#### SVERIGES METEOROLOGISKA OCH HYDROLOGISKA INSTITUT

MEDDELANDEN, SERIE D. Nr 17 COMMUNICATIONS, SERIES D. No. 17

# DEVELOPMENT OF A CONCEPTUAL DETERMINISTIC RAINFALL - RUNOFF MODEL

AV

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Ur: NORDIC HYDROLOGY. Årg. 4. Nr. 3. 1973

STOCKHOLM 1973

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Nordic Hydrology 4, 1973, 147-170

Published by Munksgaard, Copenhagen, Denmark

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# DEVELOPMENT OF A CONCEPTUAL DETERMINISTIC RAINFALL-RUNOFF MODEL

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This progress report outlines the main principles for the development of a simple conceptual rainfall-runoff model at the Swedish Meteorological and Hydrological Institute.

The HBV-2 Model is based on lumped-parameter approximations to the physical laws governing infiltration, percolation and runoff formation. The time interval is one day. The model structure includes a soil moisture storage, an upper zone storage and a lower zone storage. A procedure for evaluating the parameter values is described.

Examples of applications to several test catchments in various hydrologic settings are included.

#### THE NEEDS FOR RAINFALL RUNOFF MODELS

Conceptual models of the land phase of the hydrological cycle are needed for various purposes, including the following:

Simulation of long streamflow records for design purposes

Operational simulation of streamflow for management of water resources facilities (streamflow forecasting)

Quality control of computer processed streamflow data

#### Sten Bergström and Arne Forsman

Estimation of streamflow statistics in ungaged streams

Simulation of actual evaporation records

Simulation of records of soil moisture conditions for estimation of flood hazards and drought persistence

Assessment of the influence of man's activities on the hydrological processes

Research on the hydrological processes, including runoff, actual evaporation, infiltration, percolation, snowmelt, groundwater recharge, and soil moisture changes.

#### **OBJECTIVES OF MODEL DEVELOPMENT**

The ultimate goal is to develop a simple digital runoff model which could be used for most of the above purposes.

The model algorithm should be logical and physically relevant, and the model should simulate as accurately as possible the streamflow hydrograph. It could also give useful information on the rate of actual evaporation and on soil moisture changes.

The model should be as simple as possible and contain the minimum number of parameters required for adequate representation of the runoff. The values of the parameters should preferably be measurable or be significantly correlated to easily measurable catchment characteristics.

The input data should include only those basic data which are generally available, such as precipitation and air temperature, or data which could easily be derived from generally available data, such as potential evaporation according to Penman's formula.

#### THE SMHI PROGRAM FOR DIGITAL MODEL DEVELOPMENT

At the Swedish Meteorological and Hydrological Institute (SMHI) work began in early 1972 on the development of a simple conceptual digital model of the land phase of the hydrological cycle. The present article is a progress report on the results obtained so far, up to January 1973.

The program adopted to meet the requirements stated in the preceding section is as follows:

- 1. Development of a simple lumped-parameter model of general applicability
  - a. First a model for the snow free part of the year, then the introduction of a snowmelt sub-routine
  - b. First the application to small catchments, then to larger streams
- 2. Special applications such as to
  - a. urban runoff.
  - b. streamflow forecasting.

It was decided to begin with a very simple lumped-parameter model, partly because it was felt that the quality of the input data available did not warrant any high level of complexity.

The model would first be developed for the snow free part of the year, and not until it has been proved that the model structure adequately describes the rainfall-runoff relationship will a special snowmelt sub-routine be developed. Possible changes in parameter values and also in the model structure because of frozen ground and because of the influence of the snowcover on the runoff formation will be considered.

From the beginning, the model development was geared to small catchments, where the response is quite rapid and a large number of events occur every year.

At the present time the model is being developed and tested in a number of small catchments with short but comparatively high quality data records in different physiographic settings.

These test basins are the following (main physiographic features within brackets): Lilla Tivsjön (moraine, coniferous forest), Nolsjön (moraine, coniferous forest), Stabby (moraine and exposed bedrock, coniferous forest), Stormyra (exposed bedrock, coniferous forest), Solmyren (swamp, coniferous forest), and Sundbromark (clay, arable land). See Table 1. Additional test basins will also be used.

The first model, HBV-1, was developed for the Lilla Tivsjön Catchment (Bergström 1972a), and the second improved version, HBV-2, was developed and has so far been tested in the catchments of Nolsjön, Stabby and Stormyra (Bergström 1972b).

An attempt will be made to relate model parameters to physiographic features of the catchments.

Even in these small catchments (drainage area between 4 and 20 km<sup>2</sup>) the time interval used is one day, because the precipitation data are generally available as daily totals only.

After the model for small catchments has been developed, it will be applied

to larger streams, and possible additions and changes will be considered and tested.

After a general model for small and large streams has been developed and tested, it will be adjusted for applications in various fields such as urban hydrology and streamflow forecasting. For certain applications entirely new model structures may have to be developed.

A salient feature of the HBV-2 Model is that all rainfall is assumed to infiltrate. Also important is that only some years of a data record are used for parameter calibration. The remaining years are used for testing the model on independent data material.

Work is now being done on (1) adjustments of the model structure, (2) the estimation routine for potential evaporation, (3) the development of a snowmelt routine, and (4) an automatic parameter optimization program.

#### **DATA REQUIREMENTS**

In particular, the development of a suitable model structure and of a method for parameter evaluation, as well as the necessary testing of the model require high quality data records of some length. The actual application of the model can, of course, be done with less accurate data input.

The measurement of the most important data input, the precipitation, is regularly biased on the low side.

It is desirable, of course, to have the precipitation stations within the catchment, but this is not always possible.

The spatial variability of rainfall, particularly when there are local rain showers, introduces an additional uncertainty, which at times could be substantial.

The potential evaporation is generally computed by Penman's formula, using values of total radiation, air temperature, air humidity, wind run and duration of sunshine. The albedo of the catchment surface has to be estimated, which introduces an uncertainty.

The application of Penman's formula poses a special problem, but this will not be dealt with here.

Evaporimeter data could also be used if available.

The streamflow records may be in error by 5 per cent or more, even if the station is rated as very good.

All hydrologic data used are more or less in error and this has a bearing on the suitable level of sophistication of the model structure.

#### THE STRUCTURE OF THE HBV-2 MODEL

The main components of the model are a soil moisture zone and two reservoirs, as shown in Fig. 1. The upper zone storage is fed with water from the soil moisture zone. When available, water percolates from the upper to the lower zone storage at a constant rate PERC. Meanwhile, runoff takes place from these two storages in proportion to their contents. The lower zone is also affected directly by precipitation, and by potential evaporation on a part of it that represents lakes, rivers and outflow areas. This procedure means that during a wet period when the upper zone storage is not empty, recharge of the lower zone can be expressed as:

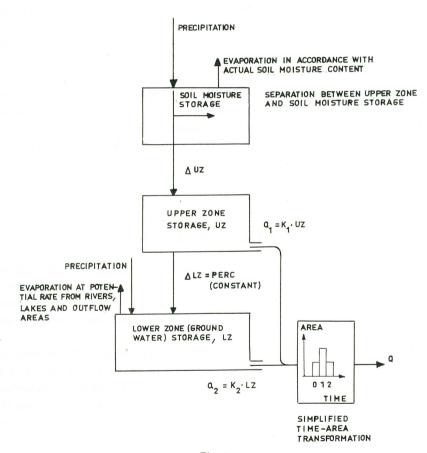


Fig. 1. The structure of the HBV-2 model.

$$NR = PERC - K_2 \cdot LZ \tag{1}$$

where:

NR = net recharge of the lower zone

PERC = percolation

K<sub>2</sub> = storage-discharge constant for the lower zone

LZ = lower zone storage

The effect of this assumption on the lower zone is shown in Fig. 2. During a wet period, LZ will increase and asymptotically approach its upper limit, which is  $PERC/K_2$ . At this point LZ has reached its equilibrium where inflow is balanced by outflow. Even though this theory is disturbed by the part of the lower zone at potential evaporation (Fig. 1), it has been found useful for the estimation of PERC.

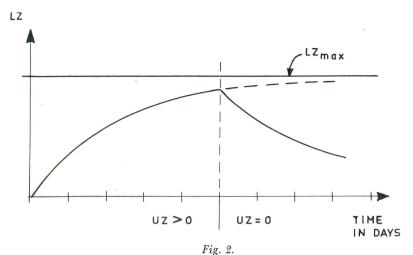
In the HBV-1 model the upper zone was excluded, meaning that the catchment reacts to excess water from the soil moisture zone more or less like a single linear reservoir.

Finally discharges from the two reservoirs are added and a simple time-area transformation is applied. This transformation is a triangular function adjusted for correct timing of peakflows (one parameter). This procedure was not included in the HBV-1 model where a simple time-lag procedure between rainfall and runoff was adopted.

#### THE SOIL MOISTURE ZONE

In the soil moisture routine, precipitation is split up into two parts. One remains in the soil moisture storage and eventually leaves the system as evaporation. The other passes through the soil moisture zone and enters the upper zone, thereby contributing to runoff or evaporation from the lower zone storage (Fig. 1).

The determination of the part entering the upper zone,  $\Delta UZ$ , is made in accordance with actual soil moisture content as shown in Fig. 3. In Fig. 4 is shown the estimation of actual evaporation from potential. Beta is a numerical parameter governing the disposal of rainfall. LP is a parameter representing the soil moisture value below which actual evaporation is impeded. FC is analogous to the difference between field capacity and wilting point. After a long dry spell, when the soil moisture content (SM) is low, rainfall will consequently add very little to runoff, while after a wet period the situation is the opposite.



Net recharge and drainage of the lower zone. UZ = Upper zone, LZ = Lower zone.

This method has the advantage of being very flexible, as no assumption of homogeneity over the catchment area is made. The optimum value of BETA in the expression  $\left(\frac{SM}{FC}\right)^{BETA}$  (Fig. 3) describes the overall response of the entire

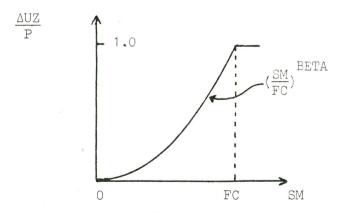
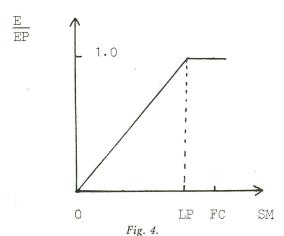


Fig. 3.

Estimation of the part of the precipitation which enters the upper zone. △UZ = contribution to upper zone. P = precipitation. SM = soil moisture content. FC = max. soil moisture content, corresponding to field capacity minus wilting point. BETA = numerical parameter governing the disposal of rainfall.



Relation between actual and potential evaporation. E = actual evaporation. EP = potential evaporation. SM = soil moisture content. LP = limit below which actual evaporation is impeded. FC = max. soil moisture content, corresponding to field capacity minus wilting point.

basin. The model has also been tested in highly inhomogenous areas with encouraging results.

To allow for changes in the soil moisture content during rainfall, precipitation is fed into the soil moisture routine millimeter by millimeter. This leads to an increase, during the rainfall, of the part contributing to runoff, a more feasible way than letting the initial soil moisture content govern the separation of rainfall throughout the day.

#### FITTING THE MODEL

As an index of agreement between observed and computed discharges, Nash & Sutcliffe (1970) suggested the following sum of squares criterion:

$$F^2 = \Sigma (q_{\text{obs}} - q_{\text{comp}})^2$$
 (2)

where

 $q_{obs} = observed discharge$ 

 $q_{comp} = computed discharge$ 

If we define an initial variance F<sub>0</sub><sup>2</sup> by:

$$F_o^2 = \Sigma (\overline{q}_{obs} - q_{obs})^2 \tag{3}$$

where  $\overline{q}_{obs} = mean$  of the observed discharge, the proportion of the initial variance accounted for by the model can be expressed as:

$$R^2 = \frac{F_0^2 - F^2}{F_0^2} \tag{4}$$

This is what Nash & Sutcliffe (1970) call the efficiency of the model.

In this investigation F<sup>2</sup> has been used for optimization of parameters. R<sup>2</sup>-values have also been computed, but it was found that this criterion must be interpreted with great caution, especially when applied to single years. Some of the years were extremely dry, and due to the low initial variance of the actual discharges, small differences between computed and observed discharges produced low values of R<sup>2</sup>.

In addition to this, visual inspection and the accumulated differences between observed and computed hydrographs have been studied concurrently.

#### PARAMETER EVALUATION

The model includes 8 parameters to evaluate as follows (Figs. 1, 3 and 4):

Three parameters in the soil moisture-evaporation routine, FC, BETA and LP.

Two storage-discharge constants  $K_1$  and  $K_2$ .

One percolation constant, PERC.

One timing parameter, T, describing the simplified time-area diagram.

The part of the area at potential evaporation represented by a constant P.

In this paper, however, six of the eight parameters are found through studies of the observed hydrograph and the catchment, rather than by optimization. This method, though admittedly not automatic, has the advantage that because fewer parameters remain to be optimized, problems of multiple maxima and dependence between the parameters are greatly reduced.

Investigations showed that the objective function is extremely flat around the optimum FC-value and strongly interacts with LP. A substantial change in FC was easily absorbed by a change in LP. Therefore a rough estimate of FC, after considerations of the soil conditions in each catchment, is sufficient.

The two storage-discharge constants are estimated from recession analysis of the observed hydrograph, Q.

If log nat Q is plotted against time during a recession period, a pronounced

knee is often detected, indicating that the upper zone is empty. This knee can occur at different levels depending on the rate of outflow from the lower zone. Its highest level during the calibration period is taken as the equilibrium point for LZ (Fig. 2) and is used for the estimation of PERC. The method is very rough but probably accurate enough as the model has proved to be only slightly sensitive to changes in PERC.

The timing parameter, T, corresponds to the time of concentration and is obtained from inspections of rainfall and runoff records. If T=2 a proportion of 1/3 and 2/3 is used between the first and the second day. Otherwise a triangular function is used as shown in Fig. 1.

The part of the basin at potential evaporation is taken from the map. Finally the two remaining parameters, LP and BETA, are easily found from the contours of the objective function, F<sup>2</sup>.

In addition to these parameter values, the model also requires initial values in the different storages at the beginning of the season: soil moisture (SM), upper zone (UZ) and lower zone storage (LZ). SM is given the value of FC while UZ and LZ are adjusted to suit the observed discharge on the first day.

#### APPLICATION OF THE MODEL TO TEST BASINS

The HBV-1 model was applied to the Lilla Tivsjön Catchment, and the HBV-2 has so far been applied to the Nolsjön, Stabby and Stormyra Catchments.

These four catchments are quite small and they are gaged by high quality stream-gaging stations equipped with weir structures and with water level recorders.

The vegation is predominantly coniferous forest and the bedrock is igneous rock (granite or gneis) overlain by a thin – less than 20 meter thick – soil layer. It should also be pointed out that the groundwater level is quite close to the ground surface.

The general catchment characteristics are shown in Table 1. The figures in the Table are only approximate.

The Lilla Tivsjön Catchment is a sub-basin within the IHD Representative Basin Kassjöån. It is situated in central Sweden in the counties of Jämtland and Medelpad, and the streamflow drains through the River Ljungan to the Baltic Sea.

The stream-gaging station is located at the outlet of the Lake Lilla Tivsjön with a lake area of 0.32 km², which tends to damp rapid streamflow fluctua-

#### Development of a Conceptual Deterministic Rainfall-runoff Model

Table 1.

	Catchments								
Catchment characteristics	Lilla Tivsjön	Nolsjön	Stabby	Stormyra					
Drainage area, km <sup>2</sup>	12.7	18.2	6.4	4.0					
Lake, per cent	2.7	1.5	0.0	0.0					
Coniferous forest, per cent	88	78	79	80					
Swamp, per cent	8	14.2	2.3	1.6					
Arable land, per cent	0.5	3.7	11	5.9					
Exposed bedrock, per cent	1	2	22	73					
Moraine, per cent	96	95	56	8					
Coarse material, per cent	1	1	0	9					
Silt and clay, per cent	0	0	18	10					
Altitude range, meters	200	55	37	67					
Difference between mean and									
minimum elevation, meters	65	21	17	32					
Average depth to groundwater									
table, meters	1-2	1	1-2	1					
Ratio, mean annual									
maximum to mean annual flow	9.7	10.5	14	16					
Mean annual flow, l/s	117	171	49	36					
Mean annual flow, l/s·km <sup>2</sup>	9.2	9.4	7.6	9					
Mean annual precipitation, mm	600	700	640	600					
Mean annual evaporation, mm	300	400	400	380					

tions. The precipitation station is at the southern end of the catchment close to the stream-gaging station.

In this early attempt the potential evaporation was taken as the long term mean monthly Penman values somewhat adjusted for the current air temperature (Wallén 1966).

Moraine soil dominates. Large areas of the forest have been clear cut. The relief is quite high with an altitude range of 200 meters.

The catchment could be considered very homogeneous.

The *Nolsjön Catchment* is a sub-basin within the Velen IHD Representative Basin. It is situated in the southern half of Sweden in the county of Västergötland. The streamflow drains to the large Lake Vättern.

The streamgaging station is at the outlet of Lake Nolsjön (lake area 0.07

km<sup>2</sup>) which seems to be too small to affect the streamflow fluctuations appreciably.

For most of the test period precipitation has been taken from one gage, Nordtorp, centrally located in the catchment. But for some years Thiessen polygon values from a number of stations outside the catchment were used.

The same method as for Lilla Tivsjön has been used for the estimation of the potential evaporation.

Coniferous forest on moraine dominates, but  $14\,$  % is swamps. The altitude range is much less than in the Kassjöån Catchment.

The whole catchment is fairly homogeneous.

The Stabby Catchment is situated close to Uppsala, which is 70 km north of Stockholm in central Sweden.

Coniferous forest on moraine soil dominates this catchment too, but a substantial part is "exposed" bedrock with partly a thin moraine cover less than 0.5 m thick.

The altitude range is less than 37 m.

This catchment is not very homogeneous. It is described in detail by Bergqvist (1971).

The precipitation data are taken from a station 4 km away from the catchment, and long term monthly means of Penman values were used as potential evaporation input (Wallén 1966).

The *Stormyra Catchment* is situated only a couple of km southeast of Stockholm.

This catchment is also dominated by coniferous forest, part of which has been clear cut, but as much as 73 % is exposed bedrock with a soil cover of coarse material less than 0.5 m thick. Forty-eight per cent has a soil cover less than 1 decimeter. The altitude range is 67 m. Ten per cent is covered by clay soil. The streamflow response to rainfall is very rapid. The catchment is quite inhomogeneous. Precipitation stations at some distance from the catchment were used together with long-term monthly means of Penman values of potential evaporation (Wallén 1966).

#### **RESULTS**

For all catchments a calibration period of 1–3 years has been used for parameter evaluation. The optimum parameter values thus obtained are presented in Table 2 and the agreement between the observed and computed hydrographs expressed as R<sup>2</sup>-values (4) is given in Table 3. In Figs. 5–12 hydrographs of

### Development of a Conceptual Deterministic Rainfall-runoff Model

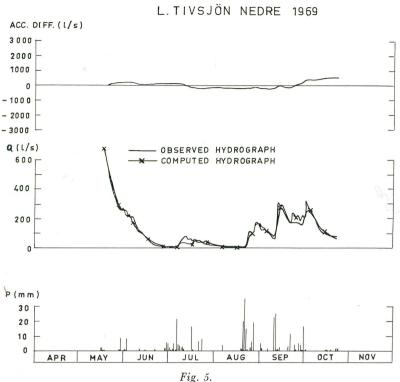
 $Table\ 2.$  Optimum parameter values for the different catchments.

	FC (mm)	LP (mm)	ВЕТА	K <sub>1</sub> (day <sup>-1</sup> )	K <sub>2</sub> (day <sup>-1</sup> )	PERC mm/day	T (days)	P º/0
Lilla Tivsjön	100	70	3.4	_	0.079	-	2*	4.6
Nolsjön	200	160	8.0	0.194	0.071	0.5	4	2.0
Stabby	100	100	7.0	0.360	0.137	0.9	2	0.5
Stormyra	50	40	4.0	0.422	0.126	0.8	2	3.0

<sup>\*</sup> A time lag of one day is used between rainfall and runoff for the HBV-1 model in Lilla Tivsjön.

 $Table \ 3.$  Results from the different catchments.

			$\mathbb{R}^2$
Lilla Tivsjön	1969	used for optimization	0.98
	1968	simulated	0.84
	1967	simulated	0.97
Nolsjön	1971	used for optimization	0.81
	1970	used for optimization	0.91
	1969	simulated	0.97
	1968	simulated	0.90
	1967	simulated	0.88
Stabby	1959	used for optimization	-0.64
,	1960	used for optimization	0.93
	1961	used for optimization	0.80
	1962	simulated	0.76
	1963	simulated	-0.90
	1964	simulated	0.84
	1965	simulated	0.93
	1966	simulated	0.64
	1967	simulated	0.62
Stormyra	1963	used for optimization	0.87



Fitted year in the Lilla Tivsjön catchment.

selected years are shown together with precipitation and the accumulated difference between the computed and observed discharges. From the Stormyra catchment only one year is included, but work is going on with further test runs.

The weakness of R<sup>2</sup> as a criterion of fit is evident for some of these tests. During 1959 in Stabby, for example, the negative value indicates an impossible model, while it can be seen from the plotting that this is an effect of the extremely dry season with very low initial variance. Conversely the high springflood recession in certain years gives a high initial variance and very high values of R<sup>2</sup> (L. Tivsjön 1967). The R<sup>2</sup> criterion seems to be less reliable in small catchments with little damping, where correct timing is more important than in larger catchments.

When relating parameter values to catchment characteristics, the parameter

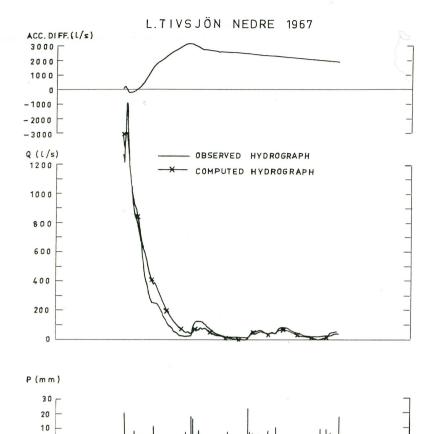


Fig. 6.
Simulated year in the Lilla Tivsjön catchment.

stability must be taken into consideration. Therefore it is felt to be premature to draw any conclusions concerning BETA and LP.  $K_1$  and  $K_2$  seem to be more or less related to the size of the catchment, and this of course is an effect of the heavy damping in larger basins. PERC could be related to the permeability in the catchment, but one must bear in mind that a lake near the outlet can damp out the effect of the two runoff components. This is the situation in the Lilla Tivsjön catchment and can be the explanation of why the HBV-1 model worked well there.

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 $Table \ 4.$  Optimum values of LP and BETA obtained from objective function contours over different periods in the Stabby catchment.

	ВЕТА	LP
1959–1961	100	7.0
1962-1964	60	3.0
1965-1967	60	3.0
1959–1967	80	5.0

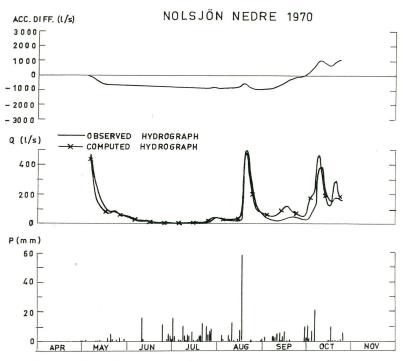


Fig.~7. Fitted year in the Nolsjön catchment.

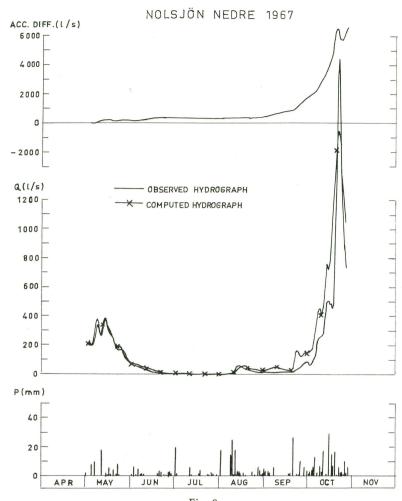
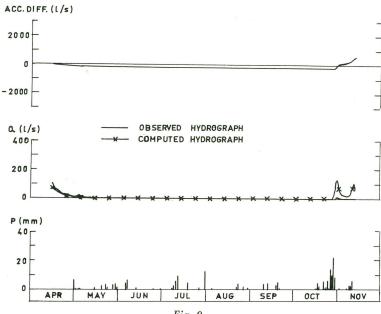


Fig. 8. Simulated year in the Nolsjön catchment.

#### STUDIES OF PARAMETERS

In the Stabby catchment a special investigation was carried out to see if the length of the calibration period was enough to get stable values of LP and BETA. Objective function contours for the three years 1959–1961 were compared with the corresponding contours for 1962–1964, 1965–1967 and for all

#### STABBY 1959



Fitted year in the Stabby catchment.

#### STABBY 1960

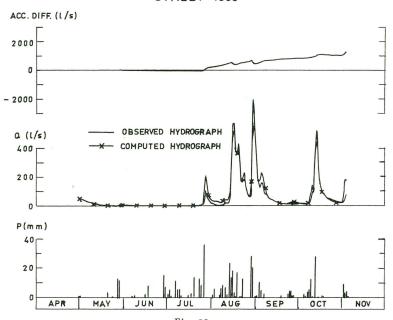
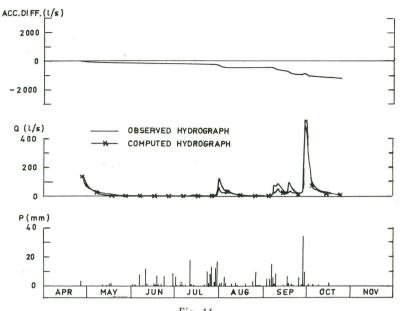


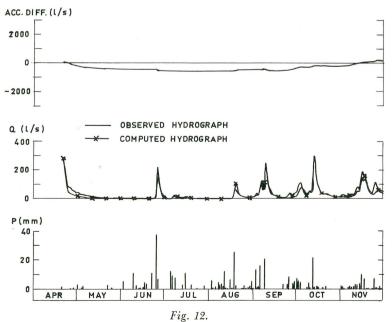
Fig. 10. Fitted year in the Stabby catchment.

#### STABBY 1965



 $\label{eq:Fig.11.} \emph{Fig. 11.}$  Simulated year in the Stabby catchment.

#### STORMYRA 1963



Fitted year in the Stormyra catchment.

#### Sten Bergström and Arne Forsman

9 years. The results are shown in Figs. 13–16, and the optimum values obtained are presented in Table 4.

The results indicate that a three-year period is not enough to get stable values of LP and BETA in Stabby. This is probably a combined effect of simplifications in the model and insufficient data. One source of error is certainly the use of standard values of potential evaporation for all years. An underestimation of potential evaporation one year can easily be corrected by a decrease in LP which will, of course, affect the stability of this parameter. The

```
BETA
               3.0
        1.0
                       5.0
                              7.0
                                     9.0
                                           11.0
                                                  13.0
                                                         15.0
                                                                17.0
                                                                        19.0
LP
20
      115.
             166.
                    208.
                           228.
                                   240.
                                          247.
                                                 251.
                                                        254.
                                                               255.
                                                                       257.
40
      123.
             118.
                    157.
                           181.
                                  194.
                                          204.
                                                 209 ..
                                                        213.
                                                               217.
                                                                       220.
60
      156.
                     99.
                           117.
                                  130.
                                          138.
                                                 145.
                                                        150.
                                                               153.
                                                                       156.
      207.
              (58%
                     54.
                            59.
                                   64.
                                           70.
80
                                                  71.
                                                         75.
                                                                76.
                                                                        78.
                     45.
                             40.
                                                         48.
100
      269.
                                    41.
                                           43.
                                                  46.
                                                                50
                                                                        52.
                            64.
                                   64.
120
      333.
             115.
                                           66.
                                                  69.
                                                         71.
                                                                73.
                                                                        76.
140
                                  117.
                                          121.
                                                 124.
                                                        128.
      395.
             170.
                    127.
                           117.
                                                               130.
                                                                      133.
160
      454.
             231.
                    191.
                           183.
                                  186.
                                          191.
                                                 197.
                                                        202.
                                                               207.
                                                                      212.
180
                                  261.
                                          269.
                                                        284.
      511.
             293.
                    259.
                           255.
                                                 277.
                                                               291.
                                                                      296.
                           328.
                                  336.
200
     563.
             355.
                    327.
                                          346.
                                                 355.
                                                        365.
                                                               372.
                                                                      378.
```

Fig. 13. Objective function (F $^2 \cdot 10^{-4}$ ) response to BETA and LP in Stabby 1959–1961.

BETA	1.0	3.0	5.0	7.0	9.0	11.0	13.0	15.0	17.0	19.0
LP 20	₹. \	22.	35.	42.	45.	46.	48.	49.	50.	50.
40	62.	15.	24.	33.	39:	41.	43.	46.	46.	46.
60	103.	1/1/2.	16.	22.	26.	31.	33.	35.	37.	38.
80	148.	50	14.	17.	<b>8</b> 1.	23.	27.	29.	29.	30
100	198.	41	15.	-26.	29.	-34.	37.	40.	42.	44.
120	248.	73.	50.	49.	53.	57.	62.	64.	67.	70.
140	296.	111.	86.	83.	87.	91.	95.	99.	102.	104.
160	345.	154.	128.	127.	131.	135.	140.	144.	148.	151.
180	389.	199.	173.	181.	180.	186.	191.	197.	201.	204.
200	430.	242.	219.	221.	229.	238.	245.	252.	257.	262.
					Fig 14					

Objective function (F2  $\cdot$  10–4) response to BETA and LP in Stabby 1962–1964.

interaction between LP and BETA is stronger than can be seen from Figs. 13–16. For high values of BETA the difference between two consecutive curves in Fig. 3 is very small and this gives a false impression of independence between LP and BETA. The high sensitivity of the objective function for the first period is caused by some extremely successful years 1960–1961. Years with more error cause a flatter topography of the objective function (Figs. 14, 15).

The sensitivity of the objective function F<sup>2</sup> to changes in other parameters has also been investigated in Stabby. The year 1960 was used as the best of the

BETA	1.0	3.0	5.0	7.0	9.0	11.0	13.0	15.0	17.0	19.0
LP 20	76x. 1	94.	127.	148.	160.	168.	174.	177.	181.	183.
40	141	82.	107	127.	142.	150.	157.	162.	165.	168.
60	187.	\ <b>\</b> \$1.	St.	305:	116,	126.	133.	140.	145.	148.
80	239.	1/4/2	86.	92.	990	105.	1095	112.	117.	119.
100	291.	123	105.	105.	110.	115.	121-	126.	129.	133.
120	344.	166.	143.	144.	148.	155.	161.	167.	172.	175.
140	396.	215.	195.	197.	202.	208.	212.	217.	222.	226.
160	446.	269.	254.	259.	267.	274.	279.	284.	287.	291.
180	494.	323.	315.	325.	337.	348.	357.	362.	367.	371.
200	540.	376.	376.	393.	411.	425.	436.	445.	452.	458.
				7	Fig. 15					

Objective function (F2 · 10-4) response to BETA and LP in Stabby 1965-1967.

BET LP	ΓA ·	1.0	3.0	5.0	7.0	9.0	11.0	13.0	15.0	17.0	19.0
20	225	. \	282.	370.	418.	445.	461.	473.	480.	486.	490.
40	326	V		288.	341.	375.	395.	409.	421.	428.	434.
60	446	W.	(169-		244.	272.	295.	311.	325.	335.	342.
80	591	. *	My of	(15)	1685	104	198.	201·	-216.	222	227.
100	7.58	3.	237.	165.	171/	180)	192	<u>-204.</u>	-244.	221	229.
120	925		354.	267.	257.	265.	278.	292.	302.	312.	321.
140	1087		496.	408.	397.	406.	420.	431.	444.	454.	463.
160	1245		654.	573.	569.	584.	600.	616.	630.	642.	654.
180	1394		815.	747.	761.	778.	803.	825.	843.	859.	871.
200	1533		973.	922.	942.	976.	1009.	1036.	1062.	1081.	1098.

Fig. 16. Objective function ( $F^2 \cdot 10^{-4}$ ) response to BETA and LP in Stabby over the whole period 1959–1967.

fitted years. In Figs. 17–19 a few examples from this investigation are shown. It is interesting to note the high interaction between LP and FC (Fig. 17), indicating that one parameter might be expressed as a function of the other. In his case the elimination of LP would simplify the model. A test of sensitivity along the axis of LP or FC could erroneously be interpreted as high sensitivity to both these parameters independently.

It is evident from Fig. 18 that quite large errors in the estimation of PERC can be accepted, but also that this parameter significantly improves the model. This is favourable as the estimation of PERC is felt to be one of the weakest points in the evaluation procedure. Fig. 19 shows a plateau on which  $K_1$  and  $K_2$  can vary considerably before any dramatic deterioration of the agreement occurs. This means that an acceptable value of these parameters can be found after a very short calibration period, perhaps only one recession event.

20 25 73 131 179 233	FC LP	60	80	100	120	140	160
40 19: 30 94 145 197 60 48: 18. 54 112 161 213 180 108 24 69 125 175 175 100 168 61 15 34 85 138 120 222 107 32 17 48 102 140 273 152 63 19 85 67 160 320 190 98 35 17 18 180 354 225 134 58 22 24 200 379 256 165 84 33 28 220							
60 48 18 54 112 161 213 80 108 21 69 125 175 175 100 168 61 15 48 51 138 120 222 107 17 48 102 140 273 152 63 9 85 67 160 320 190 98 35 17 18 180 354 225 134 58 22 24 200 379 256 165 84 33 20 220	20	25 🖍	<b>\</b> 73.	131.	179.	233.	
80 108. 21. 69. 125. 175. 100 168. 61. 15. 34. 85. 138. 120 222. 107. 17. 18. 102. 140 273. 152. 63. 12. 25. 67. 160 320. 190. 98. 35. 17. 18. 180 354. 225. 134. 58. 22. 24. 200 379. 256. 165. 84. 33. 26. 220	40	12:	1364	94.	145.	197.	
100       168.       61.       15.       34.       85.       138.         120       222.       107.       32.       17.       48.       102.         140       273.       152.       63.       49.       25.       67.         160       320.       190.       98.       35.       17.       48.         180       354.       225.       134.       58.       22.       24.         200       379.       256.       165.       84.       33.       28.         220       .       .       .       .       .       .       .       .	60	48.	<u> </u>   18;	\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\	112.	161.	213.
120       222.       107.       17.       48.       102.         140       273.       152.       63.       19.       25.       67.         160       320.       190.       98.       35.       17.       18.         180       354.       225.       134.       58.       22.       24.         200       379.       256.       165.       84.       33.       28.         220       .       .       .       .       .       .       .	80	108.	144	\\$J.	<b>\</b> 69.	125.	175.
140     273.     152.     63.     9.     85.     67.       160     320.     190.     98.     35.     17.     18.       180     354.     225.     134.     58.     22.     24.       200     379.     256.     165.     84.     33.     28.       220     .     .     .     .     .     .	100	168.	61.	<del>]][\\\\\</del> 5.	134°	85.	138.
160 320. 190. 98. 35. 17. 18. 180 354. 225. 134. 58. 22. 24. 200 379. 256. 165. 84. 33. 28. 220	120	222.	107.	PH	17.	148.	102.
180 354. 225. 134. 58. 22. 24. 200 379. 256. 165. 84. 33. 28. 220	140	273.	152.	63.	1/1/2.	85%	67.
200 379. 256. 165. 84. 83. 20. 220	160	320.	190.	98.	35!	17.	1 De
220	180	354.	225.	134.	58.	/\$5;	24.
1	200	379.	256.	165.	84.	<i>J</i> 3!	50
240	220					,	23.
	240						/31;

Fig. 17. Objective function (F2  $\cdot$  10-4) response to FC and LP in Stabby 1960.

PERC	0.00	0.30	0.60	0.90	1.20	1.50	1.80	2.10	2.40	2.70	3.00	3.30	3.60
LP T									136.				140.
20 -	124.	127.	129.	, , , ,		98.	99•	101.	102.	104.	105.	107.	108.
40	87.	90.	92.	94.	96.	-		62.	64.	67.	69.	70.	72.
60	50.	51.	53.	54.	56.	58.	60.	28:	30	32.	34.	36.	<del>-38.</del>
80	24:	24.	24.	24.	25.	26.	21.			15.	16.	17.	18.
100	20 <	19.	17.	15.	15.	14.	14.	14.	14.		22.	22.	22.
120	40_	38.	35•	32.	59	27.	25.	24.	23.	23.		44.	43.
140	74.	71.	67.	63.	59•	56.	53.	50.	48.	47.	45.		
	112.	109.	104.	98.	94.	90.	86.	83.	80.	77.	75.	74.	72.
160		146.	140.	134.	128.	122.	118.	114.	111.	108.	105.	103.	101.
180	149.	0	172.		158.	152.	147.	142.	138.	135.	132.	129.	127.
200	181.	178.	112.	10).	.,,,,,	Fig							

Fig. 18. Objective function (F2  $\cdot$  10<sup>-4</sup>) response to PERC and LP in Stabby 1960.

K 1	0.1	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55
K <sub>2</sub> [										
0.03	102.	68.	46.1	311	21./	17.	17.	20	27.	36.
0.06	101.	67.	45.	29.	20	16.	15.	19.	26.	35.
0.09	100.	66.	44.	29.	20.	15.	15.	19.	25.	34.
0.12	99.	66.	44.	29.	20.	15.	15.	19.	26.	35.
0.15	98.	66.	44.	29.	20	16.	16.	20	26.	35.
0.18	97.	65.	44.	30.	21.	17.	17.	20/.	27.	36.
0.21	97.	65.	44.	30	21.	17.	18.	2/1.	28.	37.
0.24	96.	65.	+4.	30	22.	18.	18.	/22.	29.	38.
0.27	96.	65.	44.	31.	23.	<b>\</b> 9.	19. /	23.	30	39.
0.30	95.	64.	14.	31.	23.	29.	20/	24.	31.	40.
E: ~ 10										

Fig. 19. Objective function (F $^2 \cdot 10^{-4}$ ) response to  $K_1$  and  $K_2$  in Stabby 1960.

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