A MULTILEVEL QUASI-GEOSTROPHIC MODEL FOR SHORT RANGE WEATHER PREDICTIONS

By
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A multilevel quasi-geostrophic model for short range weather predictions

Abstract

A quite generally formulated multilevel quasi-geostrophic model with possibilities to include second order terms in the vorticity equation is derived. The model includes friction, topography, latent heat and sensible heat. The treatment of the variable boundary conditions, smoothing and ellipticity control is described.

1. Introduction

Operational numerical weather predictions have for some time been performed at SMHI by use of an integrated quasi-geostrophic three-parameter model. The operational system has been described by Bengtsson and Moen (1971). The importance of a fine horizontal resolution, especially for the vertical motion pattern and the release of latent heat, is demonstrated in that article. Large scale boundary errors are eliminated in the fine-mesh model by the so-called grid-telescoping technique implying insertion of lateral boundary values produced by a course mesh, large area model.

Recently Moen (1974) has studied the effect on a developing cyclone of two possible improvements of the quasi-geostrophic model, namely a larger vertical resolution and the use of the vorticity and the thermodynamic equations in their complete form. It is shown that models with a high vertical resolution have a significantly higher instability for meteorological disturbances of short wavelengths. In order to resolve this instability about 5 vertical levels and 150 km horizontal gridlength are essential. The inclusion of small, frequently neglected terms in the vorticity and thermodynamic equations, although not energy consistent, adds features to the development of individual cyclones which are typical for the real atmosphere. The main contribution of these terms is a deepening of the cyclones and weakening of the anticyclones at the surface level.

A model with 5-6 vertical levels increases the possibilities to describe different mechanisms such as the release of latent heat, the effect of topography and the interaction between the free atmosphere and the boundary layer.

The assimilation of the increasing amount of non-synoptic observations from satellites and aircrafts also requires an analysis-forecast system with sufficiently high vertical resolution.

A quite generally formulated prediction model with possibilities to include second order terms in the vorticity equation will here be derived.

First the basic system of equations, the finite difference form and the method of solution will be defined without specification of the lower boundary condition and non-adiabatic heating. After that different physical effects, such as latent and sensible heating, friction and topography will be introduced. The treatment of lateral boundary conditions, smoothing and ellipticity control will also be described separately.

The model will be used in routine operations and will probably undergo modifications, especially in the parameterization of physical effects, as experiences are gained.

List of notations not defined in the text.

A	diffusion coefficient
c_p	specific heat at constant pressure
E	water vapor pressure at saturation
f	Coriolis parameter
g	gravity
k_E	exchange coefficient
L	latent heat
p	pressure
p_s	standard surface pressure
Q	rate of heating
q	specific humidity
q_s	-"- -"- at saturation
R	gas constant for dry air
R_v	-"- -"- -"- water vapor
s	static stability
t	time
T_s	sea surface temperature
z	height to a pressure surface
ϕ	geopotential
ω	vertical velocity in the p-system
ψ	streamfunction
ρ	density
θ	potential temperature
ζ	relative vorticity
χ	velocity potential

2. The basic system

2.1 The system of equations

The governing equations are the vorticity equation and the thermodynamic equation in the p-system

$$\begin{cases} (1) \quad \frac{\partial \zeta}{\partial t} + J(\psi, \zeta + f) = f \frac{\partial \omega}{\partial p} + W \\ (2) \quad \frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial p}\right) + J\left(\psi, \frac{\partial \phi}{\partial p}\right) + s\omega = -\frac{R}{c_p p} Q \end{cases}$$

where

$$(3) \quad W = \zeta \frac{\partial \omega}{\partial p} - \omega \frac{\partial \zeta}{\partial p} - \nabla\omega \cdot \nabla\left(\frac{\partial \zeta}{\partial p}\right) - \nabla\chi \cdot \nabla(\zeta + f)$$

is a sum of usually neglected terms of second order magnitude.

The first term in W describes, together with $f \frac{\partial \omega}{\partial p}$, the generation of vorticity by horizontal divergence. The second term in W represents generation of vorticity by vertical advection. The third term is the so-called twisting term, describing the "tilting" of horizontally oriented components of vorticity into the vertical by a non-uniform vertical motion field. The last term in W expresses the advection of vorticity by the divergent part of the wind. The velocity potential χ can be solved from the continuity equation.

$$(4) \quad \nabla^2 \chi = -\frac{\partial \omega}{\partial p}$$

The only neglected term in the vorticity equation is the divergent part of the twisting effect, the magnitude of which is one order smaller than all other terms in W.

In the thermodynamic equation (2) advection of temperature by the divergent wind is neglected and the static stability, $s = \frac{1}{\theta} \frac{\partial \phi}{\partial p} \frac{\partial \theta}{\partial p}$, is assumed to be a function of pressure only.

The rate of non-adiabatic heating, Q, will be defined in the following sections.

The relation between the mass- and the windfield is given by the balance equation

$$(5) \quad \nabla^2 \phi = \nabla \cdot (f \nabla \psi)$$

In the thermodynamic equation the relation

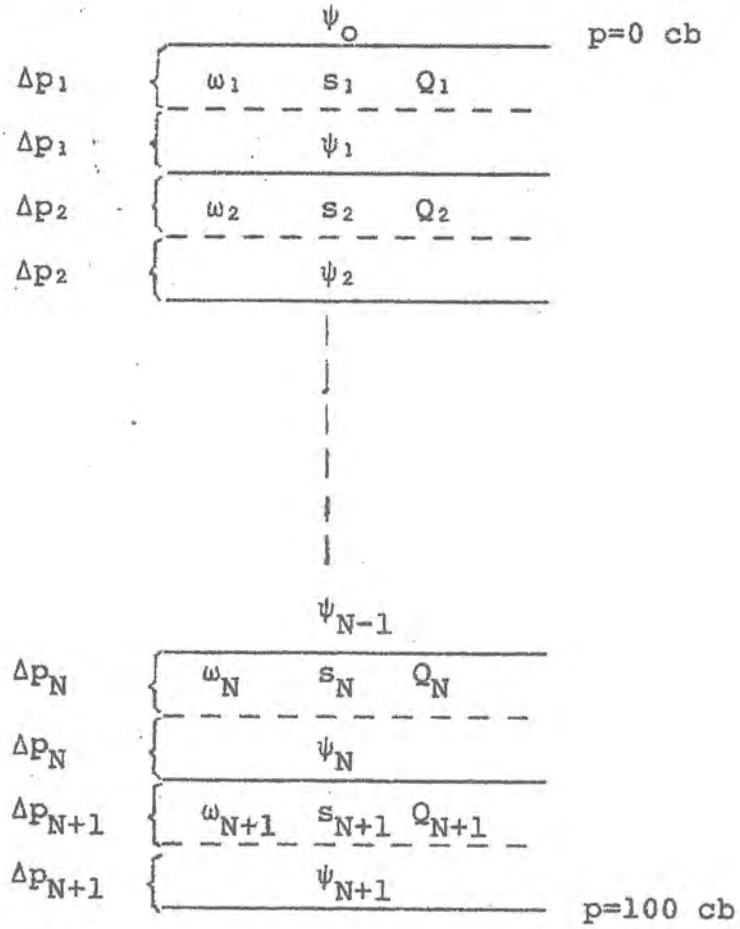
$$(6)\quad \frac{\partial\phi}{\partial p} = f_o \frac{\partial\psi}{\partial p}$$

where f_o is a constant, is used and equation (2) can be written as

$$(7)\quad \frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial p}\right) + J\left(\psi, \frac{\partial\psi}{\partial p}\right) + \frac{s}{f_o}\,\omega = -\frac{R}{f_o c_p p}\,Q$$

The two equations (1) and (7) now form a closed system in ψ and ω since ζ can be expressed in ψ and χ in ω.

The atmosphere is divided into N arbitrarily placed pressure levels between the top of the atmosphere and 100 cb. At these levels the streamfunctions ψ_n are defined. To minimize the vertical truncation error, ω_n, s_n and Q_n are placed on levels half-ways between ψ_n-levels.



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[illegible] not [illegible] ed syste [illegible] and [illegible]

The atmosphere [illegible]
pressure levels [illegible] of the atmosphere and
[illegible]

[illegible] levels.

[illegible]

The boundary conditions are

$$(8) \quad \begin{cases} \psi_o = \text{constant at } p=0 \\ \omega_{N+1} \text{ prescribed} \end{cases}$$

The commonly used upper boundary condition in similar models is $\omega=0$. However, to be able to use centered differences a ψ-level is neede at p=0.

Comparative computations with the two different upper boundary conditions have shown no differences for short and medium-long waves, whereas only small differences for ultra-long waves have been obtained.

The level where ω_{N+1} is prescribed does not coinside with the earth surface which is the case in most other models. The present arrangement can be justified by noting that the constraint on ω is best valid at the top of the boundary layer. The lowest ω-level can thus be placed at the mean position of the top of the boundary layer.

Our prognostic variables can be expressed by two vectors

$$(9) \quad \boldsymbol{\psi} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ | \\ | \\ | \\ \psi_N \end{bmatrix} \qquad \boldsymbol{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ | \\ | \\ | \\ \omega_N \end{bmatrix}$$

We first assume that W=0 and apply equation (1) on the ψ-levels and equation (7) on the ω-levels.

For n=1, 2. ..... N we get

$$
(10)\begin{cases}
\dfrac{\partial \zeta_n}{\partial t} = J_n + f\alpha_n \ (\omega_{n+1} - \omega_n) \\
\omega_n = f_o\beta_n\tilde{J}_n + f_o\beta_n(\dfrac{\partial \psi_{n+1}}{\partial t} - \dfrac{\partial \psi_n}{\partial t}) - \delta_n Q_n
\end{cases}
$$

where

$$
\begin{cases}
J_n = J(\zeta_n + f,\ \psi_n) \\
\tilde{J}_n = J(\psi_n,\ \psi_{n-1}) \\
\alpha_n = \dfrac{1}{\Delta p_n + \Delta p_{n+1}} \\
\beta_n = \dfrac{1}{2s_n \Delta p_n} \\
\delta_n = \dfrac{1}{c_p s_n p_n}
\end{cases}
$$

To be able to write this system of equations in a compressed form we introduce the following vectors (in addition to ψ and ω)

$$
\mathbf{J} = \begin{bmatrix} J_1 \\ J_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ J_N \end{bmatrix} \qquad \tilde{\mathbf{J}} = \begin{bmatrix} \tilde{J}_1 \\ \tilde{J}_2 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{J}_N \end{bmatrix}
$$

$$
\mathbf{Q} = \begin{bmatrix} Q_1 \\ Q_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ Q_N \end{bmatrix} \qquad \boldsymbol{\omega}_L = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ \omega_{N+1} \end{bmatrix}
$$

Note that $\tilde{J}_1 = 0$ due to the upper boundary condition.

The system (10) can now be written as

$$
\begin{cases}
(11) \quad \nabla^2 \frac{\partial}{\partial} \boldsymbol{\psi} - ff_o \mathbf{A}\cdot\frac{\partial}{\partial t}\boldsymbol{\psi} = \mathbf{J} + ff_o\mathbf{B}\cdot\tilde{\mathbf{J}} - f\mathbf{C}\cdot\mathbf{Q} + f\alpha_N \boldsymbol{\omega}_L \equiv \mathbf{H} \\[2em]
(12) \quad \boldsymbol{\omega} = f_o\mathbf{D}\cdot\tilde{\mathbf{J}} + f_o\mathbf{E}\cdot\frac{\partial}{\partial t}\boldsymbol{\psi} - \mathbf{F}\cdot\mathbf{Q}
\end{cases}
$$

$\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$, $\mathbf{E}$ and $\mathbf{F}$ are all NxN matrices.

$$
\mathbf{A} = \begin{bmatrix}
\alpha_1(\beta_1+\beta_2) & -\alpha_1\beta_2 & & & \mathbf{0} \\
-\alpha_2\beta_2 & \alpha_2(\beta_2+\beta_3) & -\alpha_2\beta_3 & & \\
 & \ddots & \ddots & \ddots & \\
 & & & & -\alpha_{N-1}\beta_N \\
\mathbf{0} & & & -\alpha_N\beta_N & \alpha_N(\beta_N+0)
\end{bmatrix}
$$

$$
\mathbf{B} = \begin{bmatrix}
-\alpha_1\beta_1 & \alpha_1\beta_2 & & \mathbf{0} \\
 & -\alpha_2\beta_2 & \alpha_2\beta_3 & \\
 & & \ddots & \ddots \\
 & & & \alpha_{N-1}\beta_N \\
\mathbf{0} & & & \alpha_N\beta_N
\end{bmatrix}
$$

$$C = \begin{bmatrix} -\alpha_1\gamma_1 & \alpha_1\gamma_2 & & & 0 \\ & -\alpha_2\gamma_2 & \alpha_2\gamma_3 & & \\ & & \ddots & \ddots & \\ & & & & \alpha_{N-1}\gamma_N \\ 0 & & & & -\alpha_N\gamma_N \end{bmatrix}$$

$$D = \begin{bmatrix} \beta_1 & & & 0 \\ & \beta_2 & & \\ & & \ddots & \\ 0 & & & \beta_N \end{bmatrix}$$

$$E = \begin{bmatrix} -\beta_1 & & & & 0 \\ \beta_2 & -\beta_2 & & & \\ & \beta_3 & -\beta_3 & & \\ & & \ddots & \ddots & \\ 0 & & & \beta_N & -\beta_N \end{bmatrix}$$

$$F = \begin{bmatrix} \gamma_1 & & & 0 \\ & \gamma_2 & & \\ & & \ddots & \\ 0 & & & \gamma_N \end{bmatrix}$$

The forcing function vector $\mathbf{H}$ in equation (11) is computed with W=0. In the case W≠0 a vector $\mathbf{W}$ is added to the vector H, with

$$(13)\quad W_n = \alpha_n\zeta_n\,(\omega_{n+1}-\omega_n) - \frac{\alpha_n}{4}(\omega_{n+1}+\omega_n)(\zeta_{n+1}-\zeta_{n-1}) - \\ -\frac{\alpha_n}{4}\,\nabla(\psi_{n+1}+\psi_n)\cdot\nabla(\psi_{n+1}-\psi_{n-1}) - \nabla\chi_n\cdot\nabla(\zeta_n+f)$$

Note that $\zeta_o = \nabla\psi_o = 0$.

χ_n is solved from

$$(14)\quad \nabla^2\chi_n = \alpha_n(\omega_{n+1}-\omega_n)$$

Due to the lower boundary condition, the streamfunction at the level N+1 can be solved directly from the thermodynamic equation

$$(15)\quad \frac{\partial\psi_{N+1}}{\partial t} = \frac{\partial\psi_N}{\partial t} + \tilde{\mathcal{J}}_{N+1} - \frac{\delta_{N+1}}{f_o\beta_{N+1}}\,Q_{N+1} - \frac{1}{f_o\beta_{N+1}}\,\omega_{N+1}$$

2.2 Finite differences

The equations are mapped on a polar-stereographic map with a map scale factor m. A rectangular cartesian grid with a grid length, d, is applied on the map and all derivatives are approximated by centered differences. The following finite difference operators are introduced:

$$\nabla^2 a = a_{i+1,j}+a_{i-1,j}+a_{i,j+1}+a_{i,j-1}-4a_{i,j}$$

$$\mathcal{J}(a,b) = (a_{i+1,j}-a_{i-1,j})(b_{i,j+1}-b_{i,j-1}) - \\ - (a_{i,j+1}-a_{i,j-1})(b_{i+1,j}-b_{i-1,j})$$

$$\nabla a\cdot\nabla b = (a_{i+1,j}-a_{i-1,j})(b_{i+1,j}-b_{i-1,j}) + \\ + (a_{i,j+1}-a_{i,j-1})(b_{i,j+1}-b_{i,j-1})$$

$$\Delta_\tau a = a^{\tau+1}-a^{\tau-1}$$

where i and j are grid point indices and τ is an index for time.

[illegible] is computed [illegible]. To the [illegible] W is added to the [illegible]

$$[illegible]$$

(13)

$$[illegible]$$

Note that [illegible]

χ_n is solved from

$$(14) \quad \nabla^2 \chi_n = [illegible]$$

Due to the lower boundary condition, the stream-function at the level N+1 can be solved directly from the thermodynamic equation

$$(15) \quad [illegible]$$

4.2 Finite differences

The equations are mapped on a polar-stereographic map with a map scale factor m. A rectangular cartesian grid with a grid length, d, is applied on the map and all derivatives are approximated by centered differences. The following finite difference operators are introduced:

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

where [illegible]

If the length of a time step is Δt and $\mu = (m/d)^2$ we can write

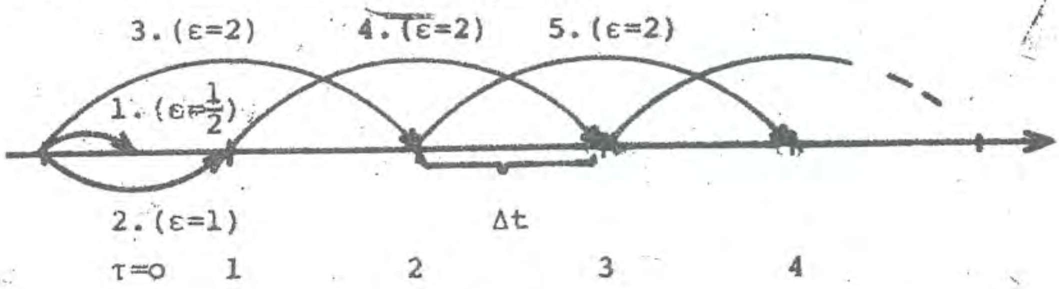
$$\nabla^2 \approx \mu\boldsymbol{\nabla}^2$$

$$J \approx \frac{\mu}{4}\mathbf{J}$$

$$\nabla a\cdot\nabla b \approx \frac{\mu}{4}\boldsymbol{\nabla}a\cdot\boldsymbol{\nabla}b$$

$$\frac{\partial}{\partial t} \approx \frac{\Delta_\tau}{\varepsilon\Delta t}$$

For a normal time step $\varepsilon=2$. For the first time step a non-centered time step is needed. The arrangement is shown in the following figure.



By introducing the finite difference operators in (11), (12), (13), (14) and (15) we get the following set of algebraic equations.

$$\boldsymbol{\nabla}^2(\Delta_\tau\boldsymbol{\psi}) - \frac{ff_o}{\mu}\mathbf{A}\cdot(\Delta_\tau\psi) = \frac{\varepsilon\Delta t}{4}\Big[\mathbf{J} + ff_o\mathbf{B}\cdot\tilde{\mathbf{J}} - \frac{4f}{\mu}\mathbf{C}\cdot\mathbf{Q} + \frac{4f\alpha_n}{\mu}\boldsymbol{\omega}_L\Big] \equiv \mathbf{H} \tag{16}$$

$$\boldsymbol{\omega} = \frac{f_o\mu}{4}\mathbf{D}\cdot\mathbf{J} + \frac{f_o}{\varepsilon\Delta t}\mathbf{E}\cdot(\Delta_\tau\boldsymbol{\psi}) - \mathbf{F}\cdot\mathbf{Q} \tag{17}$$

$$\Delta H_n = \frac{\varepsilon\Delta t}{\mu}\Big[\alpha_n\zeta_n\ \omega_{n+1}-\omega_n) - \frac{\alpha_n}{4}(\omega_{n+1}-\omega_n)(\zeta_{n+1}-\zeta_{n-1}) - \frac{\mu\alpha_n}{16}\boldsymbol{\nabla}(\omega_{n+1}+\omega_n)\cdot\boldsymbol{\nabla}(\psi_{n+1}-\psi_{n-1}) - \frac{\mu}{4}\boldsymbol{\nabla}\chi_n\cdot\boldsymbol{\nabla}(\zeta_n+f)\Big] \tag{18}$$

$$\boldsymbol{\nabla}^2\chi_n = -\frac{\alpha_n}{\mu}(\omega_{n+1}-\omega_n) \tag{19}$$

$$\Delta_\tau\psi_{N+1} = \Delta_\tau\psi_N + \varepsilon\Delta t\left[\frac{\mu}{4}\tilde{\mathfrak{J}}_{N+1} - \frac{\gamma_{N+1}}{f_o\beta_{N+1}}Q_{N+1} - \right.$$

$$(20) \qquad \left. - \frac{1}{f_o\beta_{N+1}}\omega_{N+1}\right] \equiv \Delta_\tau\psi_N + H_{N+1}$$

2.3 Uncoupling of the system

The equations expressed by the vector equation (16) are coupled together by the matrix A. As it stands, the system can not be solved for $\Delta_\tau\psi$ by solving each $\Delta_\tau\psi_n$ separately. However, by a matrix transformation the matrix A can be transformed to diagonal form. Denoting the transformation matrix by T and its invers by T^{-1} we can write

$$\mathbf{T}^{-1}\cdot\mathbf{A}\cdot\mathbf{T} = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_N \end{bmatrix}$$

or

$$(21) \qquad \mathbf{A} = \mathbf{T}\cdot \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_N \end{bmatrix} \cdot \mathbf{T}^{-1}$$

It can be shown that λ_i (i=1, 2...N) are the eigenvalues to the matrix $\mathbf{A}$ and the columns of $\mathbf{T}$ are the corresponding eigenvectors.

By introducing (21) in the system (16) and multiplying from the left by $\mathbf{T}^{-1}$ we get

$$(22) \qquad \nabla^2(\Delta_\tau\psi^*) - \frac{ff_o}{\mu}\begin{vmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_N \end{vmatrix}\cdot(\Delta_\tau\psi^*) = \mathbf{H}^*$$

where the star (*) indicates new variables achieved by the transformation

(23) $\boldsymbol{\psi}^* = \mathbf{T}^{-1}\cdot\boldsymbol{\psi}$ and $\mathbf{H}^* = \mathbf{T}^{-1}\cdot\mathbf{H}$

The new variables consist of linear combinations of the old variables and can not be assigned to specific levels.

The system (22) is now uncoupled and can be solved separately for each component $\Delta_\tau\psi_n{}^*$, (n=1, 2,...N). $\Delta_\tau\boldsymbol{\psi}$ can be obtained by the backward transformation

(24) $\boldsymbol{\psi} = \mathbf{T}\cdot\boldsymbol{\psi}^*$

2.4 Method of solution

Each Helmholtz or Poisson equation, e.g. equations (16) and (19) and the finite difference form of equation (5), is solved by a standard method of successive overrelaxation. If such an equation is formally written as

(25) $\nabla^2 a - q\cdot a = b$

where a, b and q are functions of i and j, one interation step $(\nu\rightarrow\nu+1)$ can be written as

$$a_{i,j}^{(\nu+1)} = a_{i,j}^{(\nu)} + \frac{\alpha}{4+q_{i,j}}\Big[a_{i+1,j}^{(\nu)} + a_{i-1,j}^{(\nu+1)} + \tag{26}$$

$$+ a_{i,j+1}^{(\nu)} + a_{i,j-1}^{(\nu+1)} - (4+q_{i,j}^{(\nu)}) - a_{i,j}^{(\nu)} - b_{i,j}\Big]$$

α is the overrelaxation coefficient, mainly depending upon the size of the grid area and the magnitude of q. The sequence of iterations is truncated when the value of the paranthesis is less than a prescribed tolerance. The first guess, $a_{i,j}^{o}$, is in most cases evaluated from the foregoing timestep.

The sequence of computations for one time step can be described in the following way:

Compute the forcing functions $\mathbf{H}^{\tau}$

Transform to $(\mathbf{H}^{*})^{\tau}$

Solve for $\Delta_{\tau}\boldsymbol{\psi}^{*}$

Transform to $\Delta_{\tau}\boldsymbol{\psi}$

Step forward: $\boldsymbol{\psi}^{\tau+1} = \boldsymbol{\psi}^{\tau-1} + \Delta_{\tau}\boldsymbol{\psi}$

Compute $\Delta_{\tau}\psi_{N+1}$

Step forward: $\psi_{N+1}^{\tau+1} = \psi_{N+1}^{\tau-1} + \Delta_{\tau}\psi_{N+1}$

Solve $\boldsymbol{\omega}^{\tau}$

One time step with W-terms neglected

If W-terms neglected

Solve χ_{n}^{τ}

Compute $\Delta\mathbf{H}^{\tau}$

Transform to $(\Delta\mathbf{H}^{*})^{\tau}$

Solve for $\Delta_{\tau}\boldsymbol{\psi}^{*}_{new} - \Delta_{\tau}\boldsymbol{\psi}^{*}$

Transform to $\Delta^{\tau}\boldsymbol{\psi}_{new} - \Delta_{\tau}\boldsymbol{\psi}$

Correction $\boldsymbol{\psi}^{\tau+1}_{new} = \boldsymbol{\psi}^{\tau+1} + \Delta_{\tau}\boldsymbol{\psi}_{new} - \Delta_{\tau}\boldsymbol{\psi}$

Correction $(\psi_{N+1}^{\tau+1})_{new} = \psi_{N+1}^{\tau+1} + (\Delta_{\tau}\psi_{N})_{new} - \Delta_{\tau}\psi_{N}$

Corrections to the solution for the effect of W-terms

Add 1 to τ

Next time step

3. Friction and topography

In the basic system (equations 16, 18, 19 and 20) ω_{N+1} at the lowest level is left undefined and has to be prescribed each time step. ω_{N+1} is assumed to be controlled by two effects: vertical motion resulting from the friction of the flow against the surface and the vertical motion forced by mountain obstacles.

It is assumed formally in the model that the surface of the earth coincides with the pressure level p_{N+1} (1000 mb). A more realistic assumption would lead to coefficients α_n as functions of x and y and the transformation of the system into diagonal form would be impossible.

The effect of a surface not coinciding with the level p_{N+1} can, however, be simulated by introducing it as extra forcing functions in the right hand sides of the equations.

The vorticity equation (10) for the levels N, N-1 and N-2 can be written as

$$\begin{cases} \frac{\partial \zeta_{N-2}}{\partial t} - f\alpha_{n-2}(\omega_{N-1} - \omega_{N-2}) = J_{N-2} & (27) \\ \frac{\partial \zeta_{N-1}}{\partial t} - f\alpha_{N-1}(\omega_N - \omega_{N-1}) = J_{N-1} & (28) \\ \frac{\partial \zeta_N}{\partial t} - f\omega_N \; (\quad - \omega_N) = J_N + f\alpha_N\omega_{N+1} & (29) \end{cases}$$

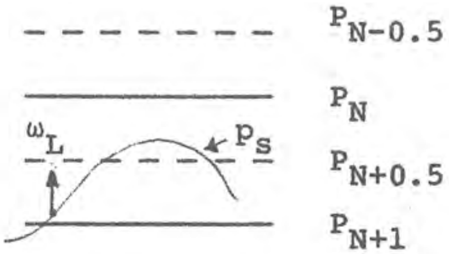
We restrict the mountains to be lower than the level (N-2). We assume that the forced vertical velocity, ω_L, at the real surface of the earth can be expressed as

$$\omega_L(p_s) = \mathbf{V}(p_s)\cdot\nabla p_s - k\zeta(p_s) \quad (30)$$

where p_s is the standard surface pressure and k is a constant. The first term describes the main orographic influence and the second term the effect of friction. A value for k can be estimated from the Ekman theory.

We consider three cases:

a) $\boxed{p_s > p_N}$



Equation (29) should in this case be written as

$$\frac{\partial \zeta_N}{\partial t} - f\alpha_N(1-\delta_1)(\omega_L-\omega_N) = J_N$$

where

$$\delta_1 = \frac{p_s-p_{N+0.5}}{p_s-p_{N-0.5}}$$

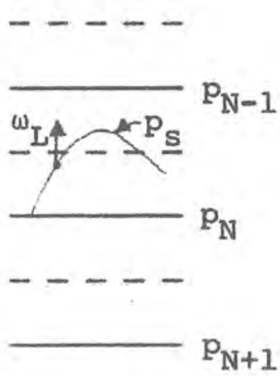
or, after rearrangement

$$\frac{\partial \zeta_N}{\partial t} - f\alpha_N(\ -\omega_N) = J_N + f\alpha_N\left[(1-\delta_1)\omega_L + \delta_1\omega_N\right]$$

Comparing with equations (29) and (30) we see that ω_{N+1} at the time τ can be approximated by

$$(31)\quad \begin{cases} \omega^\tau_{N+1} = (1-\delta_1)\omega^\tau_L + \delta_1\omega^{\tau-1}_N \text{ and} \\ \omega^\tau_L = (a_1\mathbf{v}^\tau_N + a_2\mathbf{v}^\tau_{N+1})\cdot\nabla p_s - k(a_1\zeta_N + a_2\zeta_{N+1}) \\ \begin{cases} a_1 = \dfrac{p_{N+1}-p_s}{p_{N+1}-p_N} \\ a_2 = \dfrac{p_s-p_N}{p_{N+1}-p_N} \end{cases} \end{cases}$$

b) $\boxed{p_N \geqslant p_s > p_{N-1}}$



$$\delta_2 = \frac{p_s-p_{N-0.5}}{p_s-p_{N-1.5}}$$

Comparing with equations (29)

(31)

Equation (28) should read

$$\frac{\partial \zeta_{N-1}}{\partial t} - f\alpha_{N-1}(1-\delta_2)(\omega_L-\omega_{N-1}) = J_{N-1} \quad \text{or}$$

$$\frac{\partial \zeta_{N-1}}{\partial t} - f\alpha_{N-1}(\omega_N-\omega_{N-1}) = J_{N-1} + f\alpha_{N-1}\Big[(1-\delta_2)\omega_L + $$

$$+ \delta_2\omega_{N-1} - \omega_N\Big]$$

In this case a forcing term must be added to equation (28)

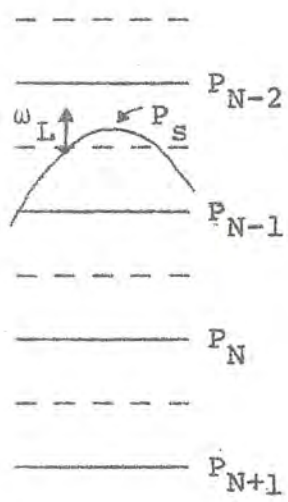
$$(32)\quad \begin{cases} \Delta H_{N-1}^{\tau} = f\alpha_{N-1}\ (1-\delta_2)\omega_L^{\tau} + \delta_2\omega_{N-1}^{\tau-1} - \omega_N^{\tau-1} \\ \omega_L^{\tau} = (a_1\mathbf{V}_{N-1}^{\tau} + a_2\mathbf{V}_N)\cdot\nabla p_s - k(a_1\zeta_{N-1}^{\tau} + a_2\zeta_N^{\tau}) \\ \begin{cases} a_1 = \dfrac{p_N-p_s}{p_N-p_{N-1}} \\ a_2 = \dfrac{p_s-p_{N-1}}{p_N-p_{N-1}} \end{cases} \end{cases}$$

We further assume that "inside" the mountain there is no vertical variations in ω, which is approximately fullfilled if

$$(33)\quad \omega_{N+1}^{\tau} = \omega_N^{\tau-1}$$

c) $\boxed{p_{N-1} \geqslant p_s > p_{N-2}}$

$$\delta_3 = \frac{p_s-p_{N-1.5}}{p_s-p_{N-2.5}}$$



In a similar way we find the extra forcing function to equation (27)

$$
(34)\quad \begin{cases}
\Delta H_{N-2}^{\tau} = f\alpha_{N-2}\left[(1-\delta_3)\omega_L^{\tau} + \delta_3\omega_{N-2}^{\tau-1} - \omega_{N-1}^{\tau-1}\right] \\
\omega_L^{\tau} = (a_1\mathbf{v}_{N-2}^{\tau} + a_2\mathbf{v}_{N-1}^{\tau})\cdot\nabla p_s - k(a_1\zeta_{N-2}^{\tau} + a_2\zeta_{N-1}^{\tau}) \\
\qquad \begin{cases}
a_1 = \dfrac{P_{N-1}-P_s}{P_{N-1}-P_{N-2}} \\
a_2 = \dfrac{P_s-P_{N-2}}{P_{N-1}-P_{N-2}}
\end{cases}
\end{cases}
$$

If we assume no vertical variation of ω "inside" the mountain, the following approximations can be made

$$
(35)\quad \begin{cases}
\Delta H_{N-1}^{\tau} = f\alpha_{N-1}(\omega_{N-1}^{\tau-1}-\omega_N^{\tau-1}) \\
\omega_{N+1}^{\tau} = \omega_N^{\tau-1}
\end{cases}
$$

With these relations for ω_{N+1} and the extra forcing functions ΔH_{N-1} and ΔH_{N-2} for the three different cases, the vorticity equations (27), (28) and (29) can be left untouched and the equations can formally be treated in the same way as described in section 2.

4. Heating from oceans

The adiabatic models provide the major characteristics of middle latitude disturbances. However, the effects of diabatic heating are by no means insignificant. Differences of amplitude and phase velocity of 10-20% are possible during a day or two as a result of sensible heat exchange or latent heat release.

The most effective source of sensible heat is the ocean area. The turbulent flux of heat per unit area from the ocean to the atmosphere through the boundary layer can be estimated by the formula

$$
(36)\quad F_Q = \rho_o\, k_E \left|\mathbf{v}_o\right| c_p(T_s-T_o)
$$

where k_E is the exchange coefficient and ρ_o is the density at 1000 mb.

[illegible] function [illegible]

[illegible]

(34)

[illegible]

If we assume [illegible] variation of [illegible] "inside" the [illegible] the following approximations can be made

[illegible]

(35)

[illegible]

which [illegible] of [illegible] the [illegible] functions [illegible] these [illegible]

[illegible] (27), (28) and [illegible] can be [illegible] are normally [illegible] in section [illegible]

[illegible]

[illegible]

In conditions of near-neutral stability $k_E = 1.25 \cdot 10^{-3}$ is fairly widely accepted value if $(T_s - T_o) \geqslant 0$. (Robinson, 1966).

The heat supplied to the atmosphere in this way is transported upwards by turbulent or convective motion. Since these processes are not described by the model, we have to assume a vertical distribution of the rate of heating per unit mass, $Q(p)$. The relation between F_Q and $Q(p)$ is

$$(37) \quad F_Q = \frac{1}{g} \int_0^{p_o} Q(p)dp$$

If we put

$$(38) \quad Q(p) = A_1 \left| \mathbf{V}_o \right| (T_s - T_o) \left(\frac{p - p_{50}}{p_o - p_{50}}\right); \quad \begin{matrix} T_s > T_o \\ p \geqslant p_{50} \end{matrix}$$

the magnitude of the coefficient A_1 can be estimated by a vertical integration of the formula between p_{50} and p_o and a comparison with equation (36).

With standard values for ρ_o, k_E, c_p and g

$$A_1 \approx 0.6 \cdot 10^{-3}$$

The modifications of formula (38) will be introduced. A coefficient A_2 will be used to guarantee a heat exchange even if the wind is zero.Similar to Benwell et alii (1971) we will also try to simulate the increase of the exchange coefficient k_E for unstable conditions by assuming that

$$k_E = 1.25 \cdot 10^{-3} \left[1 + A_3 (T_s - T_o)\right]$$

The final form of Q(p) will then be

$$(39)\quad \begin{cases} Q(p) = A_1\left[|\mathbf{V}_o| + A_2\right](T_s - T_o)\left[1 + A_3\cdot(T_s - T_o)\right]\left(\dfrac{p-p_{50}}{p_o-p_{50}}\right) \\ \qquad \begin{cases} T_s > T_o \\ p > p_{50} \end{cases} \\ Q(p) = 0 \quad \text{over land areas and} \\ \qquad\qquad \text{over ocean areas if } T_s \leqslant T_o \end{cases}$$

$$\begin{cases} A_1 = 0.6\cdot 10^{-3} \\ A_2 = 10 \\ A_3 = 0.8\cdot 10^{-1} \end{cases}$$

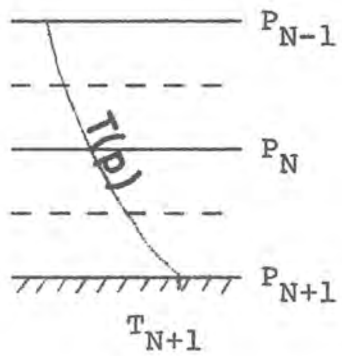
The wind speed $|\mathbf{V}_o|$ at the ocean surface is computed from ψ_{N+1} and the ocean surface temperature, T_s, is taken from monthly mean values.

The most difficult parameter to estimate in equation (39) is the air temperature at the ocean surface, T_o, since it must be extrapolated from stream functions on the levels N-1, N and N+1.

Assume that the vertical variation of the temperature in the two lowest layers can be described by

$$(40)\quad T = a + b \ln p$$

The hydrostatic equation can then be integrated in the two layers and the coefficients a and b evaluated.



Putting $p = p_{N+1}$ in equation (40) we get

$$T_{N+1} = a_{N+1}\, z_{N+1} + a_N\, z_N + a_{N-1}\, z_{N-1} = z^*$$

where

$$\begin{cases} a_{N+1} = -\frac{g}{R}\left[\frac{1}{\ln(\frac{p_{N+1}}{p_{N-1}})} + \frac{1}{\ln(\frac{p_{N+1}}{p_N})}\right] \\ a_{N-1} = -\frac{g}{R}\,\frac{\ln(\frac{p_{N+1}}{p_N})}{\ln(\frac{p_N}{p_{N-1}})\cdot\ln(\frac{p_{N+1}}{p_{N-1}})} \\ a_N = -\,(a_{N+1} + a_{N-1}) \end{cases}$$

The predicted parameters are ψ_{N+1}, ψ_N and ψ_{N-1} and we use the balance equation

$$(42)\quad \nabla^2 z_i = \frac{1}{g}\nabla\cdot(f\nabla\psi_i) \quad (i = N+1,\ N,\ N-1)$$

We finally get

$$(43)\quad \nabla^2 T_{N+1} = \frac{1}{g}\nabla\cdot(f\nabla\psi^*)$$

where

$$\psi^* = a_{N+1}\,\psi_{N+1} + a_N\psi_N + a_{N-1}\,\psi_{N-1}$$

5. <u>Release of latent heat</u>

5.1 <u>Humidity forecast</u>

The change of the specific humidity, q, in the atmosphere can be described by the following equation

$$(44)\quad \frac{\partial q}{\partial t} = -\,\mathbf{V}\cdot\nabla q - \omega\frac{\partial q}{\partial p} + A\nabla^2 q + E - P$$

where A is a dissipation coefficient
E is the addition of humidity due to evaporation
P is the loss of humidity due to precipitation

For short range predictions evaporation is of minor importance and will be neglected.

When no condensation occurs, $P = 0$, and $q < q_s$, where q_s is the specific humidity at saturation over a water surface. Equation (44) then takes the following form

$$(45) \quad \frac{\partial q}{\partial t} = - \mathbb{V}\cdot\nabla q - \omega\frac{\partial q}{\partial p} + A\nabla^2 q; \quad q < q_s$$

In the case of condensation $q = q_s$. The condensation continues as long as q_s decreases. The condensation phase can be written as

$$(46) \quad \frac{d(q-q_s)}{dt} = A\nabla^2 q; \quad q = q_s; \quad \frac{dq_s}{dt} < 0$$

where

$$\frac{d(\ )}{dt} = \frac{\partial(\ )}{\partial t} + V\cdot\nabla(\ ) + \omega\frac{\partial(\ )}{\partial p}$$

By comparing eq. (44) and (46) we see that the loss of humidity due to precipitation is

$$(47) \quad P = - \frac{dq_s}{dt}$$

The equations (45) and (46) can be combined to a single equation

$$(48) \quad \frac{\partial q}{\partial t} = - \mathbb{V}\cdot\nabla q - \omega\frac{\partial q}{\partial p} + A\nabla^2 q + \delta\cdot\frac{dq_s}{dt}$$

$$\delta = 1 \text{ when } q = q_s \text{ and } \frac{dq_s}{dt} < 0$$

$$\delta = 0 \text{ otherwise}$$

It is found that, when applied to discrete levels in a prediction model, this equation is too slow to predict the onset of condensation. Condensation starts before $q = q_s$ is reached, possibly because of the difference between saturation over water and ice surfaces and also due to the fact that the clouds often are concentrated to specific layers, with dryer layers between, in which case the mean humidity for a certain layer can be well below q_s, when condensation occurs.

We will thus use the criterion

$$(49) \quad q < \varepsilon\cdot q_s \quad (\varepsilon<1)$$

Empirically ε is found to be of the order 0.8.

The last term in equation (48) will be removed and the effect of precipitation on q will be satisfied by the use of criterion (49) at each time step. The prognostic equation for humidity thus reads after introduction of the streamfunction and velocity potential

$$(50)\quad \begin{cases} \dfrac{\partial q}{\partial t} = - J(\psi, q) - \nabla\chi \cdot \nabla q - \dfrac{\partial q}{\partial p} + A\nabla^2 q \\ q < \varepsilon q_s \end{cases}$$

Values for $\psi(x,y,p,t)$, $\chi(x,y,p,t)$ and $\omega(x,y,p,t)$ are taken from the prediction model described in section 2. As upper and lower boundary conditions for q will be used

$$\begin{cases} q = 0 \text{ above some level } p_{K-1} \text{ if } \omega_K > 0 \\ \dfrac{\partial q}{\partial p} = \dfrac{\partial q_s}{\partial p} \text{ at the level } p_{N+1} \text{ if } \omega_{N+1} < 0 \end{cases}$$

These two assumptions take care of the vertical inflow from above and from below into the "wet part" of the atmosphere. In the case of outflow ($\omega_K < 0$ and $\omega_{N+1} > 0$), $\frac{\partial q}{\partial p}$ is approximated with non-centered finite differences inside the "wet part".

5.2 Precipitation

The total rate of precipitation can be expressed as

$$(51)\quad \overline{P} = - \frac{1}{g} \int_p \delta \frac{dq_s}{dt} \cdot dp$$

where

$$\begin{cases} \delta = 1 \text{ when } q = \varepsilon q_s \text{ and } \dfrac{dq_s}{dt} < 0 \\ \delta = 0 \text{ otherwise} \end{cases}$$

An expression for $\frac{dq_s}{dt}$ will now be derived.

Defining q_s as

$$(52) \quad q_s = \frac{0.622 \cdot E}{p}$$

where E is the water vapor pressure at saturation, we can differentiate

$$(53) \quad \frac{1}{q_s} \frac{dq_s}{dt} = \frac{1}{E} \frac{dE}{dt} - \frac{\omega}{p}$$

E is eliminated from (53) by the use of Clapeyron's equation

$$(54) \quad \frac{dE}{E} = \frac{LdT}{R_V \cdot T^2}$$

Thus

$$(55) \quad \frac{1}{q_s} \frac{dq_s}{dt} = \frac{L}{R_V T} \frac{dT}{dt} - \frac{\omega}{p}$$

Assuming that the condensation takes place as a wet-adiabatic lifting process, the first law of thermodynamics can be used in the following form

$$(56) \quad - L \frac{dq_s}{dt} = C_p \frac{dT}{dt} - \frac{RT}{p} \omega ; \quad \omega < 0$$

$\frac{dT}{dt}$ can be eliminated from (55) and (56) and thus

$$(57) \begin{cases} \dfrac{dq_s}{dt} = F^{*}(p,T) \\ F^{*}(p,T) = \dfrac{q_s T}{p} \left[\dfrac{LR - C_p R_V T}{C_p R_V T^2 + q_s L^2} \right] \end{cases}$$

During condensation $\frac{dq_s}{dt} < 0$ and $\omega < 0$, so that $F^{*} > 0$, or

$$T < \frac{LR}{C_p R_V} \approx 1275^{o}K$$

This condition is always satisfied in the atmosphere.

It follows that $\omega < 0$ can replace $\frac{dq_s}{dt} < 0$ in equation (48).

The total rate of precipitation can thus be computed from

$$
(58)\quad \begin{cases}
\overline{P} = -\frac{1}{g}\int\limits_{p} \delta\omega F^{*} dp \\
\begin{cases} \delta = 1 \text{ when } q = q_s \text{ and } \omega < 0 \\ \delta = 0 \text{ otherwise} \end{cases} \\
F^{*}(p,T) = \frac{q_s T}{p}\left[\frac{LR - c_p R_v T}{C_p R_v T^2 + q_s L^2}\right] \\
q_s = \frac{0.622\, E_o}{p}\, e^{\frac{0.622\, L_o}{R_d}\left(\frac{1}{T_o} - \frac{1}{T}\right)}
\end{cases}
$$

5.3 Heating due to condensation

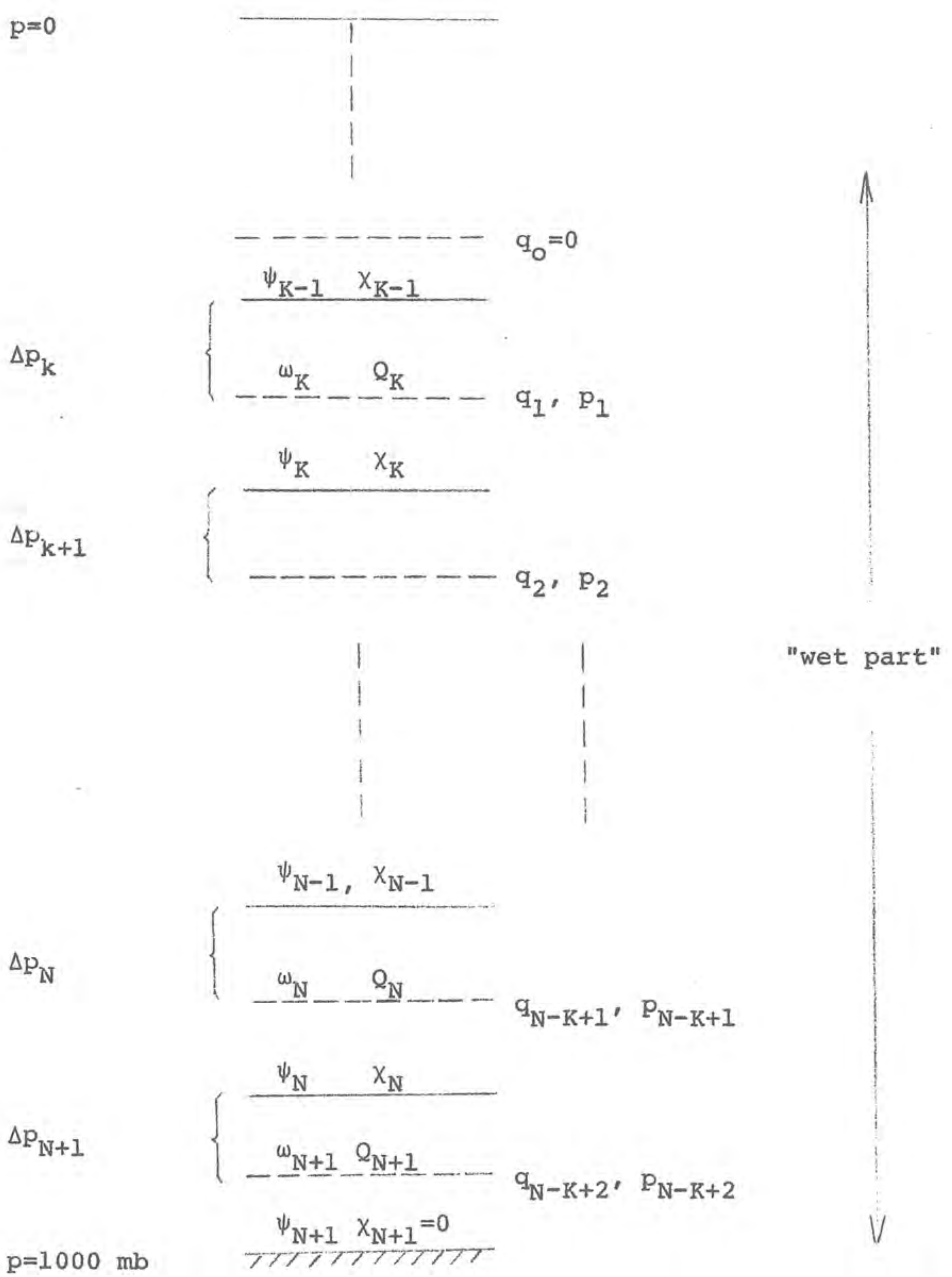
The non-adiabatic heating due to condensation

$$(59)\quad Q_n = -\delta L\, \omega_n\, F_n^{*}$$

with the same criterion for δ as in (58) is introduced in the vector $\mathbb{Q}$ in equations (11) and (12) and in Q_{N+1} in equation (15). Above a certain pressure level p_{K-1} and $Q_n = 0$.

5.4 Finite difference form

The vertical structure of the model is illustrated in the following figure.


p=0
$q_o=0$
ψ_{K-1} χ_{K-1}
Δp_k
ω_K Q_K
q_1, p_1
ψ_K χ_K
Δp_{k+1}
q_2, p_2
"wet part"
ψ_{N-1}, χ_{N-1}
Δp_N
ω_N Q_N
q_{N-K+1}, p_{N-K+1}
ψ_N χ_N
Δp_{N+1}
ω_{N+1} Q_{N+1}
q_{N-K+2}, p_{N-K+2}
ψ_{N+1} $\chi_{N+1}=0$
p=1000 mb

Horizontal and vertical finite differences are now introduced in equation (50):

$$
\begin{aligned}
q_j^{\tau+1} &= q_j^{\tau-1} + \varepsilon \Delta t \left[- \frac{\mu}{8} \left\{ J(\psi_n + \psi_{n-1}; q_j) + \right. \right. \\
(60) \qquad & \left. + \nabla(\chi_n^\tau + \chi_{n-1}^\tau) \cdot \nabla q_j^\tau - 8A\nabla^2 q_j^\tau \right\} - \\
& \left. - \alpha_n^1 \cdot \omega_n^\tau (q_{j+1}^\tau - q_{j-1}^\tau) \right]
\end{aligned}
$$

where

$$
\begin{cases}
j = n - K + 1 \\
\alpha_n^1 = \dfrac{\alpha_n \cdot \alpha_{n-1}}{\alpha_n + \alpha_{n-1}}
\end{cases}
$$

$(F^*)_n^\tau$ and $(q_s)_j^\tau$ can be computed from

$$
(61) \begin{cases}
(F^*)_n^\tau = \dfrac{(q_s)_j^\tau \cdot T_n^\tau}{p_j} \left[\dfrac{LR - C_p R_v T_n^\tau}{C_p R_v (T_n^\tau)^2 + (q_s)_j^\tau \cdot L^2} \right] \\
(q_s)_j^\tau = \dfrac{0.622\ E_o}{p_j}\, e^{\dfrac{0.622\ L_o}{R_d} \left(\dfrac{1}{T_o} - \dfrac{1}{T_n^\tau}\right)} \\
T_n^\tau = - \dfrac{f}{R} \cdot \dfrac{p_j}{2\Delta p_n} (\psi_n^\tau - \psi_{n-1}^\tau)
\end{cases}
$$

To reduce the number of arithmetic operations F^* and q_s in (61) can be tabulated for a number of (p_n, T_n)-combinations.

Upper boundary conditions:

a) $\underline{\omega_K^\tau < 0}$

In eq. (60) $\alpha_n^1 (q_{j+1}^\tau - q_{j-1}^\tau)$ is approximated by $\alpha_K (q_2^\tau - q_1^\tau)$

b) $\underline{\omega_K^\tau > 0}$

$q_o^\tau = 0$ is used in eq. (60).

Lower boundary conditions:

$$\chi^{\tau}_{N+1} = 0 \text{ and}$$

a) $\underline{\omega^{\tau}_{N+1} < 0}$

$\frac{\partial q}{\partial p}$ is approximated by $\frac{\partial q_s}{\partial p} = - \frac{q_s}{p}$

$\alpha^{1}_{n}(q^{\tau}_{j+1} - q^{\tau}_{j-1})$ is replaced by $- \frac{(q_s)^{\tau}_{N-K+2}}{p_{N-K+2}}$

b) $\underline{\omega^{\tau}_{N+1} > 0}$

In eq. (60) $\alpha^{1}_{n}(q^{\tau}_{j+1} - q^{\tau}_{j-1})$ is approximated by $\alpha_N(q_{N-K+2} - q_{N-K+1})$

The non-adiabatic heating is computed from equation (59) and the total rate of precipitation can be expressed as

$$(62) \quad \overline{P}^{\tau} = \frac{1}{gL} \int_{p} Q^{\tau} dp = \frac{2}{gL} \sum_{n=K}^{N+1} \left[Q^{\tau}_{n} \cdot \Delta p_n \right]$$

The accumulated precipitation during one time step will then be

$$(63) \quad \Delta_{\tau}\overline{P} = \frac{2\Delta t}{gL} \sum_{n=K}^{N+1} \left[Q^{\tau}_{n} \cdot \Delta p_n \right]$$

The accumulated precipitation during a certain time period is achieved by adding all $\Delta_{\tau}\overline{P}$ during the period. In the case of a non-centered start of the forecast, the second value, $\Delta_{1/2}\overline{P}$, is neglected.

6. Smoothing and filtering

In the real atmosphere there is a flow towards the shortest wavelength part of the energy spectrum. Since wavelengths shorter than $2\Delta s$ cannot be represented in a finite-difference model, it is necessary to remove energy accumulated in the shortest waves in order to avoid a spurious growth of short waves, which in other case could blow up and obscure a meteorological forecast.

Numerical operators which filter two-gridlength "noise" and only have small damping of longer wavelengths have been discussed by Shapiro (1970). By a repeated application of a smoothing operator with a certain set of involved constants, one can achieve an ideal low-pass filter which does not change waves longer than $2\Delta s$.

However, if the smoothing is applied only a small number of times during a forecast integration, of the order 10, a two-step smoothing is sufficient. Two-gridlength waves are then removed and longer waves are slightly damped at specified forecast lengths, say each sixth hour.

A more frequent use of filtering, i.e. each time step, requires at least a 4-step version of Shapiro's method.

The smoothing operator is defined as

$$\begin{aligned}\overline{a}_{ij} &= a_{ij} + \frac{S}{2}(1-S)\nabla^2 a_{ij} + \\ &+ \frac{S^2}{4}(a_{i-1,j-1} + a_{i-j,j+1} + a_{i+1,j-1} + \\ &+ a_{i+1,j+1} - 4a_{ij})\end{aligned}$$

and is applied first with $S = \frac{1}{2}$ and then with $S = -\frac{1}{2}$. The resulting response function for harmonic waves, defined as

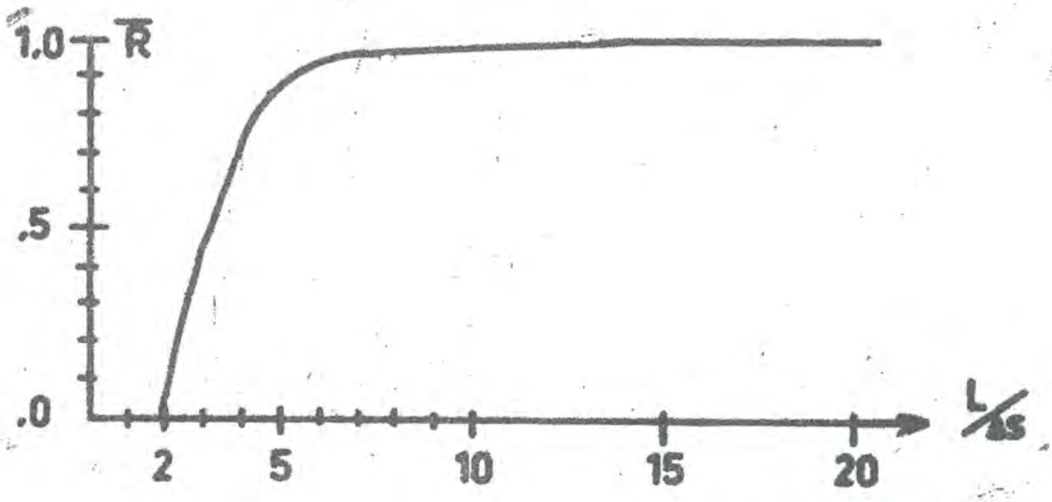
$$\overline{a} = \overline{R} \cdot a$$

has the following form

$$\overline{R}(k,\ell) = \left[1-\sin^4\left(\frac{k\Delta s}{2}\right)\right]\cdot\left[1-\sin^4\left(\frac{\ell\Delta s}{2}\right)\right] \tag{65}$$

where k and ℓ are wavenumbers in the x- and y-directions respectively.

The response function for $\ell=0$ (or $k=0$) is shown in the following figure.



7. The criterion of ellipticity

The forecast equations are of elliptic type, which implies that the relative vorticity is restricted for negative values. Since there is no guarantee that the ellipticity criterion is fullfilled for all grid-points in the initial fields or that non-elliptic points are produced during the integration due to the approximations done, an iterative procedure is used to modify a stream-function field so that the ellipticity criterion

$$(66)\quad \nabla^2\psi + \frac{f}{2} > 0$$

is valid in all points of the field.

Each point is tested with the following formula:

$$(67)\quad \mu\nabla^2\psi + \frac{f}{2} - \varepsilon_o = \delta$$

where

$$\nabla^2\psi = \psi_{i+1,j}+\psi_{i-1,j}+\psi_{i,j+1}+\psi_{i,j-1}-4\cdot\psi_{i,j}$$

$$\varepsilon_o = 0.001\cdot f.$$

If $\delta < 0$, $\psi_{i,j}$ is modified by

$$\psi_{i,j} = \psi_{i,j} - \frac{\varepsilon-\delta}{2\mu} \cdot k$$

where k is a convergence parameter.

This means that the vorticity increases by $2(\varepsilon_o-\delta)\ k$ in the point (i,j) and decreases $k \cdot 0.5\ (\varepsilon_o-\delta)$ in the four surrounding points (i+1,j), (i-1,j), (i,j+1) and (i,j-1). This procedure guarantees that δ becomes positive in the point (i,j), but not necessarily in the surrounding points. The test must therefore be repeated for all points until the criterion is valid in the whole field. With a suitable choice of k the method is convergent.

k can be found empirically and is estimated to be of the order k = 0.85.

8. Lateral boundary conditions

A simple lateral boundary condition, which has been used in the operational three-parameter balanced model at SMHI (see Bengtsson and Moen, 1971), is the prescription of the variation in time of the streamfunctions, $\Delta\psi$, and vorticities, ζ, at all boundary points. A further simplification is to compute the vorticities at the boundary from the streamfunctions by use of non-centered finite-differences instead of prescribing them. Since the purpose is to take care of only the large-scale fluctuations at the boundaries this procedure seems to be satisfactory if some smoothing is used near the boundaries in order to damp out small-scale errors.

A forecast equation at an arbitrary level can formally be written as

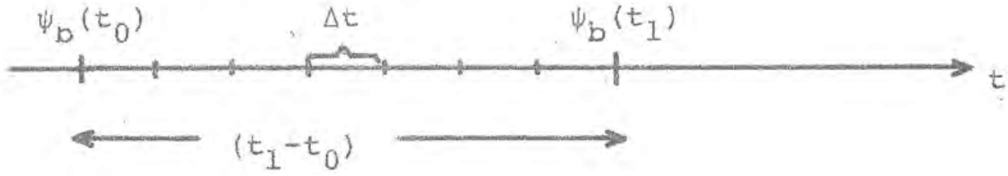
$$(68) \qquad \begin{aligned} &\nabla^2(\Delta_\tau\psi) - q \cdot (\Delta_\tau\psi) = \varepsilon \cdot \Delta t \cdot H^\tau \\ &(\Delta_\tau\psi)_b = \phi_b^\tau \end{aligned}$$

where the index b refers to boundary points.

If no information is available at boundary points, we have to assume that $\phi_b^\tau = 0$. In the case of a limited area forecast, ϕ_b^τ- values from a larger area forecast can be used.

For large-scale variations on the boundary a rather coarse resolution in time can be used, 6 or even 12 hours.

Assume that streamfunctions ψ_b, interpolated from the large area forecast, are available at the time t_o and t_1 ($t_1 - t_o$ = 6 Or 12 hours).



In the interval (t_0, t_1) the changes at the boundary can then be approximated linearly by

$$(69) \quad \left(\frac{\partial \psi}{\partial t}\right)_b \approx \frac{\psi_b(t_1) - \psi_b(t_0)}{t_1 - t_0}$$

and

$$(70) \quad \phi_b^\tau = \varepsilon \Delta t \left(\frac{\partial \psi}{\partial t}\right)_b$$

will have the same value at every time step in the interval (t_o, t_1).

By use of this relation we will now make a variable transformation of the prediction equation (68) in such a way that the new equation has the boundary condition = 0.

To achieve this we define a variable ϕ, independent of time and time step, by

$$(71) \quad \begin{cases} \nabla^2 \phi - q \cdot \phi = 0 \\ \phi_b = \left(\frac{\partial \psi}{\partial t}\right)_b \end{cases}$$

The forecast equation (68) can then be transformed to

$$(72) \quad \begin{cases} \nabla^2 (\Delta_\tau \hat{\psi}) - q \cdot (\Delta_\tau \hat{\psi}) = \varepsilon \cdot \Delta t \cdot H^\tau \\ (\Delta_\tau \hat{\psi})_b = 0 \end{cases}$$

if $\Delta_\tau \hat{\psi}$ is chosen to

$$(73) \quad \Delta_\tau \hat{\psi} = \Delta_\tau \psi - \varepsilon \cdot \Delta t \cdot \phi$$

The forward time step algoritm for ψ is then changed to

$$(74) \quad \psi^{\tau+1} = \psi^{\tau-1} + \Delta_\tau \hat{\psi} + \varepsilon \cdot \Delta t \cdot \phi$$

Equation (72) with the simple boundary condition $(\Delta_\tau \hat{\psi})_b = 0$ is solved every time step and a constant field $\varepsilon \cdot \Delta t \cdot \phi$ is added to the solution. Note that ϕ is not zero for interior points but is on the other hand unchanged during the time interval.

The ϕ-fields, different for different time intervals, can be prepared in advance, as soon as the large area forecast is computed, and the solution of these fields will therefore not delay the small area computation provided that this computation is performed at a later time. The ϕ-fields can be computed in the following way:

Assume that the height fields $Z(t_o)$ and $Z(t_1)$ are available on the larger area on an arbitrary grid.

1. Form $(\frac{\partial Z}{\partial t}) \approx \frac{Z_1(t_1)-Z_1(t_0)}{t_1-t_0}$

2. Solve the balance equation
$$\nabla^2(\frac{\partial \psi}{\partial t}) = \frac{g}{f}\left[\nabla^2(\frac{\partial Z}{\partial t}) - \frac{1}{f}\nabla f \cdot \nabla(\frac{\partial Z}{\partial t})\right]$$

3. Interpolate $(\frac{\partial \psi}{\partial t})$ to the small area grid and pick out boundary values $(\frac{\partial \psi}{\partial t})_b$.

4. Solve equation (71) and store the resulting ϕ-field.

5. Repeat the procedure for all time intervals and all pressure levels.

The stored ϕ-fields are then used during the computations on the small area as earlier described.

An inconsistency can arise at time t=0 if the Z-field on the large and small area is different on the small area boundary. This can be avoided if the small area initial fields (at t=0) are adjusted to the large area fields at and near the boundaries so that

$$Z_b^{small}(t=0) = Z_b^{large}(t=0)$$

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