.4 Space correlation

Snow depths or water equivalents of the snow pack measured at equidistant points is considered as "time series". Time series may consist of the following parts (Kendall and Stuart, 1977):

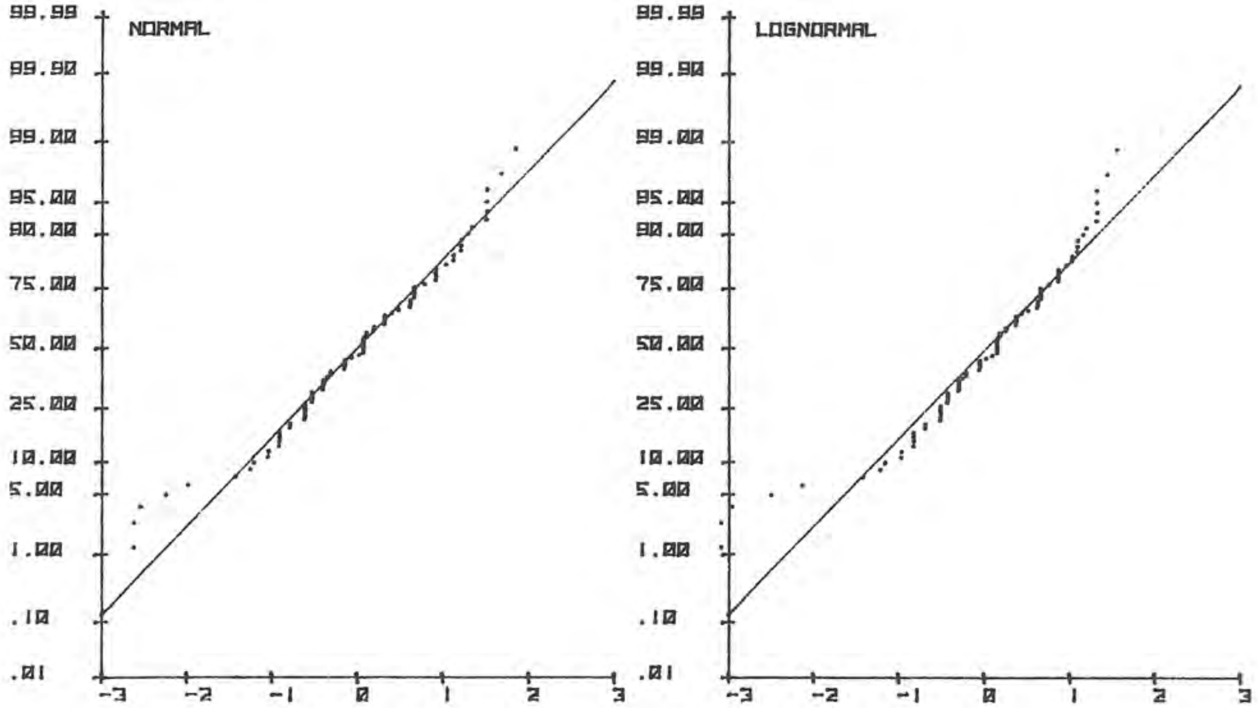
a) a trend, or a long-term movement
b) oscillations about the trend, of greater or less regularity
c) a periodic component
d) a random, unsystematic or irregular component

The decomposition of a series into the different parts enumbered above, is very often useful, but is perhaps misleading and in any case is not the ultimate object of statistical analysis. In our own case such decomposition is of no use. Important is whether we have a space dependence or we can consider the variation to be random. The origin of the dependence - trend, regular oscillations, periodicities - is an interesting information, but we are not in any way helped by removing it.

LULE FOREST - A   750314   NUMBER OF OBSERVATIONS 80   WATER EQUIVALENTS

STATISTICAL PARAMETERS

MY	138.4	SIGMA	23.4	CS	-.507
MIN.	77.0	MAX.	182.0	CV	.17



LULE FOREST - B   750314   NUMBER OF OBSERVATIONS 36   WATER EQUIVALENTS

STATISTICAL PARAMETERS

MY	352.5	SIGMA	209.1	CS	1.428
MIN.	71.0	MAX.	977.0	CV	.59

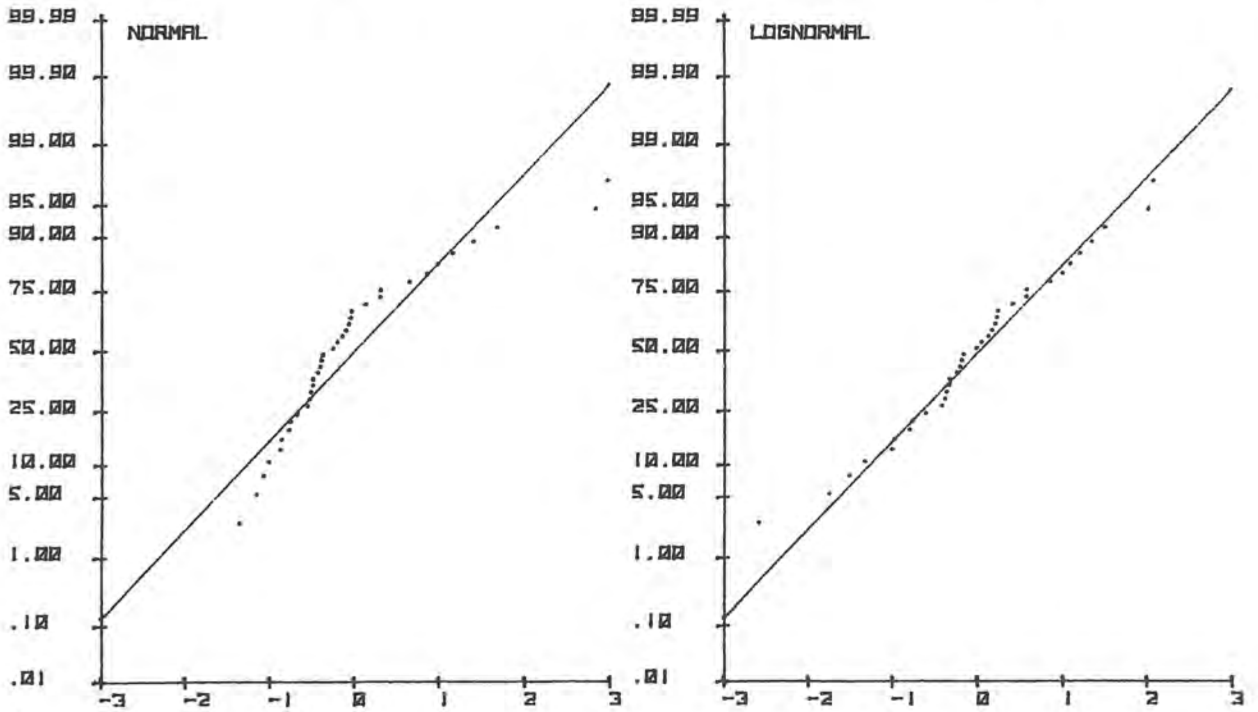


Fig. 3.7

Plots on probability paper of homogeneous samples from Upper Lule älv

Our first task is thus to test randomness of the series. We use five distribution-free tests to do this: turning points, phase lengths, difference signs, runs and reverse arrangements. The different tests are not described here and we refer to Kendall and Stuart (1977) for the three first tests and Hansen (1970) for the two others. It is not possible to specify alternative hypothesis with precision to the tests. Still we have an idea that the three first tests are sensitive for regular oscillations and periodicities while the reverse arrangement tests indicate trends. The run test can show oscillations as well as trends. If the hypothesis of randomness is rejected we go on and analyze the character of dependence by correlation functions.

3.4.1 Test of randomness

The number of snow surveys that allowed test of randomness was 22. Sampling intervals varied from 1 meter to 100 meters. Results of the tests are shown in table 3.9. For all tests except for phase lengths significance tests were made on 5% level. If the hypothesis of randomness could be rejected on this level 1 is written in the column "sign" in table 3.9 else 0. The expected number of rejections on 5%-level for 22 cases is 1.1. The probability of 4 or more rejections is 1% and the probability of 9 or more rejections is less than 0.01%. The tests indicate that there are elements of non-randomness in our series. For series with 1 m sampling intervals we have reason to believe that we have both regular oscillations and trends at hand while for longer sampling intervals only trends. We shall observe that trends are a relative conception. A trend observed for a snow route of a certain length can be a part of a periodic component with the same scale as the length of the snow route.

The tests of randomness give us a reason to continue the analysis of our series to find out the character of the non-randomness.

Table 3.9 Results of tests of randomness

		No of obs	Turning points			Difference sign			Run	R.A.[1]	Phase lengths																Sampling interval
											1		2		3		4		5		6				Tot		
		obs	obs	E	sign	obs	E	sign	sign		obs	E	obs	E	obs	E	obs	E	obs	E	obs	E	obs	E	obs	E	m
Buskbäcken	, 1979	200	111	132	1	99	99.0	0	1	1	66	82.1	29	33.9	12	10.3	2	2.2	4	0.4	1	0.1	[9] 1	0.0	114	131.9	1
-"-	, 1979	200	116	132	1	94	99.0	0	1	1	67	82.1	39	35.9	13	10.3	2	2.2	1	0.4					122	131.0	1
Tärnsjö	, 1979	200	125	132	0	96	99.0	0	1	0	77	82.1	29	35.9	11	10.3	3	2.2	2	0.4	1	0.1			123	131.0	1
-"-	, 1979	200	121	132	0	85	99.0	1	1	1	77	82.1	40	35.9	8	10.3	4	2.2							129	131.0	1
Skärkind	, 1977	100	47	65.3	1	49	49.5	0	1	0	15	40.4	8	17.6	10	5.0	4	1.1	1	0.2					40	64.3	1
-"-	, 1977	101	58	66.0	0	41	49.5	1	1	1	29	40.8	16	17.8	5	5.1	1	1.1	1	0.2	1	0.0	[7] 1	0.0	54	69.0	1
-"-	, 1977	101	61	66.0	0	43	49.5	1	1	1	36	40.8	11	17.8	8	5.1	3	1.1	1	0.2	1	0.0	1	0.0	59	65.0	1
Skärkind	, 1977	125	75	82.0	0	61	61.5	0	1	0	47	50.8	22	22.2	6	6.3	2	1.4	1	0.2					78	81.0	20
-"-	, 1977	190	128	125.3	0	82	94.0	1	1	1	86	77.9	29	34.1	9	9.8	3	2.1	1	0.4					128	124.3	20
-"-	, 1977	123	82	80.7	0	55	60.5	0	0	0	46	50.0	27	21.8	3	6.2	3	1.4							79	79.7	20
-"-	, 1978	176	112	116.0	0	85	87.0	0	1	0	67	72.1	34	31.5	7	9.0	3	2.0	1	0.3					112	115.0	20
Vuoddasbäcken	, 1978	21	14	12.7	0	9	9.5	0	0	0	12	7.5	1	3.1	1	0.8									14	11.7	25
-"-	, 1978	21	13	12.7	0	10	9.5	0	0	0	10	7.5	3	3.1	1	0.8									14	11.7	25
-"-	, 1978	21	14	12.7	0	10	9.5	0	0	0	10	7.5	1	3.1	2	0.8									13	11.7	25
-"-	, 1978	21	14	12.7	0	9	9.5	0	0	0	9	7.5	3	3.1											12	11.7	25
Solmyren	, 1978	21	12	12.7	0	8	9.5	0	1	1	5	7.5	6	3.1											11	11.7	50
-"-	, 1978	21	13	12.7	0	9	9.5	0	0	1	6	7.5	6	3.1											12	11.7	50
-"-	, 1978	21	10	12.7	0	11	9.5	0	0	0	5	7.5	2	3.1			4	0.2							11	11.7	50
-"-	, 1978	50	36	32.0	0	24	24.0	0	0	0	23	19.5	9	8.4	1	2.4									33	31.0	50
Solmyren	, 1979	21	9	12.7	1	8	9.5	0	1	1	4	7.5	3	3.1	2	0.4									9	11.7	100
-"-	, 1979	25	17	15.3	0	12	11.5	0	0	0	13	9.2	3	3.8	1	1.1									17	14.3	100
-"-	, 1979	109	73	7113	0	53	53.5	0	1	0	46	44.2	19	19.2	5	5.5	2	1.2							73	70.3	100

[1] = Reverse Arrangement

3.4.2 Correlation functions

The coherence of snow cover from point to point can be described by spatial correlation functions and structural functions. If the water equivalents of the snow pack, x_i, are given at equidistant points $\Delta\ell$ along a line, the correlation function is calculated as:

$$r_x(k\cdot\Delta\ell) = \sum_{i=1}^{N-k} (x_i-\bar{x})(x_{i+k}-\bar{x}) / \sum_{i=1}^{N} (x_i-\bar{x})^2 \quad (3.15)$$

In meteorology the structural function is often used as an alternative to the correlation function. This function describes the probable gradients over a territory.

Under the assumptions of uniformity, like in the case of the correlation function, it is only a function of the distance $k\cdot\Delta\ell$ between points and is expressed:

$$b(k\cdot\Delta\ell) = 2\sigma_x^2\left[1-\eta_x(k\cdot\Delta\ell)\right] \quad (3.16)$$

Both the structural function (specially in Soviet literature) and the correlation function are used to describe snow data. Here we shall use only the correlation function.

Correlation functions of snow depths were calculated for 12 of the series, where the number of observations were larger than 100. Examples of calculated correlation functions are given in figs 3.8 and 3.9. For 1 m sampling interval the correlation functions indicate a completely white noise process. Specially the run and reverse arrangement tests showed the opposite. We shall note that the maximum lag for the correlation function is 20 m. Oscillations of a scale larger than this can not be analyzed with the limited number of data that is at hand. Plots of measured snow depths and moving averages of these confirm the run tests. There is a tendency of piling up values above and below the mean. For sampling intervals of 20-100 m the correlation functions indicate a larger dependence. As seen from fig 3.10 where some snow surveys with approximately 20 observations are shown, the dependence may be caused by waves and trends.

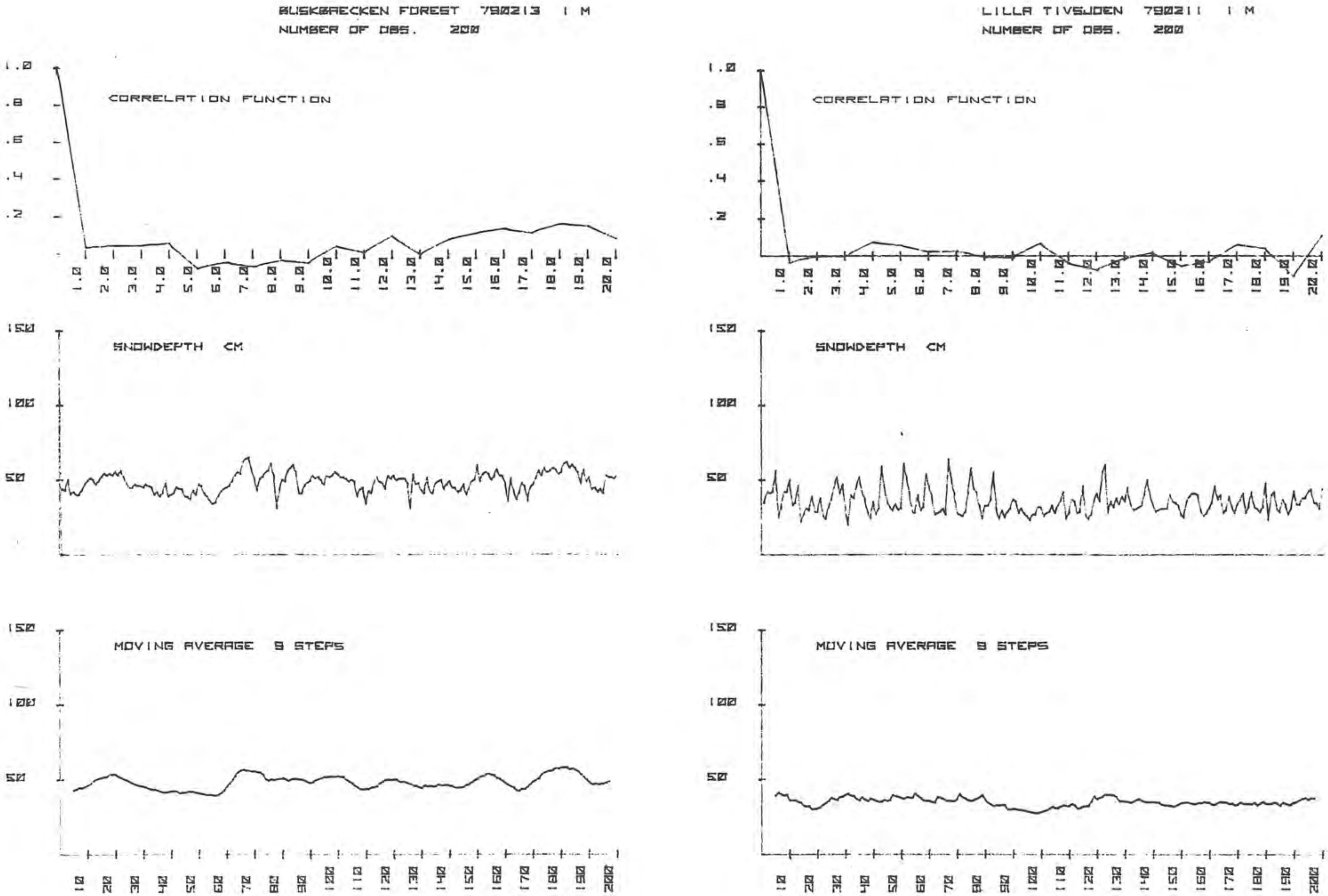


Fig. 3.8

Correlation functions; observed and smoothed snow depths for Buskbäcken and Lilla Tivsjön (equidistance 1 m)

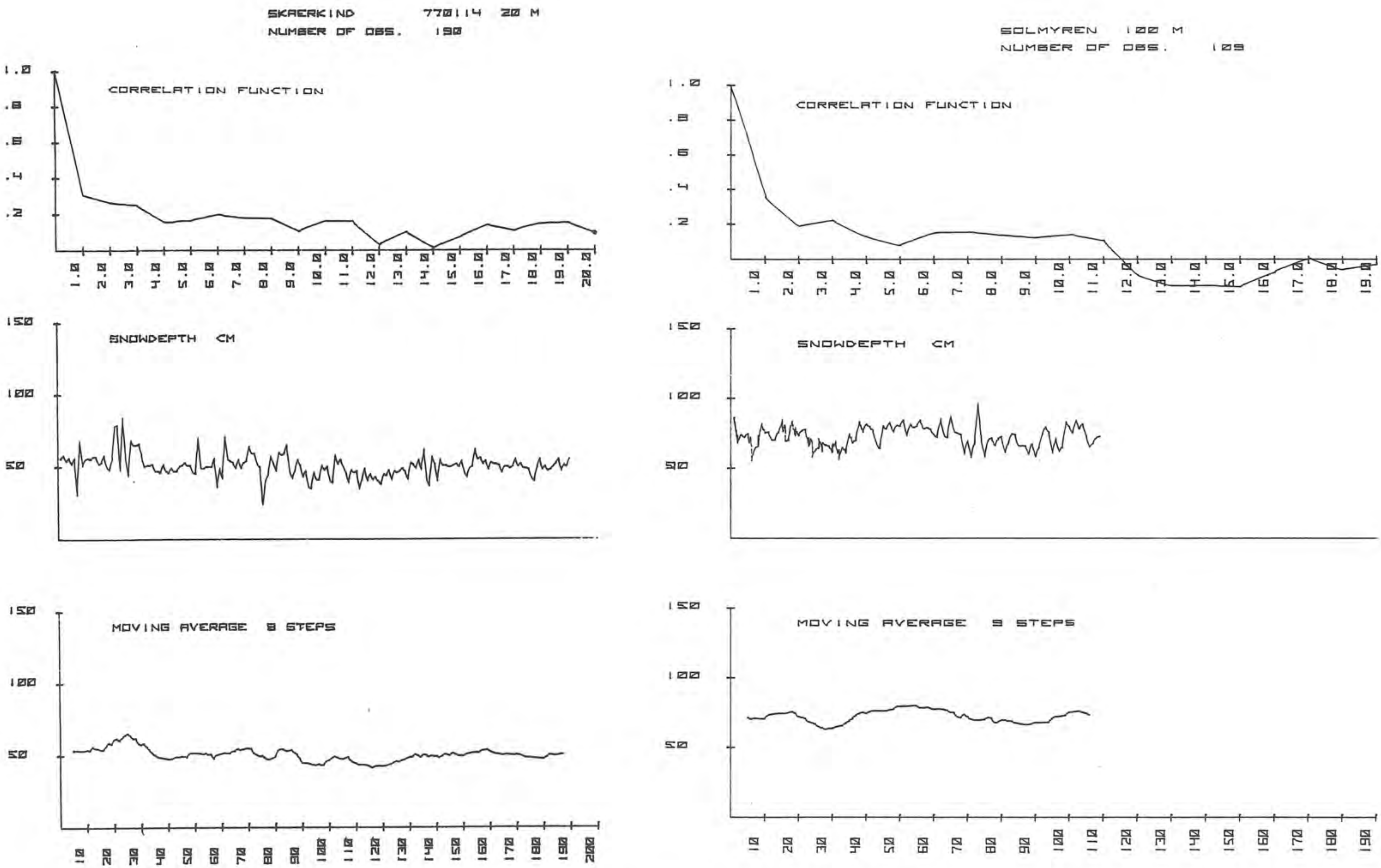


Fig. 3.9

Correlation functions; observed and smoothed snow depths for Skärkind and Solmyren (equidistance 20 m and 100 m)

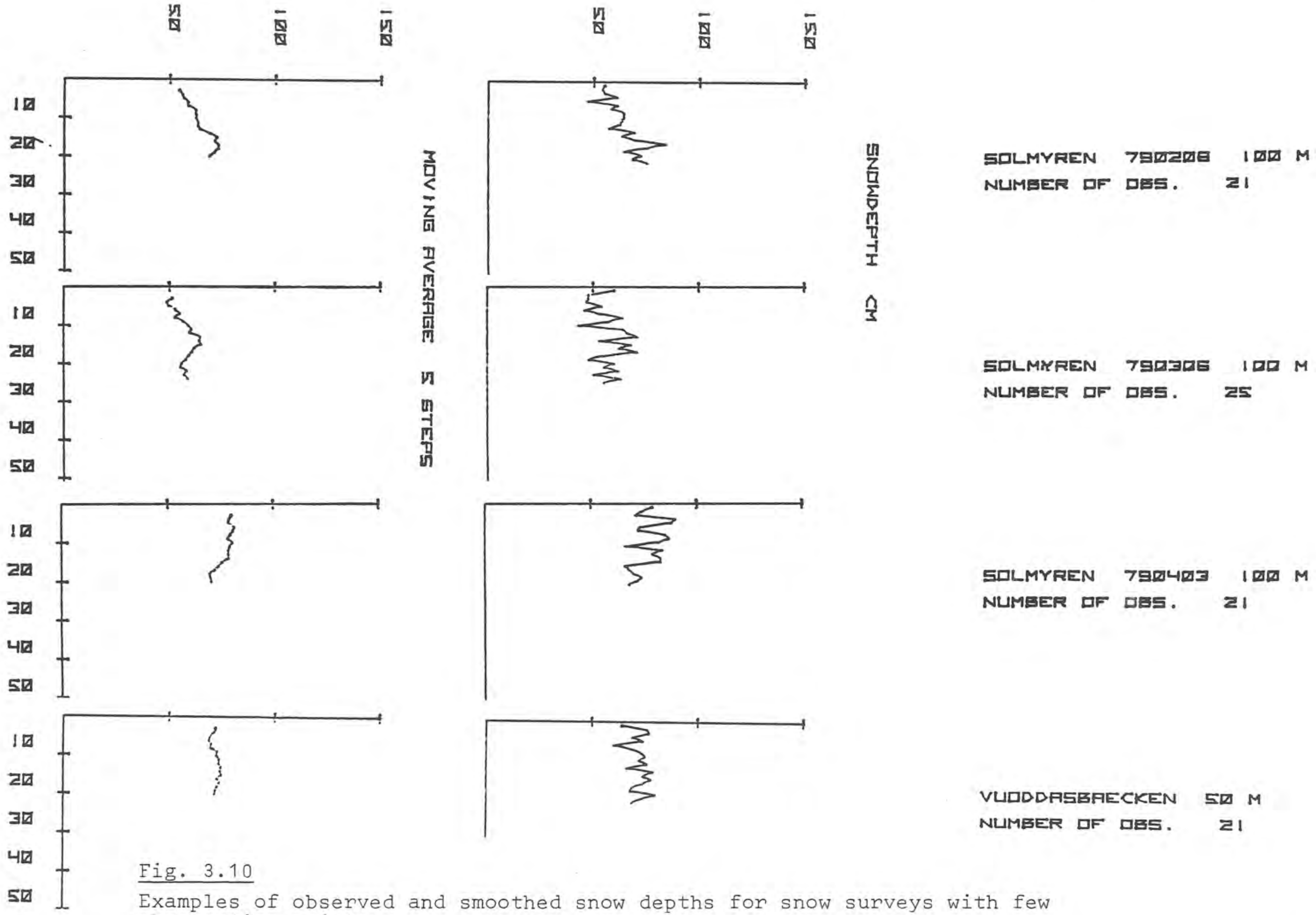


Fig. 3.10

Examples of observed and smoothed snow depths for snow surveys with few observation points

The surface of the snow pack usually looks very smooth, much smoother than the underlying ground surface. The main source to the variability of the snow pack in a microscale is thus the roughness of the ground. The immediate impression from the analysis of correlation functions is that we have a process that is almost independent. This indicates that sampling points can be chosen quite freely. 20 measurements of snow water equivalents of the snow pack with 1 m equidistance is as valuable as 20 measurements with 100 m equidistance, to estimate the areal mean. This does not agree with our understanding of how snowdepths are connected with the microscale topography. The distance between sampling points must be chosen in accordance with the micro- and mesoscale variability. We further note that to estimate areal means, it is the mesoscale variability of order 100 m that is of interest not the microscale variability.

The analysis of the given data does not give a distinct answer of how the spatial variation of the snow pack can be described by the correlation function. Further studies are needed. Important is to compare the variability of the landscape in micro- and mesoscale and its connection with the snow pack. We are inclined to assume that there is a dependence between consecutive measurements of snow depth and water equivalents of the snow pack along a snow route. The calculations of standard errors below make it necessary to give an analytical expression for the correlation function. We shall use the modified Bessel function of the second kind,

$$r(s) = k\ s\ K_1(k\ s) \qquad (3.17)$$

Gottschalk (1978) gives a theoretical background for the choice of this expression to describe spatial variation. Equation (3.17) has one parameter k. In fig 3.11 the equation is drawn for k = 5, 10, 15, 20. These values are chosen to fit approximately the empirical correlation functions (compare figs 3.8 and 3.9).

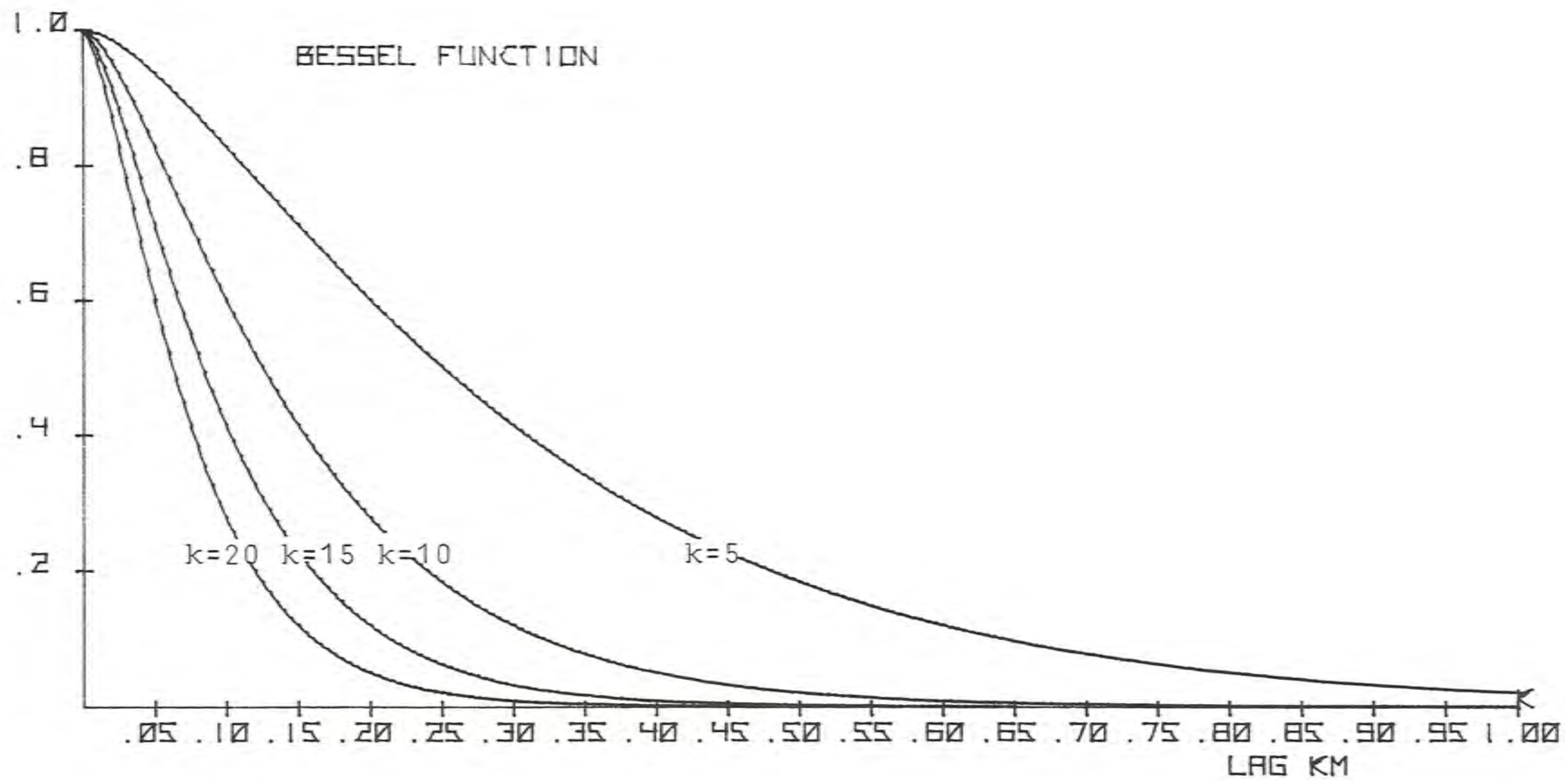


Fig. 3.11
Analytical correlation function of snow depths

4. Standard errors of snow surveys

Snow surveys as an operational routine of the hydrological service have the aim to determine the mean water equivalent of snow pack of a certain area and perhaps other parameters to describe the distribution i e the coefficient of variation. In this chapter we shall consider the standard error in the estimated mean water equivalent. We assume that the mesoscale variability of snow depths, as discussed in section 3.2, has been considered by selecting homogeneous subareas, within a total area with different land-use. The snow cover within a subarea is assumed to be homogeneous and isotropic. The space distribution is thus fully described by the covariance function.

4.1 Problem formulation

The mean water equivalent of the snow pack, m, for a certain area A is defined by:

$$m = \frac{1}{A} \iint_A f(x,y)dx\, dy \qquad (4.1)$$

where f(x,y) is a function describing the water equivalent distribution over the area A.

From N observed measurements of water equivalents at discrete points within the area the mean is estimated:

$$\hat{m} = \frac{1}{N} \sum_{i=1}^{N} f(x_i,y_i) \qquad (4.2)$$

where $f(x_i,y_i)$ is the measured water equivalent at point (x_i,y_i).

The error introduced by using $\hat{m}$ instead of m is estimated by the standard error:

$$E(m-\hat{m})^2 = E(m\, m) + E(\hat{m}\, m) - 2E(m\, \hat{m})... \qquad (4.3)$$

where E stands for expectancy.

Under assumptions of homogeneity and isotrophy we can write down (4.3) as (Kagan, 1965):

$$E(m-\hat{m})^2 = \frac{1}{A^2} \iint_A \iint_A \mathrm{cov}\{\sqrt{(x-\xi)^2+(y-\eta)^2}\}dx\, dy\, d\xi\, d\eta$$
$$+ \frac{1}{N^2} \sum_{i=1}^{N} \sum_{i=1}^{N} \mathrm{cov}\{\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}\}$$
$$- \frac{2}{NA} \sum_{i=1}^{N} \iint_A \mathrm{cov}\{\sqrt{(x-x_i)^2+(y-y_i)^2}\}dx\, dy \qquad (4.4)$$

where cov(s) is the covariance of water equivalents.

The covariance function we can write as:

$$\mathrm{cov}(s) = \sigma^2 r(s) \qquad (4.5)$$

where σ^2 is the standard deviation of water equivalents and r(s) is the correlation function. These two parameters were discussed and calculated in chapter 3. We thus have the necessary information to calculate the standard error from formula 4.4.

The integral expressions in eq (4.4) are difficult to treat numerically the way they stand. The quadriintegral can be transformed to a simple integral (Bras, Rodriguez-Iturbe, 1976)

$$\frac{1}{A^2} \iint_A \iint_A \mathrm{cov}\,(\mathbf{x},\mathbf{y})\, d\mathbf{x}\, d\mathbf{y} = \int_0^d \sigma^2 r(s) G(s) ds \qquad (4.6)$$

where G(s) is the probability density function of the distances between two randomly chosen points and d is the longest distance for the studied area. G(s) is dependent on the form of the area. For rectangles G(s) is defined as (Matern, 1960):

$$G(s) = \frac{1}{\sqrt{A}}\, g(s/\sqrt{A},\ \sqrt{\ell_1/\ell_2}) \qquad (4.7)$$

$$g(\omega,a) = 2\omega(g_1(\omega,a) + g_2(\omega a,a) + g_2(\omega/a,1/a))$$

with

$$g_1(\omega,a) = \begin{cases} \pi+\omega^2-2\omega(a+1/a) & 0<\omega<(a^2+a^{-2})^{1/2} \\ 0 & \text{otherwise} \end{cases}$$

$$g_2(\omega,a) = \begin{cases} 2\sqrt{\omega^2-1} - 2\arccos(1/\omega) + a^{-2} & 1<\omega<(1+a^4)^{1/2} \\ 0 & \text{otherwise} \end{cases} \qquad (4.4)$$

and ℓ_1 and ℓ_2 are the respective side lengths of the rectangle.

For the rectangular geometric figure the double integral can be transformed to a simple integral (Linton and Rodriguez-Iturbe, 1977) by a procedure described below.

The total rectangular area is divided into four rectangular subareas (see fig 4.1). For each subarea the integral is now written

$$\int_0^{d_1}\int_0^{d_2} r(\sqrt{x^2+y^2})\, dx\, dy \qquad (4.7)$$

where d_1 and d_2 are the sides of the subareas respectively.

By performing the substitution to polar coordinates

$$x = v \cos \Theta, \; y = v \sin \Theta$$

and noting that in this case the Jacobian is equal to v, (4.7) is written

$$\int_o^{\Theta} \int_{\Theta_a}^{\Theta_u} vr(v)d\Theta \, dv = \int_o^{D} (\Theta_u - \Theta_e) vr(v)dv \qquad (4.8)$$

where D is the length of the diagonal of respective rectangular subarea and Θ_u and Θ_e are the upper and lower limites of integration with respect to Θ. A geometric analysis gives the following dependence on v:

For $d_1 > d_2$

$$\begin{aligned} \Theta_u - \Theta_e &= \pi/2 && 0<v<d \\ \Theta_u - \Theta_e &= \pi/2 - \text{arc} \cos(d\ /v) && d_2<v<d_1 \\ \Theta_u - \Theta_e &= \text{arc} \sin(d_1/v) - \text{arc} \cos(d_2/v) && d_1<v<D \end{aligned} \qquad (4.9)$$

and for $d_2 > d_2$

$$\begin{aligned} \Theta_n - \Theta_e &= \pi/2 && 0<v<d_1 \\ \Theta_n - \Theta_e &= \text{arc} \sin(d_1/v) && d_1<v<d_2 \\ \Theta_n - \Theta_e &= \text{arc} \sin(d_1/v) - \text{arc} \cos(d_2/v) && d_2<v<D \end{aligned}$$

The double integral expression is thus transferred to a simple integral by the following formula:

$$\frac{2}{NA} \sum_{i=1}^{N} \iint_A \text{cov}\sqrt{(x-x_i) + (y-y_i)} \, dx \, dy$$
$$= \frac{2}{NA} \sum_{i=1}^{N} \sum_{k=1}^{4} \int_o^{D_{ik}} (\Theta_u - \Theta_e) v\sigma^2 r(v)dv \qquad (4.11)$$

where D_{ik} is the diagonal length corresponding to k-th rectangular subarea for coordinates (x_i, y_i).

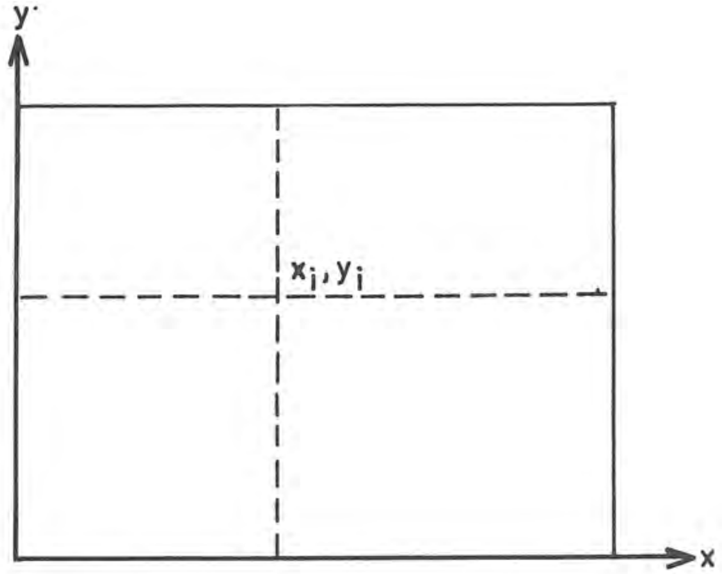


Fig. 4.1 Division into rectangular subareas of an area of the size A, corresponding to location x_i, y_i.

4.2 Estimation of standard errors of snow surveys

The fact that water equivalents of the snow pack are correlated in space makes the standard error of snow surveys dependent on the size and form of the considered area, the distance between sampling points, the geometry of the course along which sampling is done and the location of it within the area. The structure of the correlation function is, of course, also of importance. We shall throughout this chapter use the correlation function eq 3 with different κ values.

The standard error SE_i of the mean for independent observations is equal to

$$SE_i = \sigma/\sqrt{N} \qquad (4.12)$$

In the future we shall also consider the relative standard error of the mean for independent observations

$$se_i = SE_i/\sigma = 1/\sqrt{N} \qquad (4.13)$$

By the relation actual relative standard error, se, calculated from eq 4.4 considering space correlation, and relative standard error for independent observation se_i, we can define an effective number of observations N_e

$$N_e = (se_i/se)^2 N = (se)^{-2} \qquad (4.14)$$

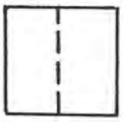
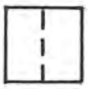
N_e is the number of independent observations that would have the same standard error as the N dependent observations.

The most effective way to distribute a fixed number of observations of snow depths over an area is to sample according to a systematic network i e gridnet of squares or triangles or to sample at random. From practical reasons this is not possible. Sampling must be done along a snow course of some tens of kilometers. Gridnets are too time-consuming, both as far as the necessary preparational arrangements to identify sampling points are concerned and the procedure of the sampling itself.

In table 4.1 relative standard error has been calculated to show the effects of size of the area, form of the area, the distance between sampling points, the number of sampling points, the location of a snow course and the geometric form of snow course. The following effects are noted.

- For equal snow courses standard error grow with increasing area
- Form of the area has small influence
- Sampling with 20 meters interval gives large redundant information. At least 50 or 100 meters is advisable for both open field and forest.
- To double the precision in the mean roughly five times the number of sampling points is needed
- Location of snow course has little influence
- Snow course as a straight line or a circle gives the smallest standard error

Table 4.1 Standard errors of snow course

	Form of the area and snow course	Area km^2	Number of sampling points	K (in eq) 3.17	Distance between sampling points	s.e.	N_e
Size of the area		10	20	5	100	0.476	4.41
		100	20	5	100	0.515	3.77
		1000	20	5	100	0.524	3.64
		10	20	15	100	0.314	10.11
		100	20	15	100	0.322	9.64
		1000	20	15	100	0.322	9.64
Form of an area		10	20	5	100	0.476	4.41
		10	20	15	100	0.314	10.14
		10	20	5	100	0.465	4.62
		10	20	15	100	0.312	10.27
Distance between sampling points		10	20	5	20	0.834	1.44
		10	20	5	50	0.629	2.53
		10	20	5	100	0.476	4.41
		10	20	15	20	0.659	2.45
		10	20	15	50	0.433	5.33
		10	20	15	100	0.314	10.14
Number of sampling points		100	10	5	100	0.680	2.16
		100	20	5	100	0.515	3.77
		100	100	5	100	0.232	18.58
		100	10	15	100	0.447	5.00
		100	20	15	100	0.322	9.64
		100	100	15	100	0.146	46.91
Location of snow course		10	20	5	20	0.834	1.44
		10	20	5	20	0.839	1.49
Geometric form of a snow course		10	20	15	100	0.314	10.14
		10	20	15	100	0.314	10.14
		10	20	15	100	0.333	9.02
		10	20	15	100	0.346	8.35

5. Conclusions

According to the results given in this report we here give some recommendations of how to carry through a snow survey and how to use obtained data. We assume that the aim of surveying is to find areal means of the snow pack.

First the area in question is divided into homogeneous subareas, considering differences in physiographic factors.

In each subarea one snow course is performed. The form of the course can be a line or a circle. Distance between sampling points should be 50-100 m. With a sampling distance of 100 m measurements of both depth and density should be made at each point. When using shorter sampling distances the frequency of density measurements may be reduced to every second or third measuring point. A suitable number of points from a practical point of view is 20. This number of points and a distance between sampling points of 100 m will give an error in areal mean, which for lowland areas (forests and open fields) is 3-6% and for the mountain region 10-20% (compare tables 3-6 and 4.1).

Analysis of variances are then applied on the snow survey data to decide the most efficient subdivision of the total area. The stratified sampling facilitates an estimation of areal mean with minimized error. The subdivision should be kept during some years until it is possible to decide the stability in time of snow cover spatial variation. After that a final subdivision is made.

To the users of snow survey data we can make the conclusion that when looking for a simple distribution function to characterize the snow cover the normal distribution will be a good approximation for homogeneous areas. Of great importance is also to note the differences in both space and time of the variability of the snow cover.

6. LITERATURE

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