NORDKLIM – Nordic co-operation within Climate activities

Extreme value analysis in the Nordic countries

– pilot studies of minimum temperature and maximum daily precipitation and a review of methods in use

H. Alexandersson, E.J. Førland, J. Helminen,
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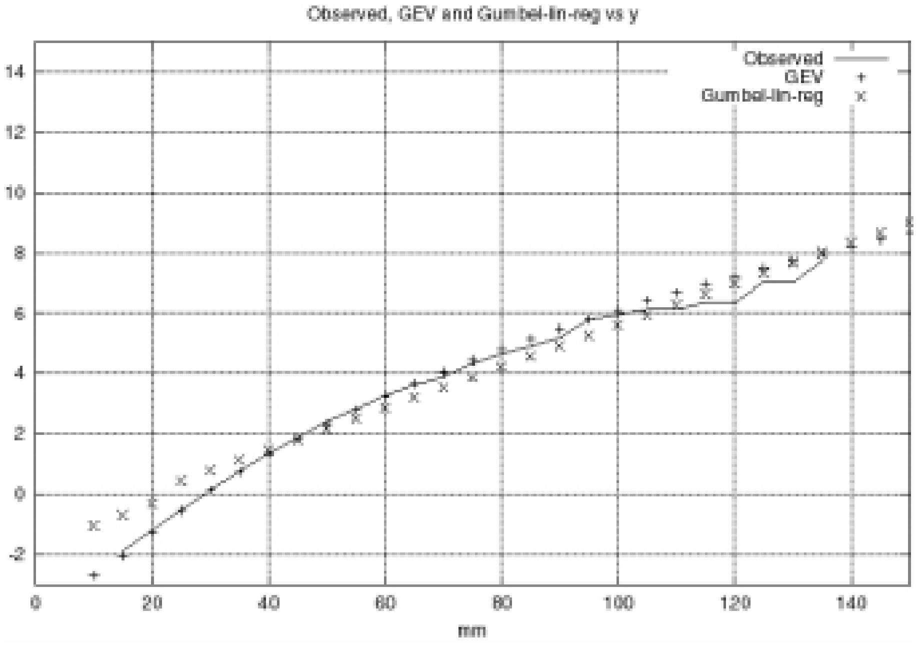

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NORDKLIM/NORDMET on behalf of the National meteorological services in
Denmark (DMI), Finland (FMI), Iceland (VI), Norway (DNMI) and Sweden (SMHI)

SUMMARY:

This report is prepared under Task 2 in the Nordic NORDKLIM project: Nordic Co-Operation Within Climate Activities. The NORDKLIM project is a part of the formalised collaboration between the NORDic METerological institutes, NORDMET.

A presentation is given of methods used in the Nordic countries to estimate recurrence values of extreme minimum temperatures and extreme 1-day precipitation amounts. At Sodankylä in Northern Finland the absolute minimum temperature during the period 1908-2000 is –49.5°C, and the return period for this value is estimated to be more than 200 years. A pilot study of extreme 1-day precipitation in a bordering area between Norway and Sweden indicated that the return period for daily point values exceeding 100 mm is ca 400 years. The dramatic extreme value (point value of 276 mm in one day) observed during a remarkable thunderstorm 30-31.August 1997 in the plateu of Fulufjället, seem to have a return period of more than 100 000 years.

KEYWORDS:

Extremes, Minimum Temperature, Daily precipitation, Extreme value distributions

SIGNATURES:

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Eirik J. Førland
NORDKLIM Activity Manager

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Head of the DNMI Climatology Division

Foreword

This report is prepared under task 2 in the Nordic NORDKLIM project: *Nordic Co-Operation Within Climate Activities.* The NORDKLIM project is a part of the formalised collaboration between the NORDic METeorological institutes, NORDMET.

The main objectives of NORDKLIM are:
1). Strengthening the Nordic climate competence for coping with increased national and international competition
*2). Improving the cost-efficiency of the Nordic meteorological services (i.e. by i*mproving procedures for standardized quality control & more rational production of standard climate statistics)
3). Coordinating joint Nordic activities on climate analyses and studies on long-term climate variations

The NORDKLIM project has two main tasks:

1. **Climate data** (Network design, Quality control, long-term datasets).
2. **Climate Applications** (Time series analysis, use of GIS within climate applications, mesoscale climatological analysis, extreme values and return periods).

A detailed description of the project is given by Førland et al.(1998).

NORDKLIM is coordinated by an Advisory Committee, headed by an Activity Manager. Each of the main tasks is headed by a Task manager.

The Advisory Committee in NORDKLIM is presently consisting of:
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Eirik J. Førland, DNMI (**Activity Manager**)
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CONTENTS

Extreme value analysis in the Nordic countries – pilot studies of minimum temperature and maximum daily precipitation and a review of methods in use.

1. Introduction

Dealing with climatic extremes is of paramount importance for the Nordic countries, and for planning purposes knowledge of recurrence values are crucial. A survey of climatic extremes in the Nordic countries, was presented by Tveito et al. (1998). In the present report focus is put on minimum temperatures and maximum 1-day precipitation. Both in Finland, Norway and Sweden minimum temperatures lower than –50°C have been recorded. Estimates of 50 year return period values of minimum temperatures in January indicated the following values: Oslo –24.4°C, Stockholm –23.1°C and Helsinki –31.9°C (Førland et al., 1998a). The highest recorded 1-day precipitation at official measuring stations in Fennoscandia is 230 mm at a station in Western Norway (Førland et al. 1998b). But as commented in the present report, there are clear evidence of events both in Finland and Sweden giving point precipitation values higher than 250 mm during 24-hours.

Within NORDKLIM Task 2 it was decided to make a pilot study using extreme value analysis. In this study we will concentrate on two specific applications. One is the use of the peak over threshold (POT) technique to handle return periods for extreme minimum temperatures. The second one is the use of the GEV (Generalised Extreme Value distribution) on daily precipitation maximum and try to find out if it is recommendable to use this three-parameter distribution instead of the simpler two-parameter Gumbel distribution. We will also give a survey of existing methods and practices within the Nordic countries concerning extreme (design) values.

2. Estimating return periods for minimum temperatures[1]

2.1 Introduction

The extreme value distribution analyses of the absolute minimum temperature in Finland have been so far carried out with the Generalized Extreme Value Distribution (GEV) method mainly on monthly values. The analysis is an annual maximum/minimum approach (AM), taking the most extreme value per year into account even for years with not very extreme absolute temperature values. On the other hand the data show that within a year one can have several physically separable and statistically independent cold episodes. These features make the use of the AM-method inefficient.

To avoid the pitfalls of the AM-method, the Peak over Threshold (POT) approach has been applied here to absolute minimum temperature observations at Sodankylä. This approach allows one to take into account all independent values exceeding a chosen threshold. Moreover the original monthly extreme value data covered the period from January 1908 to January 2001 as described in chapter 2.2. The POT-method based on the Generalized Pareto Distribution (GPD) has been reviewed in chapter 2.3. The results are presented in chapter 2.4 and discussed in chapter 2.5.

2.2 Data

The data consisted of daily absolute minimum temperatures at Sodankylä Observatory (67° 22’ N, 26° 39’ E) in Northern Finland (see Fig. 2.1) and covered the period 1.1.1908 – 31.1.2001. The daily values, however, cannot be regarded as statistically independent as they can refer e.g. to the same cold episode. Therefore monthly absolute minimum temperature was chosen to meet the requirement of statistical independence. In case of adjacent months we still had to check that the temperatures referred to two physically separate cold events. In this case we required that between the dates corresponding to the monthly absolute minimum temperature values there occurred a minimum temperature reading at least 20 degrees centigrade higher than either one of the monthly values under consideration. In statistical sense the chosen monthly values were then regarded as independent.

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[1] This section was presented at the 8th International Meeting on Statistical Climatology, 12 – 16 March 2001, Lüneburg, Germany

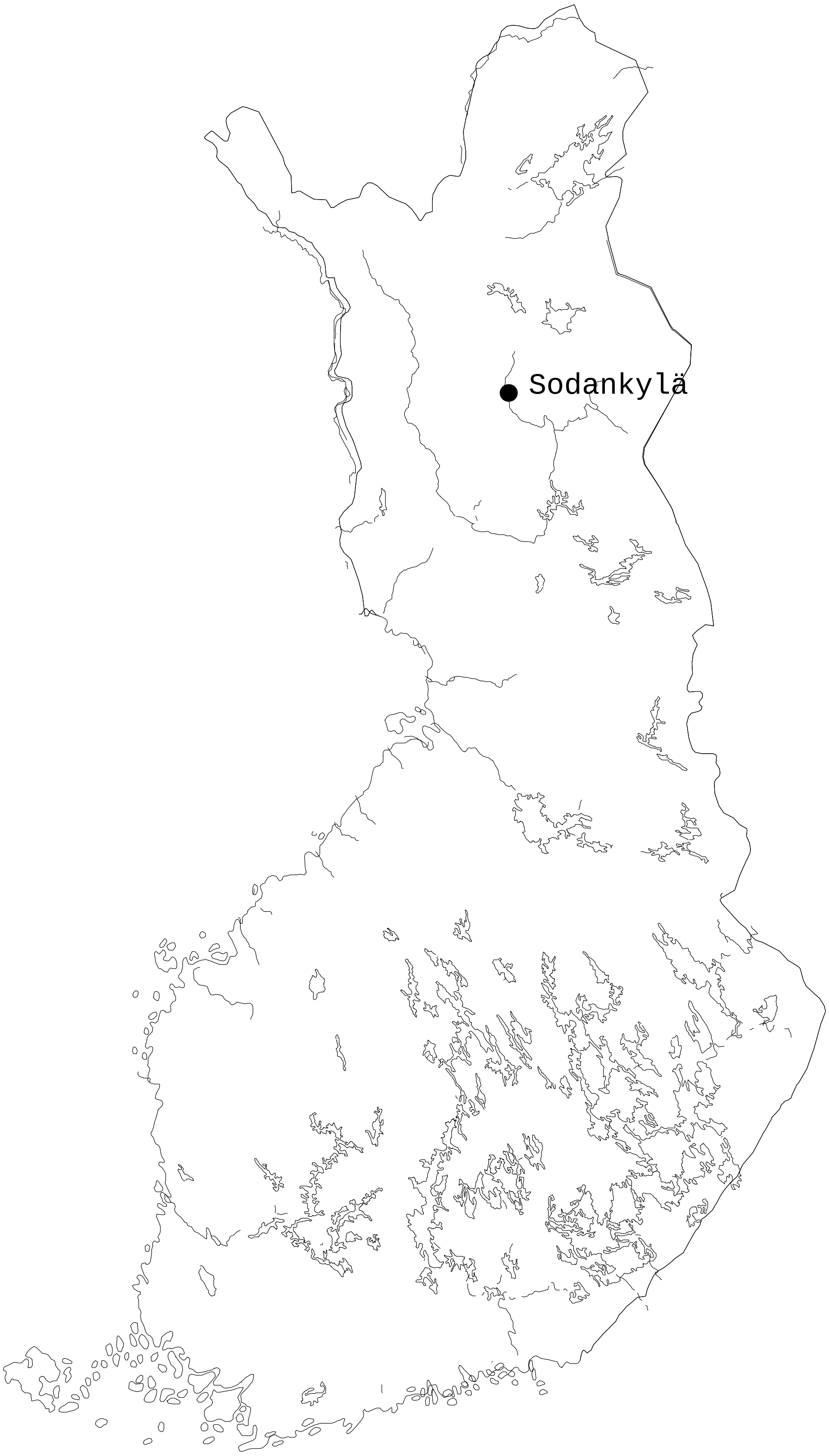

Figure 2.1. The location of Sodankylä in Northern Finland

The data set consisted of the absolute minimum temperatures of independent cold monthly values. As a new absolute minimum temperature record value was measured quite recently on 28th January 1999 two separate data sets were used, the first one for the period 1.1.1908 – 31.12.1998 (data set 1) and the second one for the period 1.1.1908 – 31.12.2000 (data set 2) to assess, how much the new record value influenced the extreme value distribution results.

2.3 Method

In order to use the limited extreme value data of the absolute minimum temperature as efficiently as possible and to improve the inferences, the peak over threshold (POT) approach was selected in line with the preparation of the data in chapter 2.2.

As the POT method turns out to be a special case of a point process characterization, where we consider a two-dimensional point process $\{(i, X_i); i = 1,\dots,n\}$, where $X_1, X_2,\dots, X_n$ is a 2D series from an unknown distribution F following Coles (1999) we represent the behaviour of the X_i at large levels in regions of the form $[t_1, t_2] \times (u, \infty)$.

Suppose F is the domain of attraction of G, i.e. there are sequences of constants a_n and b_n such that

$$\Pr\{(M_n - b_n)/ a_n \leq x\} \rightarrow G(x), \qquad (M_n = \max\{ X_1, X_2,\dots, X_n\})$$

for some non-degenerate distribution G given by

$$G(x) = \exp\{-[1 + \xi x]^{-1/\xi}\}.$$

Then for a sequence of point processes P_n on R^2

$$P_n = \{[(i/n+1);(X_i - b_n)/ a_n] : i = 1,\dots,n\}$$

away from the lower boundary the process behaves like a non-homogeneous Poisson process.

By defining the intensity measure

$$\Lambda(A) = E(\text{number of points in A})$$

and by taking into account that the process P_n converges weakly to a Poisson process away from the lower boundary and we get for the region $A = \{(t_1, t_2) \times (x, \infty)\}$

$$\begin{aligned} \exp\{-\Lambda(A)\} &= \Pr\{\text{no points in } A\} \\ &= \Pr\{M_n \le x\} \\ &\approx \exp\{-[1 + \xi x]^{-1/\xi}\}. \end{aligned}$$

Then, at high levels, the process P_n should approximate a Poisson process with intensity function given by

$$\Lambda\{(t_1, t_2) \times (x, \infty)\} = (t_2 - t_1)\,[1 + \xi x]^{-1/\xi}, \qquad (2.1)$$

which with two slight modifications can be represented for statistical purposes in the form

$$\Lambda\{(t_1, t_2) \times (x, \infty)\} = (t_2 - t_1)\,[1 + \xi(x - \mu)/\sigma]^{-1/\xi}.$$

According to Coles (1999) in the POT method we look at an explicit approximation for the conditional distribution

$$\Pr(X > u + x | X > u),$$

where u is the chosen threshold value. Then by letting $X^*_{n,i} = (X_i - b_n)/\,a_n$, for $i = 1,\ldots,n$ we have for u sufficiently large for the Poisson limit with intensity (2.1) to be a valid approximation on $[0,1] \times (u,\infty)$,

$$\begin{aligned} \Pr\{X^*_{n,i} > u + x | X^*_{n,i} > u\} &\approx \{\Lambda[(0,1) \times (u + x, \infty)]\}/\{\Lambda[(0,1) \times (u, \infty)]\} \\ &= \{1 + \xi x/[1 + \xi(u - \mu)]\}^{-1/\xi} \qquad (2.2) \end{aligned}$$

By absorbing the unknown coefficients a_n and b_n into the distribution we get

$$\Pr\{X_{n,i} > u + x | X_{n,i} > u\} = [1 + \xi x/\sigma]^{-1/\xi}, \qquad (2.3)$$

which is called the Generalized Pareto distribution (GPD). When $\xi \to 0$, GPD converges to the exponential function as a special case. It should be noted also that the GPD has no location parameter as the re-location by a_n is lost through conditioning.

In applying the GPD model to the Sodankylä absolute minimum temperature data we have used the Splus-programs of Coles (1999). As he points out the threshold choice is always a trade-off. Too low threshold values incur bias due to the invalidity of the asymptotic argument, whereas too high thresholds lead to only few excesses so the sampling variability remains high. All in all the threshold choice is not always easy and is more or less a matter of subjective judgement.

2.4 Results

In all calculations we have considered the absolute values of absolute minimum temperature at Sodankylä in centigrades. As mentioned already at the end of chapter 2.2 we have estimated the extreme value distribution for two periods.

The maximum likelihood estimates (MLE's) of the modified scale and shape parameters for period 1 are shown in Fig 2.2. According to them the threshold choice of 39 °C (corresponding to the actual value of –39 °C) seemed to be a reasonable compromise. One can see that the value and the 95 % confidence interval of the shape parameter remained negative still with the chosen threshold value, whereas for the value 40 °C the upper part of the confidence interval exceeded zero to positive values. Also the sampling variability increased considerably with threshold values higher than 39 °C.

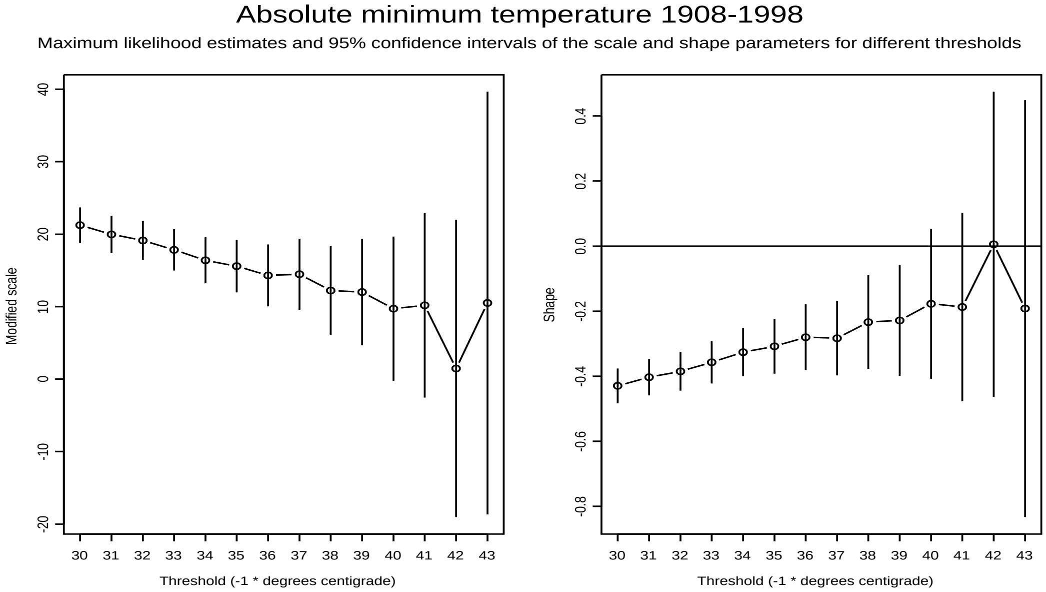

Figure 2.2. The maximum likelihood estimates of the modified scale vs. threshold and the shape vs. threshold for the absolute minimum temperature data set 1 (January 1908 – December 1998).

In Fig. 2.3 we have the GPD plot of the absolute minimum temperature at Sodankylä for the period 1908 – 1998. In this plot we have the 95 % confidence limits as well. According to the result, the extreme value distribution was of the Weibull type so that the absolute minimum temperature reaches a finite limit of some –52 °C with growing return periods.

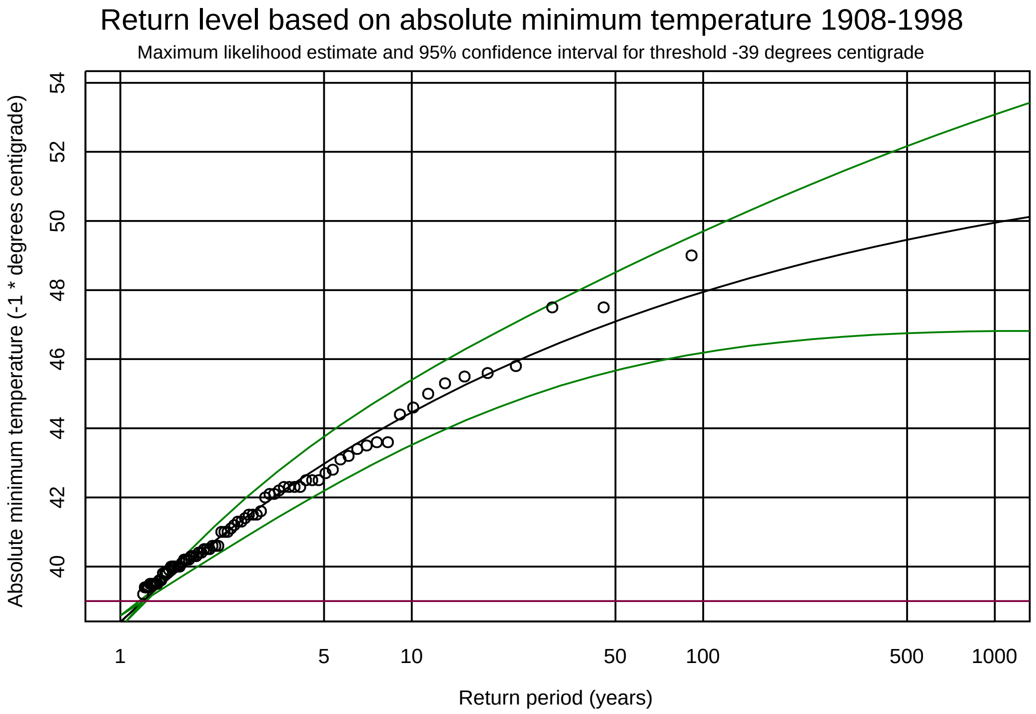

Figure 2.3. The maximum likelihood POT estimate of the extreme value distribution for the absolute minimum temperature data set 1 (January 1908 – December 1998).

The MLE's of the modified scale and shape parameters for period 2 are shown in Fig 2.4. According to them the threshold choice of 39 °C seemed to be a reasonable compromise. One can see that the value and the 95 % confidence interval of the shape parameter remained negative still with the chosen threshold value, whereas for the value 40 °C the upper part of the confidence interval exceeded zero to positive values. Also the sampling variability increased considerably with threshold values higher than 39 °C.

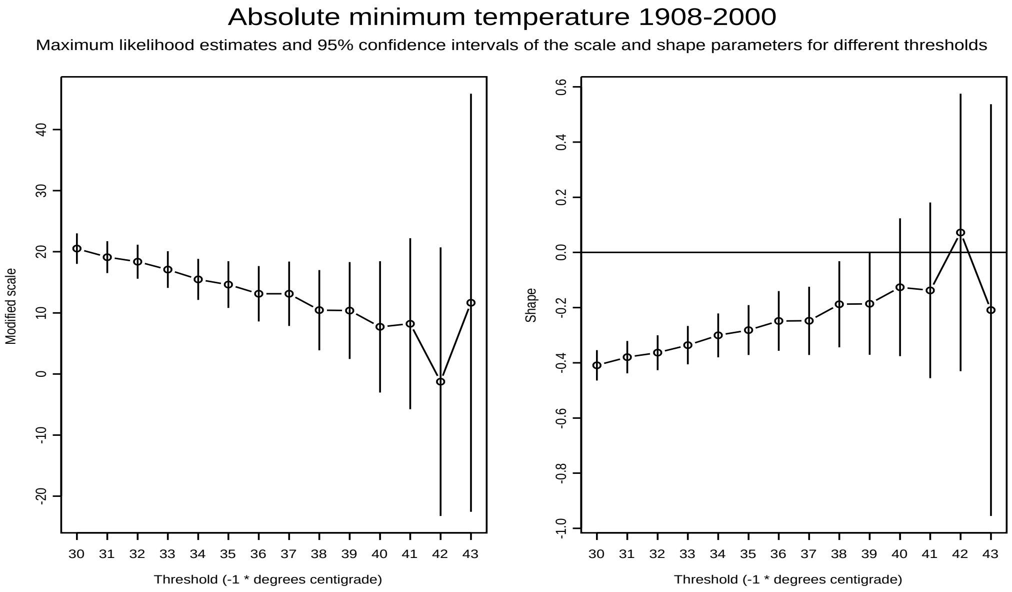

Figure 2.4. The maximum likelihood estimates of the modified scale vs. threshold and the shape vs. threshold for the absolute minimum temperature data set 2 (January 1908 – January 2001).

In Fig. 2.5 we have the GPD plot of the absolute minimum temperature at Sodankylä for the period 1908 – 2000. In this plot we have the 95 % confidence limits as well. According to the result the extreme value distribution was of the Weibull type so that the absolute minimum temperature reaches a finite limit of some –55 °C with growing return periods.

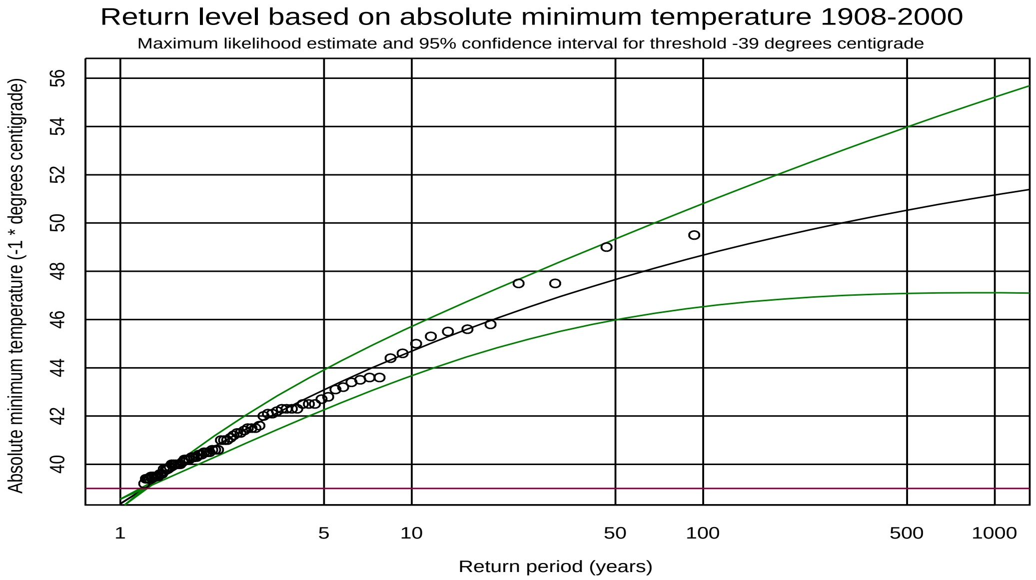

Figure 2.5. The maximum likelihood POT estimate of the extreme value distribution for the absolute minimum temperature data set 2 (January 1908 – January 2001).

Table 2.1 shows the 50-year recurrence values and the 95 % confidence limits of the absolute minimum temperature at Sodankylä as estimated for periods 1908 – 1998 and 1908 – 2000. As seen already in the limit values corresponding to long return periods, the recent absolute minimum temperature record value of 29th January 1999 shifted the results toward colder values.

Table 2.1. 50-year POT/GPD recurrence values of the absolute minimum temperature at Sodankylä for periods 1908 – 1998 and 1908 – 2000

Period	50-year recurrence value (°C)	Lower (Warmer) 95 % confidence limit (°C)	Higher (Colder) 95 % confidence limit (°C)
1908 – 1998	- 47,1	- 45,7	- 48,5
1908 – 2000	- 47,7	- 46,0	- 49,3

2.5 Discussion

It is interesting to compare the 50-year recurrence values of table 2.1 to the earlier estimates based on the Annual Minimum Series (AM) approach as applied to Sodankylä January absolute minimum temperature values for periods 1908 – 1998 and 1908 – 1999. Although year 2000 was not included in the latter data set, year 2000 with mild winter months would not have had any noticeable effect on the results.

The 50-year January recurrence values based on the AM method were as given in table 2.2 (Helminen, 1999). By comparing the results of table 2.2 with the corresponding results of table 2.1 one can see that the change between periods 1908 – 1998 and 1908 – 1999/2000 was almost the same (the former 0,7 - 0,8 °C warmer than the latter period) whereas the AM estimates were 1,2 °C and 1,1 °C warmer, respectively.

Table 2.2. 50-year AM January recurrence values of the absolute minimum temperature at Sodankylä for periods 1908 – 1998 and 1908 – 1999

Period	50-year recurrence value (°C)
1908 – 1998	- 45,5
1908 – 1999	- 46,3

These results were in line with what one would expect. First of all to consider solely January as the month of the most extreme annual absolute minimum temperatures (the coldest event, -49,5 °C on 28.1.1999) excluded the second coldest event, -49,0 °C of 5.2.1912, and also some other cold events, not having occurred in January, from the data set. Therefore we cannot from the comparison of the results of table 2.1 with those of table 2.2 draw any definite conclusions of the power of the AM-method as compared with the one of the POT-method. However, in contrast to the POT-method the strict use of just the annual absolute minimum temperature values in the AM-method ignored the possibilities to include in the data set more than one cold episode per year even if there were several independent exceptionally cold episodes during the same year. All in all the AM method leads inevitably to an inefficient use of the most extreme cases and subsequently in most cases to less extreme estimates.

On the other hand it should be emphasized that the differences between the results of table 2.1 and table 2.2 were not statistically significant. This was not surprising as the variability within all data sets was quite considerable. Nevertheless the differences might be important in practice, e.g. the combined failure risks of malfunction and maintenance of electric power lines in Northern Finland during extremely cold weather.

According to some considerations of dew point temperature readings close to the occurrence of the absolute minimum temperatures and some photographs of the event on January 29th 1999 one could get the impression that the state of saturation with a thin ice crystal layer some meters above the ground had in most cases stopped the further cooling at ground/snow surface. However, this hypothesis needs definitely more evidence before it can be accepted.

3. Estimating return periods for daily precipitation

3.1 Theory of extreme value distributions

The Generalised Extreme Value distribution (GEV) neatly summarises all three possible main types of extreme value distributions. The cumulative distribution is given by

$$F(x)=\exp[-(1-\theta(x-\mu)/\sigma)^{1/\theta}] \qquad (3.1)$$

For $\theta=0$ the GEV reduces to the Gumbel distribution

$$F(x)=\exp[-\exp(-(x-\mu)/\sigma)] \qquad (3.2)$$

Estimating θ, μ and σ in the GEV can be performed along the lines given by Buishand (1986). For the simpler Gumbel distribution one can use the method of moments or linear regression using the observed cumulative frequency and noting that Eq. 3.2 can be linearised by taking logarithms twice. The latter method gives much larger weights to the right hand tail (the most extreme values), which will be apparent later. We will, nevertheless, mainly focus on the GEV and the question if the parameter θ differs significantly from zero in section 3.4.

3.2 Existing methods in the Nordic countries

3.2.1 Norway

In Norway a modified version (Førland & Kristofferssen, 1989) of the British M5-method (NERC, 1975) is used for estimating extreme precipitation. The basic value for the estimations in Norway is the 24h precipitation with a return period of 5 years (M5(24h)). By this method extreme precipitation values (MT) with a return period of T years can be estimated as:

$$MT = M5*\exp\{c*[\ln(T-0.5)-1.5]\} \qquad (3.3)$$

where M5 is the 5 year return period value, and the factor c is given by

$$c= 0.3584 - 0.0473*\ln(M5) \qquad (M5\ \varepsilon <25,350>) \qquad (3.4)$$

A detailed methodology is worked out to estimate extreme precipitation for different return periods and for durations from 1 hour to 30 days (Førland, 1992). For flood estimation for dam design in Norway, precipitation values with return periods of 1000 years as well as estimates for Probable Maximum Flood (PMF) are needed (Sælthun & Andersen, 1986). To deal with the necessary inputs to flood modelling, the Norwegian methodology for estimating extreme precipitation is extended to produce estimates for return periods of 1000 years as well as for Probable Maximum Precipitation (PMP) (Førland & Kristofferssen, 1989, Førland, 1992). An example of presentation extreme precipitation for a precipitation station in South-eastern Norway is given in table 3.1. In addition estimates by the M5-method (NERC), also estimates using the Gumbel and Hershfield methods are presented in the table.

Table 3.1. Example of estimates of extreme 24h precipitation at DNMI

Station no.	St. Name	Established	Closed down	Elevation (m a.s.l.)	County	Community
6550	ØRBEKKEDALEN	1896	-	513	HEDMARK	ELVERUM

Data period: 1895 - 2000

Estimated maximum precipitation amounts (mm) during a 24 h period

Return Period (yrs)	Method	Annual	Dec - Feb	Mar - May	Jun - Aug	Sep - Nov
5	GUMBEL	52	21	29	47	41
10	GUMBEL	60	24	34	56	47
50	GUMBEL	77	31	45	74	62
100	GUMBEL	84	34	49	82	69
1000	GUMBEL	108	43	65	107	90
5	NERC	52	21	29	47	41
10	NERC	59	24	33	53	47
50	NERC	78	35	46	71	63
100	NERC	88	40	53	81	72
1000	NERC	131	66	85	121	110
PMP	NERC	240	144	173	228	212
PMP	HERSHFIELD	246	-	-	-	-

Maps of the basic value M5(24h) are presented in the Norwegian National Atlas (Førland, 1993). This map is reproduced in Figure 3.1. As for annual precipitation there are strong regional gradients and large geographical variations in the M5(24h) values. The M5(24h) station values varies from 25 mm at Skjåk in Central Norway to more than 150 mm at some stations in Western Norway. The Norwegian methodology for estimating extreme precipitation is adapted for use in a GIS-environment. Estimates of extreme precipitation with arbitrary duration (1h-30 days) and arbitrary return period (2 –1000 years and also PMP) can be presented for any site or watershed in Norway (Tveito & Førland, 1996). An example of point values for 24h precipitation with return period of 100 years is presented in Figure 3.2.

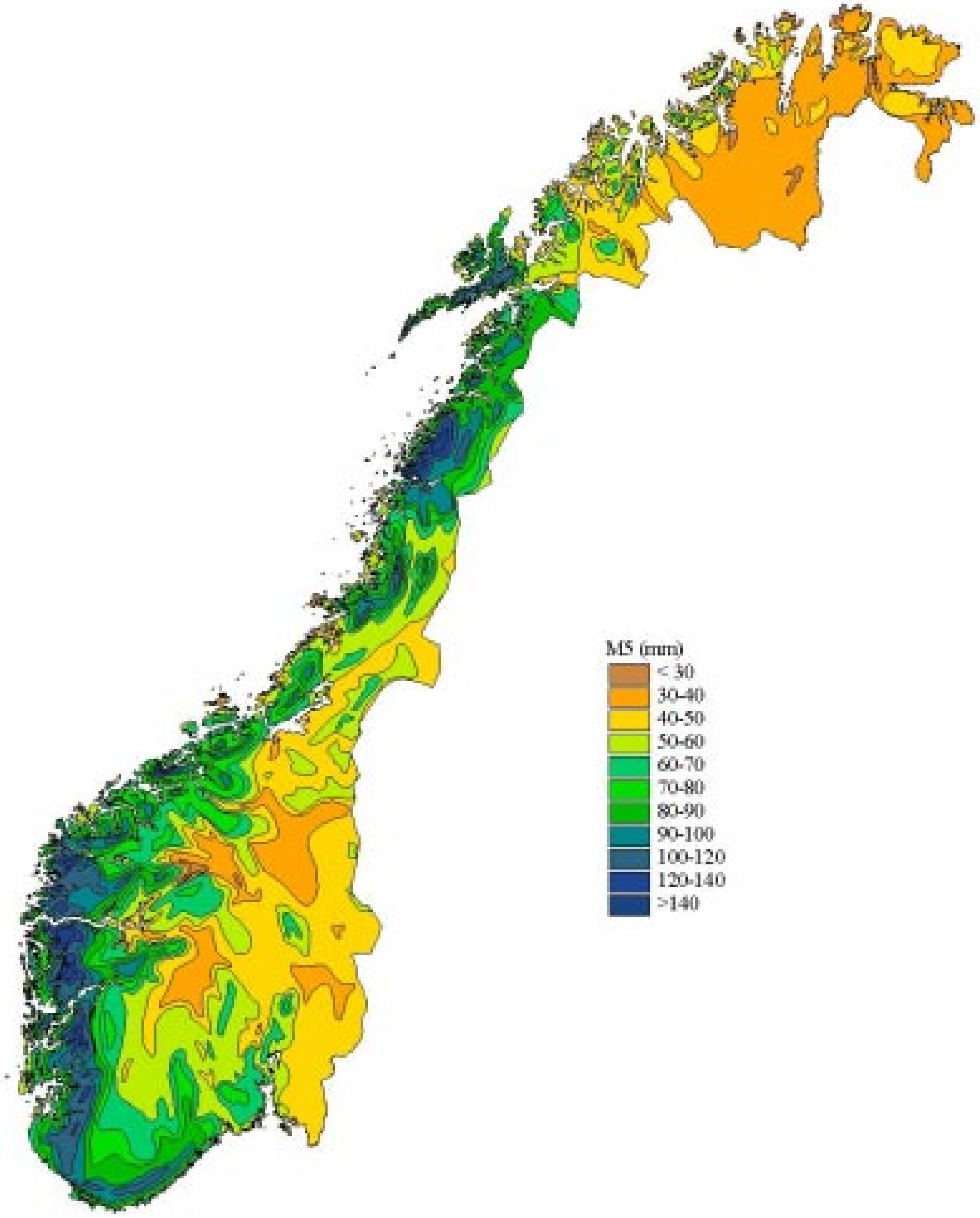

Figure 3.1 Geographical distribution in Norway of 24h precipitation values with 5-years return period

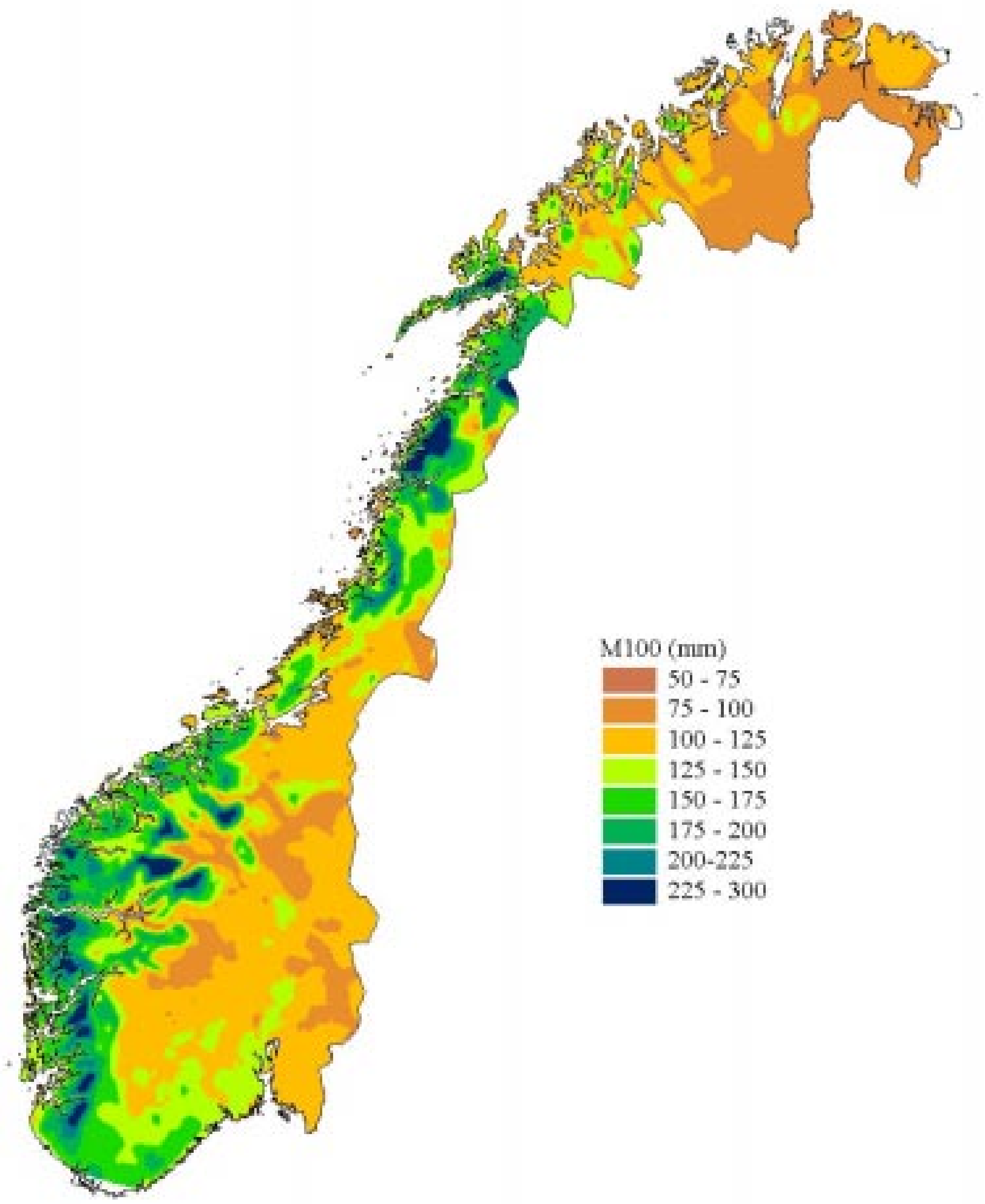

Figure 3.2 Geographical distribution in Norway of 24h precipitation values with 100-years return period

3.2.2 Iceland

Also in Iceland the M5 method (see section 3.2.1) is used for estimating extreme precipitation values (Eliasson, 2000). In Iceland, the factor c in eq. (3.4) is approximated by $c=0.78/(1/C_v+0.72)$, where C_v is the coefficient of variation. M5 maps for Iceland have been prepared by the Engineering Research Institute of the University of Iceland. Eliasson (op.cit.) concludes that “the M5 method is a very quick and effective method to estimate rainfall in order to obtain design values or in other hydrological estimation. This method is an invaluable tool in hydrological design for both small and large structures and may be the only rational solution when observations are inadequate to support continuous simulation methods”.

3.2.3 Sweden

In Sweden analyses of area rainfall events are analysed and then these values are combined into a large data set (Vedin and Eriksson, 1986). Return periods are estimated from this combined data set without using theoretical distributions. Sweden, which is a less heterogeneous country than Norway concerning precipitation, is divided into three main regions. For larger rivers it is sequences comprising 14 days that are used for design calculation purposes. The set of extreme area rainfall events is updated continuously and a more recent graph of 1000 km^2 cases with at least 90 mm within 24 hours can be found in Vedin et al. (1999). For shorter times than days a regression method was developed using pluviograph data (Dahlström, 1979). This method is very much in use in Sweden for design purposes in urban drainage systems. It is a rather robust method that uses mean precipitation for May, July and August as input data (predictors).

3.2.4 Finland

A study of extreme point and areal precipitation totals in Finland was based on 40 years of precipitation observations (Solantie & Uusitalo, 2000). Based on annual point precipitation maxima, design areal 1-, 5- and 14-day precipittaion total for Finnish regions were determined for six two-monthly periods. For an area of 2500 km^2 , the 10 000 year return period event was estimated by first identifying the highest (P_{max}) and 10 percentile ($P_{10\%}$) values (corresponding to return periods of 58 and 11 years) of the annual maxima point precipitation totals that occurred over 50*50 km grid-squares. P_{max} and $P_{10\%}$ were computed for each of the 150 grid-squares comprising Finland and then averaged.

It was found that the July-August design precipitation totals were higher than the maximum precipitation totals observed in Finland. However, there is an example of a rainfall event that achieves the 1-day areal design precipitation total for surface areas smaller than about 500 km^2. This event (Toholampi, 31 July, 1994) is an example of a rainfall event that was locally very heavy, but was poorly represented by the observational station network. The official station Toholampi Oravala measured only 51 mm on 31 July 1994, while there is evidence that within some kilometres of that station a much higher precipitation (exceeding 250 mm) had occurred the same day.

4. Joint analysis of extreme precipitation in a bordering area between Norway and Sweden

4.1. Data set

It is well known that one of the largest problems to apply extreme value analysis is the lack of large data sets. One recommendation to get around this problem is to put data together from several stations within regions with fairly uniform climate. This has mainly been applied on precipitation data and it is sometimes called combination method. In this study we have created a large data set by using 24-hour precipitation maximum (R24x) from an area around the southerly part of the Norwegian-Swedish border. More exactly data from Hedmark, Akershus and Østfold in Norway and from western Värmland and western Dalarna in Sweden was put together in a data set comprising about 2300 station-years. The period used was 1961-2000. All values are for the fixed 24 hours time period 06 UTC actual day to 06 UTC next day. Annual precipitation within this area varies from about 600 mm in valleys in the northerly parts to almost 1000 mm on the highest terrain in the southerly and westerly parts of this area. However, the region is quite homogeneous concerning precipitation amounts in weather situations typically responsible for a large part of the 24-hour maximum values. Thus we feel that it is justified to use this combined data set for some tests.

4.2 Basic features of data

More precisely data is organised in monthly columns with a maximum value for each year and month. To see the annual variation of data Figure 4.1 gives mean values and standard deviations of R24x for the whole set of observations. There is a rather clear summer and early autumn peak with values above 20 mm. Minimum occurs in late winter and early spring with values close to 10 mm. Standard deviation follows the mean value although on a lower level so that standard deviation divided by mean value (coefficient of variation) is fairly constant for all months. However, it varies a little from 0.49 in August and September to 0.55-0.58 during the dryer period January to April. Although we have as much as 2300 values for each month the annual variation shows some minor jerks that might disappear with an even larger data set. Now we do not have 2300 independent values but if we had that the standard deviation of for example the October value would be as low as 0.18 mm (sample standard deviation divided by the square root of 2300).

The annual maximum naturally is quite considerably larger than individual monthly values. The mean value is 34.57 mm and the standard deviation is 12.26 mm. In the proceeding text we will only deal with annual maximum, as it is rare with practical examples where monthly or seasonal maximum is used for design purposes.

4.3 Results

Figure 4.2 shows the observed cumulative frequency curve using 5 mm boxes. From 100 mm and upwards the curve practically coincides with the upper maximum level of 100% but there are in fact a few observations above 100 mm. They are listed below in Table 4.1.

Table 4.1. List of maximum 1-day precipitation values above 100 mm.

Climate nr	Station	Precipitation (mm)	Date
10247	Järpliden, Värmland (S)	136.2	02.Jul 1986
11223	Storbron, Dalarna (S)	130.7	30.Aug.1997
60	Linnes, Hedmark (N)	122.6	30.Aug.1997
9253	Charlottenberg, Värmland (S)	121.3	24.Aug.1996
600	Gløtvola, Hedmark (N)	111.5	06.Sep.1985
9253	Charlottenberg, Värmland (S)	104.4	30.Aug.1997

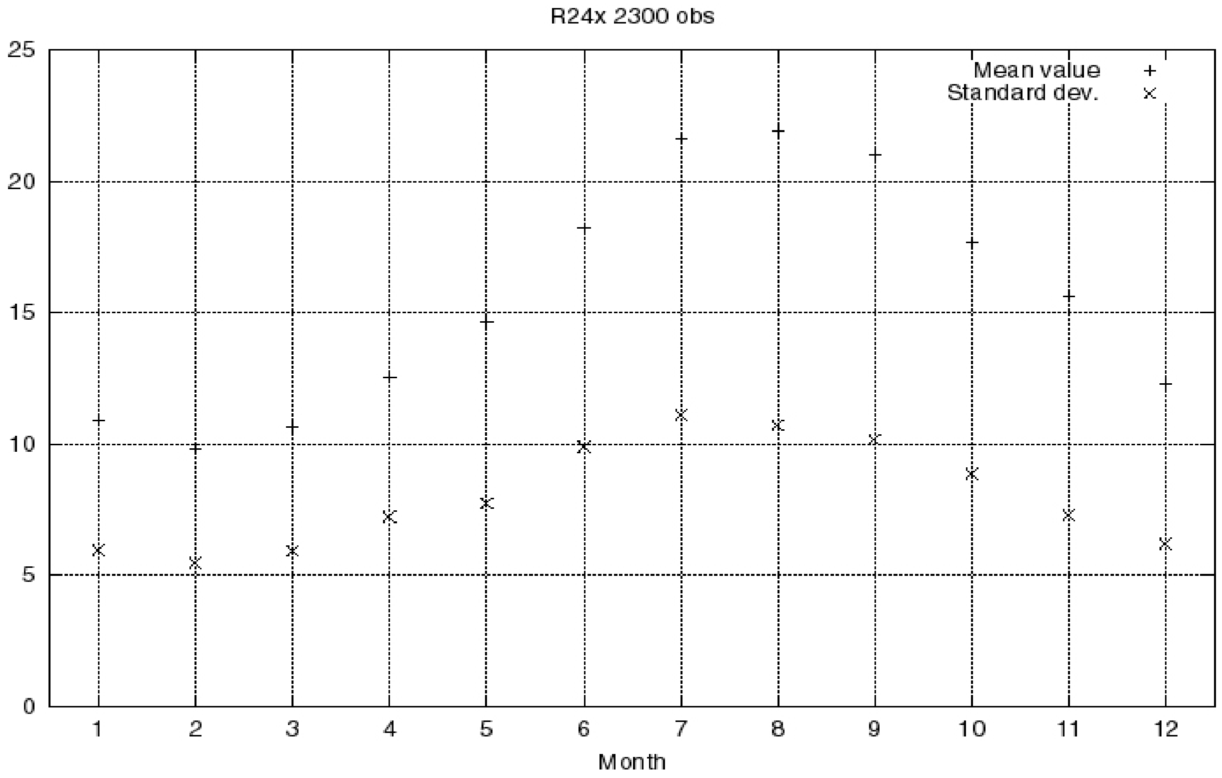

Figure 4.1 Mean value and standard deviation of maximum 24h precipitation based on 2300 observations

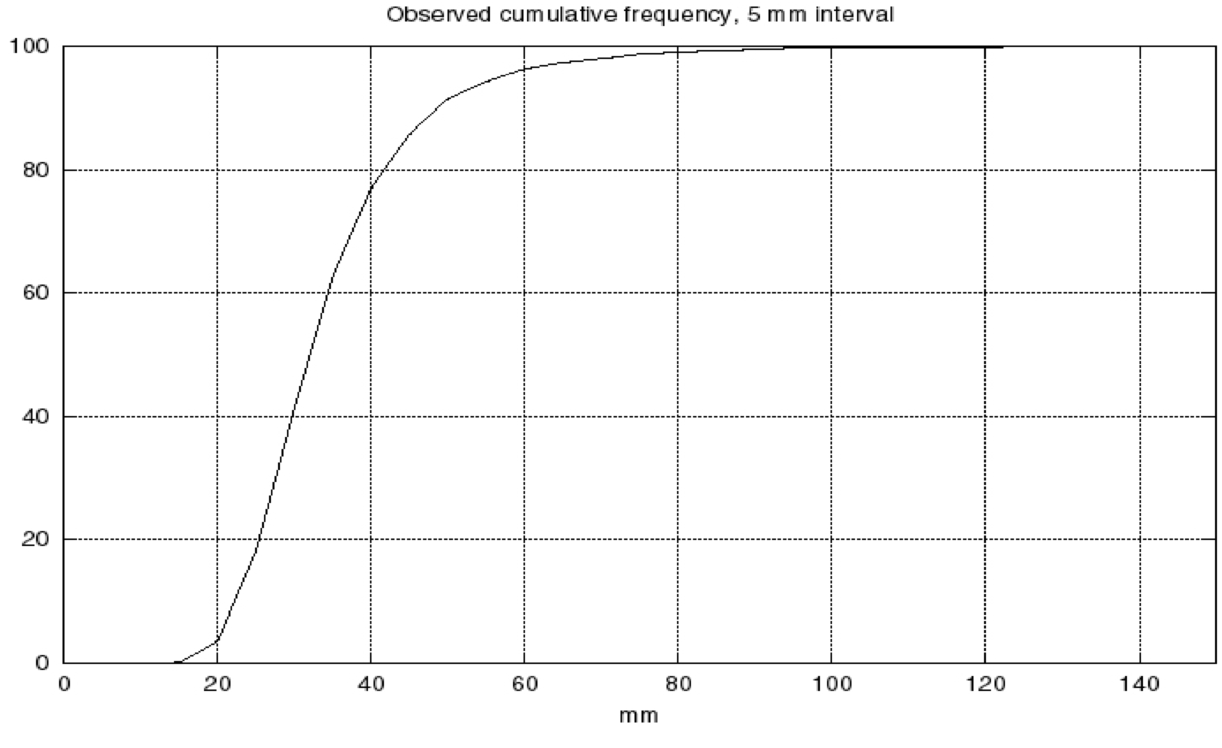

Figure 4.2 Cumulative frequency of observed maximum 24h precipitation

During the remarkable thunderstorms 30-31 of August 1997 private measurements with a simple plastic gauge – emptied frequently - on the plateau of Fulufjället at the only habitated place (a fishing camp) reached 276 mm (Vedin et al., 1999, Alexandersson et al., 2000). Along the eastern and southern slopes of Mount Fulufjället, water, stones and trees tumbling down the slopes caused incredible damage. The value 276 mm was from about noon the 30:th to noon the 31:st of August. The corresponding value for this shifted 24-hour period for Storbron was 137 mm. In the worst affected parts of the area it is estimated to have fallen about 300-400 mm during 24 hours.

The list of six values above 100 mm gives a direct estimate of the average frequency as 6/2300=0.26% or a return period of about 380 years. However, if data from 1961-1995 had been used a considerably lower frequency and higher return period would have been obtained showing how uncertain this estimate really is.

Counting the values above 75 mm gives a direct estimate of the frequency of 30/2300=1.30% or a return period of 77 years. There is some small risk that a value of above 75 mm is masked by another higher value from the same year (or even the same month) at a station, which is another complication when using these data.

Figure 4.3 shows how well the GEV fits to the observed cumulative distribution. The three parameters have the following numerical values: $\theta=-0.124$, $\mu=29.005$ and $\sigma=7.775$. In this plot double logarithms are taken of the observed and theoretical cumulative distribution. Then the Gumbel curve becomes a straight line. More precisely the vertical axis is obtained as

$$Y=-\ln(-\ln(F(x))) \qquad (4.1)$$

In Fig. 4.3 the Gumbel distribution is obtained by the method of moments, and it is quite clear that it fails to describe the right hand side tail of the distribution. The GEV seems more appropriate to use. In Figure 4.4 the Gumbel distribution plotted is obtained from the regression technique indicated earlier. It is clearly better than the method of moments for very high values but still the GEV seems superior in an overall respect. It is θ that gives the curvature of the line in this representation. Thus we have a preliminary answer to our main

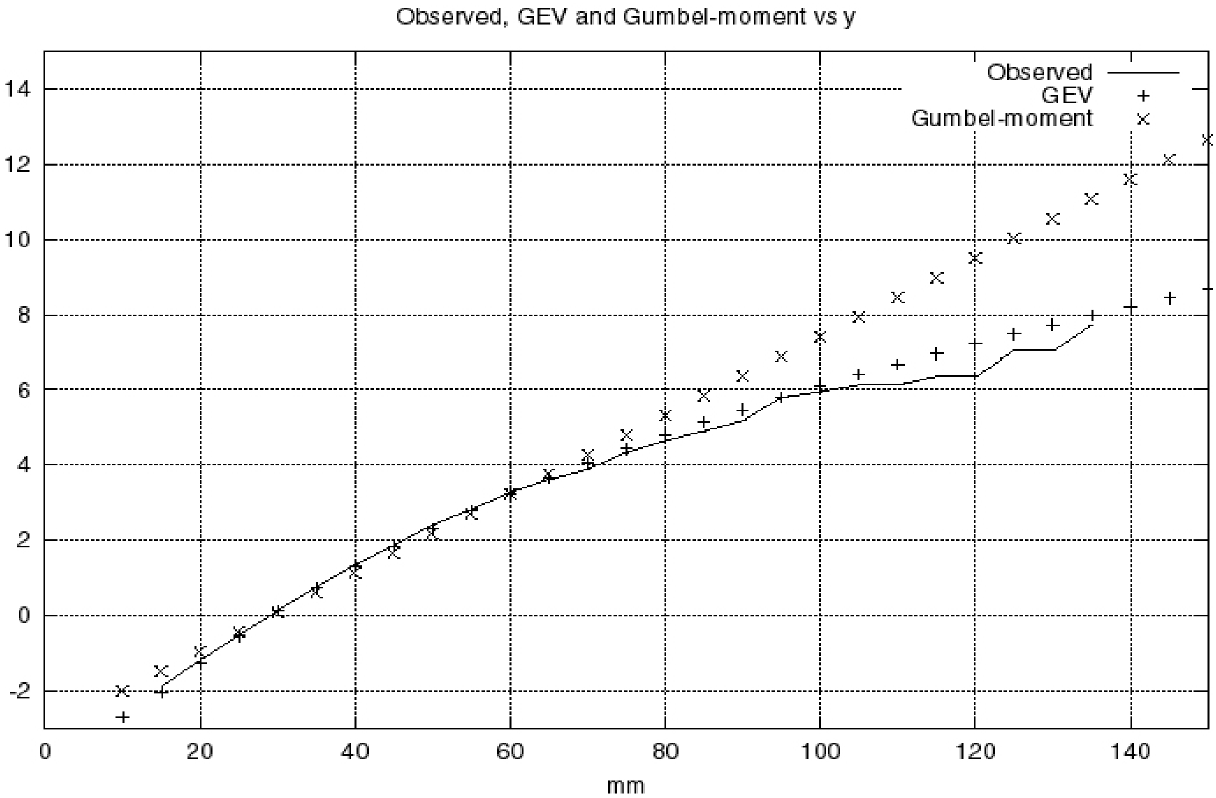

Figure 4.3 Distribution of R24x based on observations, GEV and Gumbel-moment methods

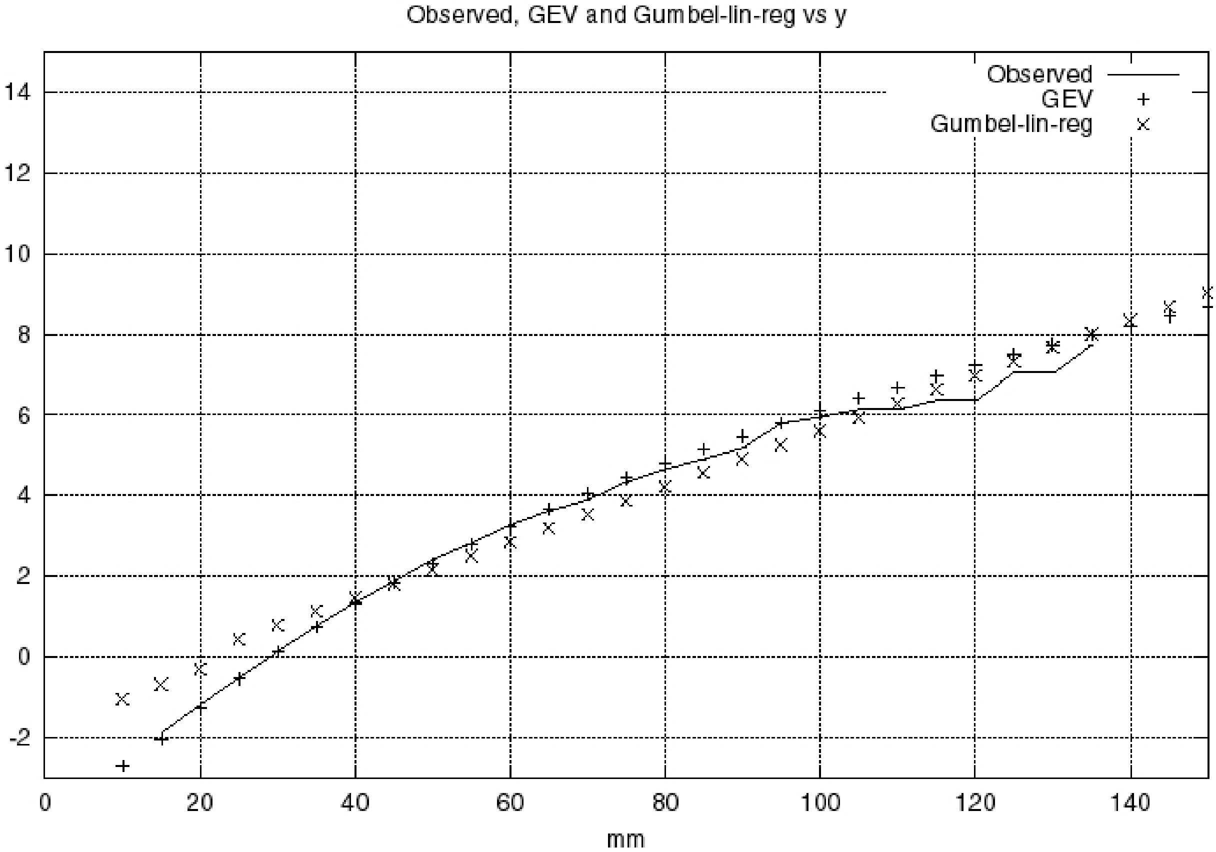

Figure 4.4 Distribution of R24x based on observations, GEV and Gumbel-regression methods

question: Yes we need θ to describe our data properly and to make extrapolations – to some extent – meaningful. Rigorous tests of the hypothesis $\theta=0$ exist (Hosking, 1984, van Montfort and Gomes, 1985) but has not been implemented yet.

In Table 4.2 return periods for various threshold values (75, 100, 150 and 276 mm) are listed. The return periods are estimated by different methods. The last row shows estimates by the M5-method used in Norway. In the actual sub-area, a typical M5 (24 hours) value is 43 mm (cf. Table 4.1), and this value is used in estimating return periods from Eq. (3.3) and (3.4).

Table 4.2. Return periods in years from observations and theoretical frequency distributions.

Observations or model	75 mm	100 mm	150 mm	276 mm
Observations	77	383	-	-
GEV	85	448	5790	390000
Gumbel-moment	123	1676	317000	$2\cdot10^{11}$
Gumbel-linear	49	271	8400	$1\cdot10^{8}$
M5-method	98	480	4500	135000

The GEV and the M5-methods give estimates that looks reasonable also for the most extreme values. Both for 75 and 100 mm the GEV-method gives slightly larger return periods than direct estimates from the observations. This can be seen in Figs 4.3 and 4.4 as some discrepancies between the crosses and the full line. From about 65 mm to the right end of observations, except at 95 mm, there is the same tendency.

5. Conclusions

The results show that the POT-analysis of the *absolute minimum temperature* values for the period January 1908 – January 2001 at Sodankylä gave some 1 °C colder estimates for the recurrence values than an earlier GEV-analysis based just on data of January for the same period at the same location (cf. table 2.1 & table 2.2). Whether the difference is equally large for the case, where the GEV-analysis would be applied to the data sets of this study (i.e. daily data sets of January 1908 – December 1998 and January 1908 – January 2001) remains open. However it is plausible that the effectiveness of the POT-method leads to more extreme recurrence values as compared with the GEV-results.

The Generalised Extreme Value distribution forms a good theoretical framework to handle a combined data set of about 2300 observations of *precipitation maximum values* for a reasonably homogeneous area in central Scandinavia. Extrapolation indicates that the return period for 150 mm or more at a specific average site within this area is about 5000-6000 years. This is of the same order of magnitude as the estimates by the M5-method used in Norway. Although estimations to such long return periods are very uncertain, these are probably the best estimates available. In NORDKLIM it will be considered whether these methods should be applied also to other sub areas within the Nordic countries

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